Nonperturbative aspects of axial vector vertex in the global color symmetry model

Hong-shi Zong,[1,2,*] Xiang-song Chen,[1] Fan Wang,[1] Chao-hsi Chang,[2,3] and En-guang Zhao[2,3]
[1]*Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China*
[2]*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China*
[3]*Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, People's Republic of China*
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The relation between the axial vector current of current quark and that of constituent quark has been studied within the framework of the global color symmetry model. Gluon dressing of the axial vector vertex and the quark self-energy function are described by the inhomogeneous Bethe-Salpeter equation in the ladder approximation and the Schwinger-Dyson equation in the rainbow approximation, respectively.

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Being a dressed particle of sea quarks and gluons, the constituent quark naturally behaves differently from the fundamental current quark appearing in the QCD Lagrangian, most evidently by a much larger mass. The dynamical or constituent mass arises owing to the spontaneous chiral symmetry breaking in QCD. A realistic and a phenomenological mechanism of chiral symmetry breaking is provided by the instantons [1–3] and the global color symmetry model (GCM) [4], respectively. There are evidences that GCM provides a successful description of various nonperturbative aspects of strong interaction physics, such as QCD vacuum and hadronic phenomena at low energies [4–7]. In Refs. [8–10], it was shown how the vector current of current quark is related to that of the constituent quark. In this paper, we follow the approach of Refs. [8–10] to study the axial vector current, which is closely related to the quark spin operator in QCD (note that $\overline{q}\vec{\gamma}\gamma^5 q = q^\dagger\vec{\Sigma}q$) and in turn related to the nucleon spin structure.

In the case of the chiral limit (zero current mass for the light quarks), we consider the Euclidean action of the GCM in an external axial vector field $\mathcal{A}_{\mu 5}(x)$:

$$S_{GCM}[\overline{q},q;\mathcal{A}] = \int d^4x\{\overline{q}(x)[\gamma\cdot\partial_x + i\gamma_\nu\gamma_5\mathcal{A}_{\nu 5}(x)]q(x)\} + \int d^4x d^4y\left[\frac{g_s^2}{2}j_\mu^a(x)D_{\mu\nu}^{ab}(x-y)j_\nu^b(y)\right], \tag{1}$$

where $j_\mu^a(x) = \overline{q}(x)\gamma_\mu(\lambda_c^a/2)q(x)$ denotes the color octet vector current and $g_s^2 D_{\mu\nu}^{ab}(x-y)$ is the dressed gluon propagator as input in the GCM. For convenience, we will employ the Feynman-type gauge $D_{\mu\nu}^{ab}(x-y) = \delta_{\mu\nu}^{ab}D(x-y)$ for the model gluon propagator.

Introducing an auxiliary bilocal field $B^\theta(x,y)$ and applying the standard bosonization procedure the generating functional of GCM,

$$\mathcal{Z}[\mathcal{A}] = \int \mathcal{D}\overline{q}\mathcal{D}q\, e^{-S_{GCM}[\overline{q},q;\mathcal{A}]} \tag{2}$$

can be rewritten in terms of the bilocal fields $B^\theta(x,y)$,

$$\mathcal{Z}[\mathcal{A}] = \int \mathcal{D}B^\theta e^{-S_{eff}[B^\theta;\mathcal{A}]} \tag{3}$$

with the effective bosonic action

$$S_{eff}[B^\theta,\mathcal{A}] = -\mathrm{Tr}\ln\mathcal{G}^{-1}[B^\theta;\mathcal{A}] + \int d^4x d^4y\frac{B^\theta(x,y)B^\theta(y,x)}{2g_s^2 D(x-y)} \tag{4}$$

and the quark operator

$$\mathcal{G}^{-1}[B^\theta;\mathcal{A}] = [\gamma\cdot\partial_x + i\gamma_\nu\gamma_5\mathcal{A}_{\nu 5}(x)]\delta(x-y) + \Lambda^\theta B^\theta(x,y). \tag{5}$$

The matrices $\Lambda^\theta = D^a\otimes C^b\otimes F^c$ are determined by Fierz transformation in Dirac, color, and flavor space of the current-current interaction in Eq. (1), and are given by

$$\Lambda^\theta = \frac{1}{2}\left\{1_D, i\gamma_5, \frac{i}{\sqrt{2}}\gamma_\nu, \frac{i}{\sqrt{2}}\gamma_\nu\gamma_5\right\}\otimes\left\{\frac{4}{3}1_C, \frac{i}{\sqrt{3}}\lambda_C^a\right\}\otimes\left\{\frac{1}{\sqrt{3}}1_F, \frac{1}{\sqrt{2}}\lambda_F^a\right\}. \tag{6}$$

One might expect that the complete set of the 16 Dirac matrices $\{1_D, i\gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$ must be employed in this description. However, by limiting the gluon two-point function $g_s^2 D(x-y)$ to diagonal components in Lorentz indices, the tensor $\sigma_{\mu\nu}$ is excluded.

In the mean-field approximation, the fields $B^\theta(x,y)$ are substituted simply by their vacuum value $B_0^\theta(x,y)$, which is defined as $\delta S_{eff}/\delta B|_{B_0} = 0$ and is given by

$$B_0^\theta[\mathcal{A}](x,y) = g_s^2 D(x-y)\mathrm{tr}_{DC}[\Lambda^\theta\mathcal{G}_0[\mathcal{A}](x,y)], \tag{7}$$

where the notation tr_{DC} includes trace over the Dirac and color indices and $\mathcal{G}_0^{-1}(x,y)$ denotes the inverse propagator with the self-energy $\Sigma(x,y) = \Lambda^\theta B_0^\theta(x,y)$ in the external background field $\mathcal{A}_{\mu 5}(x)$. Because of the external axial vector field, the quark propagator and self-energy are not translationally invariant.

*Email address: zonghs@chenwang.nju.edu.cn

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It should be noted that both $B_0^{\theta}(x,y)$ and $\mathcal{G}_0^{-1}(x,y)$ are implicitly dependent on the external background field $\mathcal{A}_{\mu5}$. If the external field $\mathcal{A}_{\mu5}$ is switched off, $\mathcal{G}_0[\mathcal{A}]$ goes into the dressed quark propagator $G\equiv\mathcal{G}_0[\mathcal{A}_{\mu5}=0]$, which has the decomposition

$$G^{-1}(p)=i\gamma\cdot p+\Sigma(p)=i\gamma\cdot pA(p^2)+B(p^2) \tag{8}$$

with

$$\Sigma(p)=\int d^4xe^{ip\cdot x}[\Lambda^{\theta}B_0^{\theta}(x)] = \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}g_s^2D(p-q)\gamma_\nu G(q)\gamma_\nu, \tag{9}$$

where the self-energy functions $A(p^2)$ and $B(p^2)$ are determined by the rainbow Dyson-Schwinger equation (DSE)

$$[A(p^2)-1]p^2=\frac{8}{3}\int\frac{d^4q}{(2\pi)^4}g^2D(p-q)\frac{A(q^2)p\cdot q}{q^2A^2(q^2)+B^2(q^2)},$$

$$B(p^2)=\frac{16}{3}\int\frac{d^4q}{(2\pi)^4}g^2D(p-q)\frac{B(q^2)}{q^2A^2(q^2)+B^2(q^2)}. \tag{10}$$

This dressing comprises the notion of "constituent quark" by providing a dynamical mass $M(p^2)=B(p^2)/A(p^2)$, reflecting a vacuum configuration with dynamical chiral symmetry breaking (D_χSB). The order parameter for chiral symmetry breaking is the quark or chiral condensate [11]

$$\langle:\bar{q}q:\rangle_\mu=(-)Z_4(\mu^2,\Lambda^2)\mathrm{tr}_{DC}\int^{\Lambda}\frac{d^4q}{(2\pi)^4}[G_0(q)], \tag{11}$$

where μ is the renormalization mass scale used in solving the quark DSE, $Z_4(\mu^2,\Lambda^2)$ is the mass renormalization constant, and Λ is the cutoff used in the translationally invariant regularization of the quark DSE. By definition, had the quark propagator only the "slash" term, the trace over the spinor indices understood in this loop would give an identical zero. Therefore, chiral symmetry breaking implies that a massless quark develops a nonzero dynamical mass (i.e., a "nonslash" term in the propagator).

In coordinate space the dressed axial vector vertex $\Gamma_{\mu5}(x,y;z)$ is given as the functional derivative of the inverse quark propagator $\mathcal{G}_0^{-1}[\mathcal{A}]$ with respect to the external field $\mathcal{A}_{\mu5}$:

$$\Gamma_{\mu5}(y_1,y_2;z)=\left[\frac{\delta\mathcal{G}_0^{-1}[\mathcal{A}](y_1,y_2)}{\delta\mathcal{A}_{\mu5}(z)}\right]_{\mathcal{A}_{\mu5}=0}. \tag{12}$$

If we take the functional derivative in Eq. (5) and put it into Eq. (12), we have

$$\Gamma_{\mu5}(y_1,y_2;z)=i\gamma_\mu\gamma_5\delta(y_1-y_2)\delta(y_1-z)+\left[\frac{\delta\Sigma[\mathcal{A}](y_1,y_2)}{\delta\mathcal{A}_{\mu5}(z)}\right]_{\mathcal{A}_{\mu5}=0}. \tag{13}$$

The second term on the right-hand side of Eq. (13) can be determined by employing the stationary condition Eq. (7), which, after reversing Fierz transformation, can be cast into

$$\Sigma[\mathcal{A}](y_1,y_2)=\frac{4}{3}g_s^2D(y_1,y_2)\gamma_\nu\mathcal{G}_0[\mathcal{A}](y_1,y_2)\gamma_\nu. \tag{14}$$

In order to find an expression for $[\delta\mathcal{G}_0[\mathcal{A}]\times(y_1,y_2)/\delta\mathcal{A}_{\mu5}(z)]_{\mathcal{A}_{\mu5}=0}$ in terms of the quark propagator $\mathcal{G}_0[\mathcal{A}]$, one expands $\mathcal{G}_0^{-1}[\mathcal{A}]$ in powers of $\mathcal{A}$ as follows:

$$\mathcal{G}_0^{-1}[\mathcal{A}]=\mathcal{G}_0^{-1}[\mathcal{A}]\Big|_{\mathcal{A}_{\mu5}=0}+\frac{\delta\mathcal{G}_0^{-1}[\mathcal{A}]}{\delta\mathcal{A}}\Big|_{\mathcal{A}_{\mu5}=0}\mathcal{A}+\cdots = G^{-1}+\mathcal{A}_{\mu5}\Gamma_{\mu5}+\cdots, \tag{15}$$

which leads to the formal expansion

$$\mathcal{G}_0[\mathcal{A}]=G-G\mathcal{A}_{\mu5}\Gamma_{\mu5}G+\cdots. \tag{16}$$

Here only the first-order dependence of $\mathcal{G}_0[\mathcal{A}]$ upon $\mathcal{A}_{\mu5}$ is of interest and this will generate the nonperturbative axial vector vertex.

Substituting Eqs. (14) and (16) into Eq. (13), we have the inhomogeneous ladder Bethe-Salpeter equation (BSE) of axial vector vertex, which reads

$$\Gamma_{\mu5}(y_1,y_2;z)=i\gamma_\mu\gamma_5\delta(y_1-y_2)\delta(y_1-z)-\frac{4}{3}g_s^2D(y_1-y_2)\int du_1du_2\gamma_\nu G(y_1,u_1)\times\Gamma_{\mu5}(u_1,u_2;z)G(u_2,y_2)\gamma_\nu. \tag{17}$$

Fourier transform of Eq. (17) leads then to the momentum space form of the inhomogeneous BSE,

$$\Gamma_{\mu5}(P,q)=i\gamma_\mu\gamma_5-\frac{4}{3}\int\frac{d^4K}{(2\pi)^4}g_s^2D(P-K)\times\gamma_\nu G\left(K+\frac{q}{2}\right)\Gamma_{\mu5}(K,q)G\left(K-\frac{q}{2}\right)\gamma_\nu. \tag{18}$$

Contracting both sides with q_μ one finds the chiral limit axial vector Ward-Takahashi identity (WTI) [11]:

$$q_\mu\Gamma_{\mu5}(P,q)=G^{-1}\left(P+\frac{q}{2}\right)\gamma_5+\gamma_5G^{-1}\left(P-\frac{q}{2}\right). \tag{19}$$

As was shown above, both the rainbow DSE (10) and the ladder BSE (18) can be consistently derived from the action of the GCM in an external axial vector field $\mathcal{A}_{\mu5}$.

From Eq. (18) it is evident that the $\Gamma_{\mu 5}(P,q)$, in addition to its Lorentz axial four-vector structure, is a four-by-four matrix in Dirac indices and is in general a function of two nonorthogonal four-vectors, P and $q(P\cdot q\neq 0)$.

In the framework of the GCM, from the Lorentz structure, the most general form for the vertex function $\Gamma_{\mu 5}(P,q)$(in the chiral limit) which fulfills Eq. (18), reads

$$\Gamma_{\mu 5}(P,q)=\gamma_5\Lambda^{(1)}_{\mu}(P,q)+i\gamma_5\gamma_\nu\Lambda^{(2)}_{\nu\mu}(P,q)+\frac{i\gamma_\nu\varepsilon_{\mu\nu\alpha\beta}P_\alpha q_\beta\Lambda^{(3)}(P,q)}{\eta^2}+\gamma_5\frac{q_\mu\Gamma_\pi(P,q)}{q^2}, \tag{20}$$

where $\Lambda^{(1)}_{\mu}(P,q)$, $\Lambda^{(2)}_{\nu\mu}(P,q)$, and $\Lambda^{(3)}(P,q)$ are regular as $q^2\to 0$ and this form admits the possibility of at least one pole term in the axial vector vertex [in the GCM, one often considers the γ_5 part of the Bethe-Salpeter (BS) amplitude for a pseudoscalar $q\bar{q}$ bound state]. From Eq. (20), it is easy to find that $\sigma_{\mu\nu}$ is absent; this is because the gluon two-point function $g_s^2D(x-y)$ is limited to diagonal components in Lorentz indices. Further, the scalar matrix 1_D can be excluded by considering that the corresponding term has the form $1_D\Lambda_{\mu 5}$, where the matrix structure has been completely factored. In order to contribute to the axial vector vertex, this term must form a four axial vector, which requires $\Lambda_{\mu 5}$ to be an axial vector. It is immediately evident that this is impossible since there is an insufficient number of vectors (P and q) to combine with the tensor $\varepsilon_{\mu\nu\alpha\beta}$ to form an axial vector. The matrix structure in Eq. (20) is now explicit and the quantities $\Lambda^{(i)}$ $(i=1,2,3)$ are dimensionless functions with Lorentz structure indicated by their indices. The constant η has the dimension of mass.

Substituting Eq. (20) into Eq. (18) and equating putative pole terms, we have the homogeneous BSE in the ladder approximation for a pseudoscalor $q\bar{q}$ bound state [12]:

$$\Gamma_\pi(P,q)=\frac{4}{3}\int\frac{d^4K}{(2\pi)^4}g_s^2D(P-K)\gamma_\nu G\left(-K-\frac{q}{2}\right)\times\Gamma_\pi(K,q)G\left(K-\frac{q}{2}\right)\gamma_\nu, \tag{21}$$

where P is the relative and q the total momentum of the quark-antiquark pair. Then for $q=0$, Eq. (21) becomes

$$\Gamma_\pi(P,0)=\frac{16}{3}\int\frac{d^4K}{(2\pi)^4}g^2D(P-K)\frac{\Gamma_\pi(K,0)}{K^2A^2(K^2)+B^2(K^2)}, \tag{22}$$

which, in comparing with Eq. (10), is seen to have solutions $\Gamma_\pi(P,0)\propto B(P^2)$. Thus the crucial result emerges that in the chiral limit the pseudoscalar meson Eq. (21) has solutions for $q^2=0$; that is, these mesons are massless (Goldstone) modes, and furthermore their form factor Γ_π is the quark mass function $B(P^2)$. This important result was first obtained in a four fermion model by Nambu and Jona-Lasinio [13] and further studied by Munczek [14] in the very close connection between the exact Schwinger-Dysa and BS equations.

The above result can also be obtained directly through the use of WTI. Substituting Eq. (20) into the left-hand side of Eq. (19) along with Eq. (8) on the right, and equating terms of order $(q)^0$, one obtains the following chiral limit relation:

$$\Gamma_\pi(P,0)\propto B(P^2). \tag{23}$$

In perturbative theory, $B(P^2)\equiv 0$ in the chiral limit [15]. The appearance of a $B(P^2)$ nonzero solution of Eq. (10) in the chiral limit signals D_χSB: one has dynamically generated a quark mass term (in the absence of a seed mass) which ensures the presence of massless (Goldstone) modes in the spectrum. Therefore, it is the WTI that ensures the presence of the pseudoscalar Goldstone modes as a consequence of DχSB.

The further reduction of the $\Lambda^{(i)}$ $(i=1,2,3)$ to a set of invariant functions is achieved through the use of the symmetry transformation of the vertex $\Gamma_{\mu 5}(P,q)$ under γ_5 and charge conjugation $C=\gamma_2\gamma_4$. The transformation properties are determined directly from the Eq. (18) and are given by

$$\gamma_5\Gamma_{\mu 5}(P,q)\gamma_5=-\Gamma_{\mu 5}(-P,-q),$$

$$C\Gamma_{\mu 5}(P,q)C^{-1}=\Gamma^t_{\mu 5}(-P,q),$$

where t denotes a matrix transpose. From the general form given above and Eq. (20), it is clear that $\Lambda^{(2)}_{\nu\mu}(P,q)$ and $\Lambda^{(3)}(P,q)$ are even in both P and q, while $\Lambda^{(1)}_{\mu}(P,q)$ is odd in q and even in P. The quantities $\Lambda^{(i)}$ can therefore be written as

$$\Lambda^{(1)}_{\mu}(P,q)+\frac{q_\mu\Gamma_\pi(P,q)}{q^2}=\frac{q_\mu}{\eta}\lambda^L_1+\frac{P^T_\mu}{\eta}\frac{P\cdot q}{q^2}\lambda^T_1,$$

$$\Lambda^{(2)}_{\nu\mu}(P,q)=\frac{P_\nu q_\mu}{\eta^2}\frac{P\cdot q}{q^2}\lambda^L_2+\frac{P_\nu p^T_\mu}{\eta^2}\lambda^T_2-\frac{q_\nu q_\mu}{q^2}\lambda^L_3-\left(\delta_{\nu\mu}-\frac{q_\nu q_\mu}{q^2}\right)\lambda^T_3+\frac{q_\nu P^T_\mu P\cdot q}{\eta^4}\lambda^T_4,$$

$$\Lambda^{(3)}(P,q)=\lambda^T_5.$$

The eight scalar dimensionless coefficients λ^L_i $(i=1,2,3)$ and λ^T_i $(i=1,2,3,4,5)$ depend on P^2, q^2, and C^2_{Pq}, where $C_{Pq}\equiv P\cdot q/(Pq)$ is the direction cosine between P and q. $p^T_\mu\equiv P_\mu-P\cdot qq_\mu/q^2$ is the vector transverse to q_μ, i.e., $q_\mu p^T_\mu=0$. The advantage of the above decomposition lies in the fact that the longitudinal components λ^L_i $(i=1,2,3)$ are determined automatically from the quark propagator G by means of WTI.

By means of the axial vector WTI (19) and Eq. (20), we have the longitudinal components [λ^L_i $(i=1,2,3)$],

$$\lambda^L_1=\frac{\eta}{q^2}[B(P^2_+)+B(P^2_-)],$$

$$\lambda_2^L=\frac{\eta^2}{P\cdot q}[A(P_-^2)-A(P_+^2)],$$

$$\lambda_3^L=\frac{1}{2}[A(P_+^2)+A(P_-^2)],$$

where $P_\pm\equiv P\pm q/2$.

This leaves only the five independent transversal components λ_i^T ($i=1,2,3,4,5$) to be determined as solutions of inhomogeneous BSE (18). In order to do this, one has to project out the single λ_i^T for each i which can be done by multiplying Eq. (20) with appropriate Dirac matrices and taking traces. Performing the same operations on the right-hand side of Eq. (18) finally leads to a set of five coupled inhomogeneous integral equations for λ_i^T:

$$\frac{P_\mu^T}{4\eta}\mathrm{tr}[\gamma_5\Gamma_{\mu5}(P,q)]=\frac{(P^T)^2}{\eta^2}\frac{P\cdot q}{q^2}\lambda_1^T,$$

$$-\frac{i}{4}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\mathrm{tr}[\gamma_\nu\gamma_5\Gamma_{\mu5}(P,q)]=\frac{(P^T)^2}{\eta^2}\lambda_2^T-3\lambda_3^T,$$

$$-\frac{i}{4}\frac{P_\mu^TP_\nu^T}{(P^T)^2}\mathrm{tr}[\gamma_\nu\gamma_5\Gamma_{\mu5}(P,q)]=\frac{(P^T)^2}{\eta^2}\lambda_2^T-\lambda_3^T,$$

$$-\frac{i}{4}\frac{P_\mu^Tq_\nu}{(P^T)^2}\mathrm{tr}[\gamma_\nu\gamma_5\Gamma_{\mu5}(P,q)]=\frac{P\cdot q}{\eta^2}\lambda_2^T+\frac{q^2P\cdot q}{\eta^4}\lambda_4^T,$$

$$\mathrm{tr}[\gamma_\nu\Gamma_{\mu5}(P,q)]=4i\epsilon_{\mu\nu\alpha\beta}P_\alpha q_\beta\eta^{-2}\lambda_5^T,$$

where the tr is over the Dirac indices. The left-hand sides of the above equations are evaluated using the inhomogeneous BSE for $\Gamma_{\mu5}$ (18). Based on the above five coupled integral equations and Eq. (18), in principle, by means of numerical studies, we can get the nonperturbative axial vector vertex $\Gamma_{\mu5}$, which is useful for the calculation of nucleon spin in the constituent quark model. So far, we have shown how the axial vector current of the current quark is related to that of the constituent quark within the framework of the global color symmetry model.

In order to have a qualitative understanding of the nonperturbative aspect of the axial vector vertex, a particularly simple and useful model of the dressed gluon two-point function [16] is used as follows:

$$g^2D((P-K)^2)=3\eta^2\pi^4\delta^{(4)}(P-K). \tag{24}$$

Then Eq. (10) may be solved to give

$$B(P^2)=(\eta^2-4P^2)^{1/2},\ \ A(P^2)=2\ \ \text{for}\ \ P^2\leqslant\frac{\eta^2}{4},$$

and

$$B(P^2)=0,\ \ A(P^2)=\frac{1}{2}\left[1+\left(1+\frac{2\eta^2}{P^2}\right)^{1/2}\right]\ \ \text{for}\ \ P^2\geqslant\frac{\eta^2}{4}.$$

Here the scale parameter η is a measure of the strength of the infrared slavery effect.

With the model of the dressed gluon propagator specified in Eq. (24), the inhomogeneous ladder axial vector vertex equation is

$$\Gamma_{\mu5}(P,q)=i\gamma_\mu\gamma_5-\frac{\eta^2}{4}\gamma_\nu G\left(P+\frac{q}{2}\right)\Gamma_{\mu5}(P,q)G\left(P-\frac{q}{2}\right)\gamma_\nu. \tag{25}$$

Using Eq. (25), the above five coupled integral equation can be further reduced to the following five coupled algebraic equations:

$$\begin{aligned}&\frac{P\cdot q}{q^2}\lambda_1^T-[\alpha(P_+^2)\beta(P_-^2)-\alpha(P_-^2)\beta(P_+^2)]\eta^3\lambda_3^T\\&=-\eta\Bigg\{-\left[\left(P^2-\frac{q^2}{4}\right)\alpha(P_+^2)\alpha(P_-^2)+\beta(P_+^2)\beta(P_-^2)\right]\eta\frac{P\cdot q}{q^2}\lambda_1^T+\left[\left(P^2+\frac{P\cdot q}{2}\right)\alpha(P_+^2)\beta(P_-^2)\right.\\&\left.-\left(P^2-\frac{P\cdot q}{2}\right)\alpha(P_-^2)\beta(P_+^2)\right]\lambda_2^T+\left[\left(P\cdot q+\frac{q^2}{2}\right)\alpha(P_+^2)\beta(P_-^2)-\left(P\cdot q-\frac{q^2}{2}\right)\alpha(P_-^2)\beta(P_+^2)\right]\frac{P\cdot q}{\eta^2}\lambda_4^T\Bigg\},\end{aligned} \tag{26}$$

$$\lambda_3^T\left[1-\frac{\eta^2}{4}\left(2P^2-\frac{q^2}{2}\right)\alpha(P_+^2)\alpha(P_-^2)+\frac{\eta^2}{2}\beta(P_+^2)\beta(P_-^2)\right]=1-\frac{q^2}{2}(P^T)^2\alpha(P_+^2)\alpha(P_-^2)\lambda_5^T, \tag{27}$$

$$\lambda_5^T\left[1-\frac{\eta^2}{2}\beta(P_+^2)\beta(P_-^2)-\frac{\eta^2}{2}\left(P^2-\frac{q^2}{4}\right)\alpha(P_+^2)\alpha(P_-^2)\right]=\frac{\eta^4}{2}\alpha(P_+^2)\alpha(P_-^2)\lambda_3^T, \tag{28}$$

$$\frac{P\cdot q}{\eta^2}\lambda_2^T+q^2\frac{P\cdot q}{\eta^4}\lambda_4^T=\frac{\eta^2}{2}\Bigg\{-\frac{1}{\eta}\frac{P\cdot q}{q^2}\lambda_1^T\left[\left(P\cdot q+\frac{q^2}{2}\right)\alpha(P_+^2)\beta(P_-^2)-\left(P\cdot q-\frac{q^2}{2}\right)\alpha(P_-^2)\beta(P_+^2)\right]$$
$$-\left[P\cdot q\left(P^2-\frac{q^2}{4}\right)\right]\frac{\lambda_2^T}{\eta^2}\alpha(P_+^2)\alpha(P_-^2)-\left[\frac{P\cdot q}{\eta^2}\lambda_2^T+q^2\frac{P\cdot q}{\eta^4}\lambda_4^T\right]\beta(P_+^2)\beta(P_-^2)+2P\cdot q\lambda_3^T\alpha(P_+^2)\alpha(P_-^2)$$
$$-\left[2(P\cdot q)^2-q^2P^2-\frac{q^4}{4}\right]\frac{P\cdot q}{\eta^4}\lambda_4^T\alpha(P_+^2)\alpha(P_-^2)\Bigg\}, \tag{29}$$

$$\lambda_2^T=\frac{\eta^4}{2}\Bigg\{-\frac{\lambda_1^T}{\eta}[\alpha(P_+^2)\beta(P_-^2)-\alpha(P_-^2)\beta(P_+^2)]-\frac{1}{\eta^2}\lambda_2^T\beta(P_+^2)\beta(P_-^2)+\left[2\lambda_3^T-\frac{2}{\eta^4}(P\cdot q)^2\lambda_4^T-\frac{\lambda_2^T}{\eta^2}\left(P^2+\frac{q^2}{4}\right)\right]$$
$$\times\alpha(P_+^2)\alpha(P_-^2)-\frac{q^2}{\eta^2}\lambda_5^T\alpha(P_+^2)\alpha(P_-^2)\Bigg\}, \tag{30}$$

where the α and β are defined as the vector and scalar parts of the quark Green function G.

For the region $P_\pm^2\leqslant\eta^2/4$, the analytic solution for Eqs. (26)–(30) has the simple form

$$\lambda_1^T=\frac{q^2}{P\cdot q}\left[\frac{4\eta[B(P_-^2)-B(P_+^2)]}{\eta^2-4P^2+5q^2-B(P_+^2)B(P_-^2)}\right],\quad \lambda_2^T=-\frac{q^2}{\eta^2}\lambda_4^T,$$
$$\lambda_3^T=2\frac{[2\eta^2-4P^2+q^2-B(P_+^2)B(P_-^2)]}{3\eta^2-8P^2+6q^2}, \tag{31}$$
$$\lambda_4^T=-\frac{32\eta^4}{[\eta^2-4P^2+5q^2-B(P_+^2)B(P_-^2)][3\eta^2-8P^2+6q^2]},\qquad \lambda_5^T=\frac{8\eta^2}{3\eta^2-8P^2+6q^2}.$$

At relative momentum $P=0$, the nonperturbative axial vector vertex from Eq. (31) can be written as

$$\Gamma_{\mu5}(0,q)=\gamma_5\frac{2q_\mu(\eta^2-q^2)^{1/2}}{q^2}-\frac{2}{3}(i\gamma_5\gamma_\mu)$$
$$-\frac{4}{3}\frac{q_\mu}{q^2}(i\gamma_5\gamma\cdot q). \tag{32}$$

It is clear now that the axial vector vertex has one, and only one, pole at $q^2=0$. This pole in the longitudinal part of the vertex is associated with the pion. This result confirms the work of Ref. [16], i.e., this model does not support a massive axial vector meson.

To summarize, we have studied the nonperturbative aspect of the axial vector vertex. This employs a consistent treatment of dressed quark propagator G and the dressed axial vector vertex $\Gamma_{\mu5}$, which are both determined from the model quark-quark interaction by the rainbow DSE for G and the inhomogeneous ladder BSE for $\Gamma_{\mu5}$. Finally we want to stress that our vertex model satisfies the criteria provided by Ref. [14]. Therefore, the present paper may serve as a useful illustration of the general techniques mentioned in Ref. [14].

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[1] C.G. Callan, Jr., R. Dashen, and D.J. Gross, Phys. Rev. D **17**, 2717 (1978).
[2] E.V. Shuryak, Nucl. Phys. **B203**, 93 (1982); Phys. Rep. **115**, 151 (1984); T. Schafer and E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
[3] D.I. Dyakonov and X. Yu Petrov, Nucl. Phys. **B245**, 259 (1984); **B272**, 457 (1986).
[4] R.T. Cahill and C.D. Roberts, Phys. Rev. D **32**, 2419 (1985).
[5] M.R. Frank and T. Meissner, Phys. Rev. C **53**, 2410 (1996); M.R. Frank and C.D. Roberts, *ibid.* **53**, 390 (1996).
[6] C.D. Roberts and A.G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994); P.C. Tandy, *ibid.* **39**, 117 (1997), and references therein.
[7] Xiao-fu Lü, Yu-xin Liu, Hong-shi Zong, and En-guang Zhao, Phys. Rev. C **58**, 1195 (1998); Hong-shi Zong, Xiao-fu Lü, Jian-zhong Gu, Chao-hsi Chang, and En-guang Zhao, *ibid.* **60**,

055208 (1999); Hong-shi Zong, Xiao-hua Wu, Xiao-fu Lü, Chao-hsi Chang, and En-guang Zhao, hep-ph/0109112.
[8] M.R. Frank, Phys. Rev. C **51**, 987 (1995).
[9] M.R. Frank and P.C. Tandy, Phys. Rev. C **49**, 478 (1994).
[10] T. Meissner and L.S. Kisslinger, Phys. Rev. C **59**, 986 (1999).
[11] P. Maris, C.D. Roberts, and P.C. Tandy, Phys. Lett. A **420**, 267 (1998).
[12] R.T. Cahill, C.D. Roberts, and J. Praschifka, Phys. Rev. D **36**, 2804 (1987).
[13] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[14] H.J. Munczek, Phys. Rev. D **52**, 4736 (1995).
[15] P. Pascual and R. Tarrach, *QCD: Renormalisation for the Pracitioner* (Springer-Verlag, Berlin, 1984).
[16] H.J. Munczek and A.M. Nemirovsky, Phys. Rev. D **28**, 181 (1983).