The Microscopic Model of Composite Fermion Type Excitations for $\nu = \frac{1}{m}$ Edge States

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We derive a microscopic theory of the composite fermion type quasiparticles describing the low-lying edge excitations in the fractional quantum Hall liquid with $\nu = 1/m$. Using the composite fermion transformation, one finds that the edge states of the system in a disc sample are described by the Calogero-Sutherland-like model (CSLM) in the one-dimensional limit. This result presents the consistency between one-dimensional and two-dimensional statistics. It is shows that the low-lying excitations, indeed, have the chiral Luttinger liquid behaviors because there is a gap between the right- and left-moving excitations of the CSLM.

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It is well-known that the bulk states of integer and fractional quantum Hall effects (IQHE and FQHE), in some sense, are very like to insulator states because there is an energy gap between the excitation state and the ground state. However, the transport behaviors of the quantum Hall states are remarkably different from those of the insulator. This difference leads to the recognization of the specific behavior of the quantum Hall gapless edge states [1]. A chiral Fermi liquid description to the edge states of IQHE has been provided by Halperin [1]. However, the Fermi liquid notion could not be applied to FQHE for either the bulk or the edge states since the strong correlations among the electrons in a high magnetic field. A very thoroughly investigation for the edge states of FQHE has been done by Wen from the chiral Luttinger liquid point of view [2]. Recently, the edge excitations of FQHE were studied by several groups numerically [3] or in accordance with the Calogero-Sutherland-like model (CSLM) [4] as well as the composite fermion (CF) picture in the Hartree approximation [5]. In this Letter, however, we will give a microscopic model of the CF-type excitations for the edge excitations while the bulk states lie on the mean-field states. Starting from the Hamiltonian of the two-dimensional interacting electrons in high magnetic field and using the composite particle transformation [6–9], one transforms the system to a CF (or composite boson) system if the statistics parameter is chosen as an even integer $\tilde{\phi}$ (or an odd m) . We would like to consider the low-lying excitations. For $\nu = 1/m$, the bulk states are gapful quantum Hall liquids. So, they can be decoupled to the low-lying excitation sector of the quantum state space. The only low-lying excitations are edge excitations. The edge of the system is considered as a circular strip with an infinitesimal width near the boundary of the quantum Hall liquid droplet. We will begin with the CF model. A Fermi-liquid-like theory could be applied to the FQHE both of the bulk and edge states [9,5]. Using the Fermi liquid notion to the CF system, we have N^e CF-type quasiparticles. Grafting Halperin's single particle argument for the edge states of IQHE to the present case, one can get a microscopic model of the CF-type excitations. In the one dimension limit, we see that the edge excitations could be described by the CSLM [10] perturbed by the interaction between the CFs. One has argued before by one of us, cooperating with Wu, the CSLM can be thought as the fixed point of the Luttinger liquid as the ideal Fermi gas versus the Fermi liquid [11]. This implies that the edge state of the FQHE could be described by the Luttinger liquid. However, one shows that the right- and left- moving sectors opens a gap $|n|\hbar\omega_c^*$ because of the existence of the magnetic field. This implies that the edge excitations are chiral, which is consistent with the chiral Luttinger liquid description of the edge excitations [2]. Our result also gives the self-consistency between one- and two-dimensional fractional statistics, which improves the observation that the CSLM could be mapped into an anyon model on a ring [13]. The short range interaction between the CFs will not effect the exponents. The Coulomb interaction provides a branch of charge plasma excitations.

The two-dimensional interacting electrons which are polarized by a high magnetic field are governed by the following Hamiltonian

$$H_{\rm el} = \sum_{\alpha=1}^{N} \frac{1}{2m_b}[\vec{p}_\alpha - \frac{e}{c}\vec{A}(\vec{r}_\alpha)]^2 + \sum_{\alpha<\beta} V(\vec{r}_\alpha - \vec{r}_\beta) + \sum_{\alpha} U(\vec{r}_\alpha), \quad (1)$$

where $V(\vec{r})$ is the interaction between electrons. m_b is the band mass of the electron and $U(\vec{r})$ is the external potential. The composite particle transformation will bring us to a good starting point to involve in the FQHE physics as many successful investigations told us [6–9]. We begin with the CF transformation which reads

$$\Phi(z_1,...,z_N) = \prod_{\alpha<\beta}\left[\frac{z_\alpha - z_\beta}{|z_\alpha - z_\beta|}\right]^{\tilde{\phi}} \Psi(z_1,...,z_N), \quad (2)$$

where Φ is the electron wave function. The CF consists of an electron attached by $\tilde{\phi}$ flux quanta. By using the CF theory, the bulk behavior of the FQHE has been well-understood [14,9]. We, now, would like to study the microscopic theory of the CF edge excitations. The partition function of the system is given by

$$Z=\sum_{N^e}C_N^{N^e}\int_{\partial}d^2z_1....d^2z_{N^e}\int_B d^2z_{N^e+1}...d^2z_N \quad (3)$$
$$\times\Big(\sum_\delta|\Psi_\delta|^2e^{-\beta(E_\delta+E_g)}+\sum_\gamma|\Psi_\gamma|^2e^{-\beta(E_\gamma+E_g)}\Big),$$

where we have divided the sample into the edge ∂ and the bulk B. E_g is the ground state energy and E_δ are the low-lying gapless excitation energies with δ being the excitation branch index. E_γ are the gapful excitation energies. At $\nu=1/\tilde\phi$, the low-lying excitations are everywhere in the sample and we do not consider this case here. We are interested in the case $\nu=\frac{1}{\tilde\phi+1}=1/m$, where the bulk states are gapful. The low-lying excitations are confined in the edge of the sample. For convenience, we consider a disc geometry sample here. The edge potential is postulated with a sharp shape. The advantage of the CF picture is we have a manifestation that the FQHE of the electrons in the external field B could be understood as the IQHE of the CFs in the effective field B^* defined by $B^*\nu^*=B\nu$. For the present case, $B^*=B/m$ and $\nu^*=1$. The energy gap in the bulk is of the order $\hbar\omega_c^*$ with the effective cyclotron frequency $\omega_c^*=\frac{eB^*}{m^*c}$ (m^* is the effective mass of the CF). Hereafter, we use the unit $\hbar=e/c=2m^*=1$ except the explicit expressions. By the construction of the CF, the FQHE of the electrons can be described by the IQHE of the CFs [14] while the electrons in the $\nu=1/\tilde\phi$ field could be thought as the CFs in a zero effective field. Thus, a Fermi-liquid like theory could be used [9] and we have a set of CF-type quasiparticles. Applying the single particle picture, which Halperin used to analyze the edge excitations of the IQHE of the electrons, to the edge excitations of the CFs, one could have a microscopic theory of the quasiparticles at the edge. In the low-temperature limit, the domination states contributing to the partition function are those states that the lowest Landau level of the CF-type excitations is fully filled in the bulk but only allow the edge CF-type excitations to be gapless because the gap is shrinked in the edge due to the sharp edge potential. The other states with their energy $E_\gamma+E_g$ open a gap at least in the order of $\hbar\omega_c^*$ to the ground state. In the low-temperature limit, $k_BT\ll\hbar\omega_c^*$, the effective partition function is

$$Z\simeq\sum_{\delta,N^e}C_N^{N^e}\int_\partial d^2z_1...d^2z_{N^e}|\Psi_{e,\delta}|^2e^{-\beta(E_\delta(N^e)+E_{g,b})} \quad (4)$$
$$=\sum_{N^e}C_N^{N^e}\mathrm{Tr}_{(\mathrm{edge})}e^{-\beta(H_e+E_{g,b})},$$

where the trace runs over the low-lying set of the quantum state space for a fixed N_e and, according to the single particle picture, $\Psi_{e,\delta}$ are the edge many-quasiparticle wave functions . $E_\delta(N^e)$ is the eigen energy of the edge quasiparticle excitations and $E_{g,b}$ is the bulk state contribution to the ground state energy. For the disc sample, the edge quasiparticles are restricted in a circular strip near the boundary with its width $\delta R(\vec r)\ll R$ while the radius of the disc is R. The edge Hamiltonian of CFs reads

$$H_e=\sum_{i=1}^{N^e}[\vec p_i-\vec A(\vec r_i)+\vec a_e(\vec r_i)+\vec a_b(\vec r_i)]^2 \quad (5)$$
$$+\sum_{i<j}V(\vec r_i-\vec r_j)+\sum_i U_{eff}(\vec r_i),$$

where the external potential U_{eff} is the effective potential including the interaction between the edge and bulk particles. We suppose the potential is an infinity wall for $r\geq R$ and induces a constant electric field for $r<R$. The statistics gauge field $\vec a$ is given by

$$\vec a_e(\vec r_i)=\frac{\tilde\phi}{2\pi}\sum_{j\neq i}\frac{\hat z\times(\vec r_i-\vec r_j)}{|\vec r_i-\vec r_j|^2}, \quad (6)$$
$$\vec a_b(\vec r_i)=\frac{\tilde\phi}{2\pi}\sum_a\frac{\hat z\times(\vec r_i-\vec r_a)}{|\vec r_i-\vec r_a|^2},$$

where a is the index of the bulk electrons. Taking the polar coordinate $x_i=r_i\cos\varphi_i,\ y_i=r_i\sin\varphi_i$, the vector potential $A_\varphi(\vec r_i)=\frac{B}{2}r_i$ and $A_r(\vec r_i)=0$. In the mean-field approximation, $a_{r,b}(\vec r_i)=0$ and $a_\varphi(\vec r_i)=B_{\tilde\phi}r_i/2$. Substituting the polar variations and the vector potential to H_e while using the mean-field value of $\vec a$, one has

$$H_e=\sum_i\Big[-\frac{\partial^2}{\partial r_i^2}+(-\frac{i}{r_i}\frac{\partial}{\partial\varphi_i}-\frac{B^*}{2}r_i)^2 \quad (7)$$
$$+\frac{\tilde\phi^2}{4R^2}\sum_i(\sum_{j\neq i}\cot\frac{\varphi_{ij}}{2})^2$$
$$-\frac{\tilde\phi}{R}\sum_{i<j}\cot\frac{\varphi_{ij}}{2}\cdot i(\frac{\partial}{\partial r_i}-\frac{\partial}{\partial r_j})-\frac{1}{R}\frac{\partial}{\partial r_i}\Big]$$
$$+V+U+O(\delta R/R).$$

Now the eigen problem at the edge could be solved by taking a trial wave function,

$$\Psi_e(z_1,...,z_{N^e})=\exp\{\frac{it}{2}\sum_{i<j}\frac{r_i-r_j}{R}\cot\frac{\varphi_{ij}}{2}\} \quad (8)$$
$$\times f(r_1,...,r_{N^e})\Psi_s(\varphi_1,...,\varphi_{N^e}),$$

where $t=\sqrt{\tilde\phi^2+\tilde\phi}-\tilde\phi$. The wave function f is symmetric and Ψ_s is anti-symmetric in the particle exchange. In the one-dimensional limit, $\delta R/R\to0$ and a shift of the ground state energy, it is easy to see that the wave function $\Psi_s(\varphi_1,...,\varphi_{N^e})$ satisfies the Schrodinger equation, $H_s\Psi_s=E_\varphi\Psi_s$ with

$$H_s=H_{cs}+V, \quad (9)$$

$$H_{cs}=\sum_i(i\frac{\partial}{\partial x_i}+\frac{B^*}{2}R)^2+\frac{g\pi^2}{L^2}\sum_{i<j}\Big[\sin(\frac{\pi x_{ij}}{L})\Big]^{-2}, \quad (10)$$

where $x_{ij} = x_i - x_j$, $\varphi_i = \frac{2\pi x_i}{L}$ and $L = 2\pi R$ is the length of the boundary. The Hamiltonian (10) is the CSLM Hamiltonian with a constant shift to the momentum operator. And the coupling constant

$$g = 2(\tilde{\phi}^2 + \tilde{\phi}) = 2(m^2 - m), \quad (11)$$

where $m = \tilde{\phi} + 1$ is an even integer. So we see that the consistency between one- and two-dimensional statistics because the particle's statistics in the CSLM [15] and anyons could be described by the same parameter m. The problem is exactly soluble and the wave functions are known with the Bethe ansatz form or more exactly are given by the Jack polynomials [16]. The eigen-energy is $E_\varphi = \sum_i (n_i - \frac{B^*}{2}R^2)^2/R^2$ where n_i satisfy the Bethe ansatz equations

$$n_i = I_i + \frac{1}{2}\sum_{j\neq i}(m-1)\mathrm{sgn}(n_i - n_j). \quad (12)$$

To see the low-lying excitations at the edge of the quantum Hall liquid droplet, we first turn off the interaction between electrons and will switch it on later. It is known that the CSLM has two-branches low-lying excitations, the left- and right-moving gapless sound waves. However, we will see the magnetic field leads to the right-moving modes to be ranged out of the low-lying state sector. To see that, we substitute the azimuthal wave function to the trial wave function (9) and then the radial wave function satisfies

$$\sum_i \left[-\frac{\partial^2}{\partial r_i^2} + (\frac{n_i}{r_i} - \frac{B^*}{2}r_i)^2\right] g + U_{eff}g + O(\frac{\delta R}{R}) = Eg, \quad (13)$$

with $g(r_1, .., r_{N^e}) = e^{-r_i/2R}f$. One finds that the many-body problem could be reduced to the single-particle one except the n_i are related by the Bethe ansatz equations (12). Notice that it is consistent with the discussion to IQHE edge state if $\tilde{\phi} = 0$ [1]. We consider the external electric field lacks first. In the harmonic potential approximation, one see that the single-particle wave function $g(r_n - r)$ with $r_n = \sqrt{2|n|/B^*}$ satisfies

$$\left[-\frac{d^2}{dy^2} + \omega_c^{*2}y^2\right] g_+(y,s) = \varepsilon g_+(y,s), \quad (14)$$

for $n \geq 0$, where $y = r - r_n$ and $s = R - r_n$ ($O(\delta R/R)$ could give r_n a minor change). For $n \leq 0$,

$$\left[-\frac{d^2}{dy^2} + \omega_c^{*2}y^2 + |n|\omega_c^*\right] g_-(y,s) = \varepsilon g_-(y,s). \quad (15)$$

Comparing (14) and (15), we see that there is a gap $|n|\hbar\omega_c^*$ between states g_+ and g_-. The states with the negative n are ranged out of the low-lying state sector. The width of the wave function g_+ is several times of R_c^*, the cyclotron radius of the CF in the effective field. (14) has been discussed by Halplerin years ago [1] and one sees that the eigen-energy if $r_n = R$ is $\varepsilon^R_{n,\nu^*} = (2(\nu^* - 1) + \frac{3}{2})\hbar\omega_c^*$ since the wave function vanishes at $r = R$ while $\varepsilon_{n,\nu^*} = ((\nu^* - 1) + \frac{1}{2})\hbar\omega_c^*$ for $R - r_m \gg R_c^*$, a harmonic oscillator energy and coinciding with the mean field theory applied to the bulk states. The gapless excitations appear when $|R - r_n| \sim R_c^*$. Using a perturbative calculation, one has the bare exciation energy is $\varepsilon_0 \sim v_c^* m^* \omega_c^*(r_n - R)$ with the cycltron velocity v_c^* of a CF. Now, turn the electric field on. A second order perturbative calculation shows that [17]

$$\varepsilon_0 = \delta\varepsilon_{n,\nu^*} = v_F^*(p_n - p_R) + \frac{1}{2m^*}(p_n - p_R)^2, \quad (16)$$

where $p_n = m^* r_n \omega_c^*$ and the Fermi velocity $v_F^* = v_d^* + b v_c^*$ with $v_d^* = m\frac{cE}{B}$ and b an order one constant. And $p_R = m^* R\omega_c^*$, which shows the boundary of the droplet behaves like the Fermi surface [18]. We see the linear dispersion. Since $v_c^* \ll v_d^*$, the CF sound wave velocity is $v_F^* \approx v_d^* = m v_d$ where v_d is the CF drift velocity (the current velocity). The relation between the current velocity and the sound wave velocity resembles the Haldane's velocity relations in the Luttinger liquid, $v_J = e^{2\varphi}v_s$, if we identify $m = e^{-2\varphi}$ [12]. Note that another zero point of (16) is at $p_n = p_R - 2m^* v_F^*$ where is actually deep inside of the bulk. So, we see that there is, indeed, only one branch low-lying excitations ($\nu^* = 1$) of the edge states, which behaves like the chiral Luttinger liquid (For details see [17]).

Now, let's make the relation to the macroscopic theory. In terms of the partition function (5), there is a most probable edge CF number $\bar{N}^e$ which is given by $\delta Z/\delta N^e = 0$. $\bar{N}^e = \int dx \rho(x)$ with the edge density $\rho(x) = h(x)\rho_e$ [2]. Here $h(x)$ is the edge deformation and ρ_e is the average density of the bulk electrons. One assumes the Fourier transformation of $h(x) = \sum_n e^{in\varphi}h_n$. Then the Fourier components of the density is $\rho_n = h_n\rho_e$. Now, we identify $h_n = r_n - R$. One finds that

$$\delta\varepsilon_{n,1} = 2\pi m v_d \rho_n, \quad (17)$$

and then the effective Hamiltonian reads

$$H_{eff} = \sum_{n>0}\rho_{-n}\delta\varepsilon_{n,1} = 2\pi m v_d \sum_{n>0}\rho_{-n}\rho_n, \quad (18)$$

which is the Hamiltonian used by Wen from the hydrodynamic point of view [2].

Now, let's switch on the interaction between CFs. First, we consider the short range interaction. In general, the CSLM pluses a short range interaction is no longer soluble. So, we treat it as a perturbation to the CSLM. One sees that it varies the value of n but does not renormalize the exponent m because the step-function phase shift is robust to the short range perturbation [17]. So the short range interaction does not affect the exponent m. This is consistent with the chiral Luttinger liquid consideration [19]. The Coulomb interaction may take more space to discuss [17]. Here we only give our recent result in a limit case. The one-dimensional Hamiltonian problem (9) for $N^e = 2$ could be exactly solved if $V = \frac{\alpha\pi}{L|\sin(\pi x_{ij}/L)|}$ in the large L limit. Actually, (9) is the same as the radial Hamiltonian of a three-dimensional electron scattering by a negative Coulomb interaction [20]. The phase shift, then, is given by

$$\theta(k) = \mathrm{sgn}(k)\tilde{\phi}\pi - 2\mathrm{arg}\Gamma(m + i/k). \tag{19}$$

By using the dilute gas limit, $x_1 \ll x_2 \ll ... \ll x_{N^e}$, the many-body problem could be solved asymptotically by the Bethe ansatz with the wave function

$$\Phi_s(x_1,..,x_{N^e}) = \sum_P A(P)\exp\{i\sum_i k_{P_i}x_i \tag{20}$$
$$-i\sum_{i<j} F(k_{P_i} - k_{P_j}, x_i - x_j)\},$$
$$F(k,x) = \frac{\alpha}{2k}\ln(\left[\tan\frac{\pi x}{2L}\right]\frac{2\pi k}{L}).$$

The asymptotic Bethe ansatz equation is given by $n_i = I_i + \frac{1}{2\pi}\sum_{j\neq i}\theta(k_i - k_j)$. If we assume the Coulomb interaction is dramatically renormalized such that $\alpha L \ll 1$. In the thermodynamic limit and the zero temperature, n_i could be solved by iteration and one has $n = \tilde{n} + C_n\ln(p_n - p_F)$, where $\tilde{n}$ is regular if $p_n - p_F$ tends to zero and C_n is a constant proportional to αL. Hence, the radial equation reads

$$-\frac{d^2g_+(y,s)}{dy^2} + \omega_c^{*2}y^2g_+(y,s) \tag{21}$$
$$+C_n'y\ln(p_n - p_R)g_+(y,s) = \varepsilon g_+(y,s),$$

where $y = r - r_{\tilde{n}}$ and $s = R - r_{\tilde{n}}$. We have identified $p_F = p_R$ here. By taking the approximation with $y \simeq s$ in the logarithm term, one see that the dispersion relation, after adding the external electric field,

$$\delta\varepsilon_{\tilde{n},\nu^*} = (p_n - p_R)(v_d^* + A_n\ln(p - p_R)), \tag{22}$$

with A_n proportional to αL. We see that a branch of the excitations with the form $q\ln q$, which is the plasma excitations caused by the Coulomb interaction [19].

Before concluding this paper, we would like to point out that the theory presented here is only for the single branch edge excitations of the $\nu = 1/m$ FQHE. This procedure could be generalized to the many branch case, say $\nu = \frac{p}{\tilde{\phi}p+1}$. The one-dimensional Hamiltonian for CFs is

$$H = \sum_{I,i_I} -\frac{\partial^2}{\partial x_{i_I}^2} + \frac{\pi^2}{2L^2}\sum_{I,J;i_I\neq j_J} g_{IJ}\left[\sin\frac{\pi(x_{i_I} - x_{j_J})}{L}\right]^{-2}, \tag{23}$$

where $I, J = 1, ..., p$ are the branch indices and $g_{IJ} = 2(K_{IJ}^2 + K_{IJ})$. The matrix K is given by $K_{IJ} = \tilde{\phi} + \delta_{IJ}$, which is introduced by Wen and Zee [21].

In conclusion, we have derived the microscopic model of the CF-type excitations for the edge excitations in the $\nu = 1/m$ FQHE with the sharp edge by using the CF picture. The edge excitations are chiral because the magnetic field suppresses one branch of the CSLM excitations. The self-consistency between one-dimensional and two-dimensional statistics is exhibited.

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