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Hessence: a new view of quintom dark energy

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Abstract

Recently a lot of attention has been given to building a dark energy model in which the equation-of-state parameter w can cross the phantom divide $w = -1$. One of the models to realize crossing the phantom divide is called the quintom model, in which two real scalar fields appear, one is a normal scalar field and the other is a phantom-type scalar field. In this paper we propose a non-canonical complex scalar field as the dark energy, which we dub 'hessence', to implement crossing the phantom divide, in a similar sense as the quintom dark energy model. In the hessence model, the dark energy is described by a single field with an internal degree of freedom rather than two independent real scalar fields. However, the hessence is different from an ordinary complex scalar field, we show that the hessence can avoid the difficulty of the Q-ball formation which gives trouble to the spintessence model (an ordinary complex scalar field acts as the dark energy). Furthermore, we find that, by choosing a proper potential, the hessence could correspond to a Chaplygin gas at late times.

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1. Introduction

A lot of cosmological observations, such as SNe Ia [1], WMAP [2], SDSS [3], Chandra X-ray Observatory [4] etc, reveal some cross-checked information on our universe. They suggest that the universe is spatially flat, and consists of approximately 70% dark energy with negative pressure, 30% dust matter (cold dark matter plus baryons), and negligible radiation, and that the universe is undergoing an accelerated expansion.

To accelerate the expansion, the equation-of-state parameter $w \equiv p/\rho$ of the dark energy must satisfy $w < -1/3$, where p and ρ are its pressure and energy density, respectively.

The simplest candidate for the dark energy is a tiny positive time-independent cosmological constant Λ, for which $w = -1$. Another possibility is quintessence [5, 6], a cosmic real scalar field that is displaced from the minimum of its potential. With the evolution of the universe, the scalar field slowly rolls down its potential. To be definite, we consider the action[4]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{\mathrm{DE}} + \mathcal{L}_m\right), \tag{1}$$

where g is the determinant of the metric $g_{\mu\nu}$, $\mathcal{R}$ is the Ricci scalar, $\mathcal{L}_{\mathrm{DE}}$ and $\mathcal{L}_m$ are the Lagrangian densities of the dark energy and matter, respectively. The Lagrangian density for the quintessence is

$$\mathcal{L}_{\mathrm{DE}} = \tfrac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi), \tag{2}$$

where φ is a real scalar field. Considering a spatially flat FRW universe and assuming that the scalar field φ is homogeneous, one has the equation-of-state parameter of quintessence as

$$w = \frac{\dot\varphi^2/2 - V(\varphi)}{\dot\varphi^2/2 + V(\varphi)}. \tag{3}$$

It is easy to see that $-1 \leqslant w \leqslant +1$ for quintessence. On the other hand, the observations cannot exclude the possibility of phantom matter with $w < -1$ [7–9]. One way to realize the phantom matter is a scalar field with a 'wrong' sign kinetic energy term. The Lagrangian density for the phantom scalar field is given by

$$\mathcal{L}_{\mathrm{DE}} = -\tfrac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi). \tag{4}$$

Its equation-of-state parameter

$$w = \frac{-\dot\varphi^2/2 - V(\varphi)}{-\dot\varphi^2/2 + V(\varphi)}, \tag{5}$$

clearly, one has $w \leqslant -1$ with $\rho = -\dot\varphi^2/2 + V(\varphi) > 0$.

Actually, by fitting the recent SNe Ia data, marginal (2σ) evidence for $w(z) < -1$ at $z < 0.2$ has been found [10]. In addition, many best fit values of w_0 are less than -1 in various data fittings with different parametrizations (see [11] for a recent review). The present data seem to favour an evolving dark energy with w being below -1 around the present epoch from $w > -1$ in the near past [12]. Obviously, the equation-of-state parameter w cannot cross the phantom divide $w = -1$ for quintessence or phantom alone. Recently, some efforts have been made to build a dark energy model whose equation-of-state parameter can cross the divide $w = -1$. Although some variants of the k-essence [13] look possible to give promising solutions, a no-go theorem, shown in [14], shatters this kind of hope: it is impossible to cross the phantom divide $w = -1$, provided that the following conditions are satisfied: (1) classical level, (2) GR is valid, (3) single real scalar field, (4) arbitrary Lagrangian density $p(\varphi, X)$, where $X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ is the kinetic energy term, and (5) $p(\varphi, X)$ is a continuous function and is differentiable enough. Thus, to implement the transition from $w > -1$ to $w < -1$ or vice versa, it is necessary to give up at least one of conditions mentioned above.

Obviously, the simplest way to get around this no-go theorem is to consider a two real scalar fields model, i.e. to break the third condition. In [15], Hu considered a phenomenological model with two real scalar fields (see also [23]) and showed that it is possible to cross the phantom divide $w = -1$. Feng, Wang and Zhang in [12] proposed a so-called quintom model which is a hybrid of quintessence and phantom (thus the name quintom). Naively, one may consider a Lagrangian density [12, 16]

$$\mathcal{L}_{\mathrm{DE}} = \tfrac{1}{2}(\partial_\mu\phi_1)^2 - \tfrac{1}{2}(\partial_\mu\phi_2)^2 - V(\phi_1, \phi_2), \tag{6}$$

[4] We adopt the metric convention as (+, −, −, −) throughout this paper.

where ϕ_1 and ϕ_2 are two real scalar fields and play the roles of quintessence and phantom respectively. Considering a spatially flat FRW universe and assuming the scalar fields ϕ_1 and ϕ_2 are homogeneous, one obtains the *effective* pressure and energy density for the quintom

$$p = \tfrac{1}{2}\dot{\phi}_1^2 - \tfrac{1}{2}\dot{\phi}_2^2 - V(\phi_1, \phi_2), \tag{7}$$

$$\rho = \tfrac{1}{2}\dot{\phi}_1^2 - \tfrac{1}{2}\dot{\phi}_2^2 + V(\phi_1, \phi_2). \tag{8}$$

And then, the corresponding *effective* equation-of-state parameter is given by

$$w = \frac{\dot{\phi}_1^2 - \dot{\phi}_2^2 - 2V(\phi_1, \phi_2)}{\dot{\phi}_1^2 - \dot{\phi}_2^2 + 2V(\phi_1, \phi_2)}. \tag{9}$$

It is easy to see that $w \geqslant -1$ when $\dot{\phi}_1^2 \geqslant \dot{\phi}_2^2$ while $w < -1$ when $\dot{\phi}_1^2 < \dot{\phi}_2^2$. The cosmological evolution of the quintom model without direct coupling between ϕ_1 and ϕ_2 was studied by Guo *et al* [16]. They showed that the transition from $w > -1$ to $w < -1$ or vice versa is possible in this type of quintom model.

In many of the existing quintom-type models [12, 15, 16, 23, 24, 35], they invoke two independent real scalar fields to describe the dark energy. However, it is also natural to consider a *single* field with an *internal degree of freedom* to describe the dark energy. For the spintessence model of dark energy [17–20] with a single complex scalar field, it suffers from the problem of Q-ball formation [17, 19, 21]. For the so-called $SO(1, \eta)$ model of dark energy [25], Wei *et al* extended the unit imaginary number i to a new parameter i_η and constructed an extended complex scalar field as dark energy.

In fact, by a new view of the quintom model, we propose a non-canonical complex scalar field, which we dub 'hessence', to play the role of quintom. The hessence is similar to the extended complex scalar field proposed in [25] in some sense. However, the motivation and emphasis here are different from those in [25]. The hessence could be viewed as a new window to look into the unknown internal world of the mysterious dark energy. In addition, like the case of canonical complex scalar field, the hessence has a *conserved charge*. In section 2 we will discuss some aspects of the hessence model. In section 3, we will show that, different from the case of ordinary complex scalar field, the hessence can avoid the difficulty of Q-ball formation which gives trouble to the spintessence. (A Q-ball is a kind of nontopological soliton whose stability is guaranteed by some conserved charge.) In section 4, we show that, by choosing a proper potential, the hessence could correspond to a Chaplygin gas at late times. A brief summary and some discussions are presented in section 5.

2. Hessence

2.1. Motivation

Consider a non-canonical complex scalar field as the dark energy,

$$\Phi = \phi_1 + \mathrm{i}\phi_2, \tag{10}$$

with a Lagrangian density

$$\mathcal{L}_{\mathrm{DE}} = \tfrac{1}{4}[(\partial_\mu \Phi)^2 + (\partial_\mu \Phi^*)^2] - V(\Phi, \Phi^*). \tag{11}$$

Obviously, this Lagrangian density is identified with equation (6) in terms of two real scalar fields ϕ_1 and ϕ_2. By this formalism, however, the dark energy is described by a single field rather than two independent fields. The physical content is changed. On the other hand, this

Lagrangian density of the hessence is different from the case of a canonical complex scalar field Ψ whose Lagrangian density is given by

$$\mathcal{L}_{\rm DE} = \tfrac{1}{2}(\partial^\mu\Psi^*)(\partial_\mu\Psi) - V(|\Psi|), \tag{12}$$

where $|\Psi|$ is the absolute value of Ψ, namely $|\Psi|^2 = \Psi^*\Psi$. Thus, we give this non-canonical complex scalar field a new name 'hessence' (the meaning of this name will be clear below) to make a distinction with the canonical complex scalar field. In fact, the canonical complex scalar field was considered as a variant of quintessence for several years, while it was dubed 'spintessence' [17–20]. The spintessence also has a conserved charge. However, it is overlooked as a viable candidate of dark energy because it is troubled by the Q-ball formation [17, 19, 21] which we will discuss in section 3. We find that it is suggestive to compare the hessence with the spintessence since they are similar in many aspects. (Of course, they also have many differences which are crucial to make the hessence avoid the difficulty of Q-ball formation.)

The most interesting feature of a complex scalar field different from a real scalar field is that the complex scalar field has a conserved charge due to internal symmetry. It is suggestive to review the case of canonical complex scalar field first. In terms of $\Psi = \psi_1 + {\rm i}\psi_2$, the Lagrangian density of the canonical complex scalar field, equation (12), becomes

$$\mathcal{L}_{\rm DE} = \tfrac{1}{2}(\partial_\mu\psi_1)^2 + \tfrac{1}{2}(\partial_\mu\psi_2)^2 - V(|\Psi|).$$

It is invariant under the transformation

$$\psi_1 \to \psi_1\cos\alpha - \psi_2\sin\alpha, \qquad \psi_2 \to \psi_1\sin\alpha + \psi_2\cos\alpha,$$

which also keeps $|\Psi|^2 = \psi_1^2 + \psi_2^2$ unchanged. Here α is a constant. On the other hand, in terms of $\Psi = \psi\,{\rm e}^{{\rm i}\eta}$, where $\psi = |\Psi|$ is the amplitude and η is the phase angle, this transformation is equivalent to

$$\psi \to \psi, \qquad \eta \to \eta + \alpha,$$

which means a phase displacement. According to the well-known Noëther theorem, this symmetry leads to a conserved charge. In the light of a canonical complex scalar field, it is easy to find that the hessence also has a similar symmetry. One can verify that the kinetic energy terms

$$\tfrac{1}{4}[(\partial_\mu\Phi)^2 + (\partial_\mu\Phi^*)^2] = \tfrac{1}{2}(\partial_\mu\phi_1)^2 - \tfrac{1}{2}(\partial_\mu\phi_2)^2$$

of the hessence are invariant under the transformation

$$\phi_1 \to \phi_1\cos\alpha - {\rm i}\phi_2\sin\alpha, \qquad \phi_2 \to -{\rm i}\phi_1\sin\alpha + \phi_2\cos\alpha, \tag{13}$$

which also keeps $\phi_1^2 - \phi_2^2$ unchanged. Then, if the potential of the hessence $V(\Phi, \Phi^*)$ or $V(\phi_1, \phi_2)$ depends on the quantity $\Phi^2 + \Phi^{*2}$ or $\phi_1^2 - \phi_2^2$ only, the Lagrangian density of the hessence is invariant under this transformation above. In this case, the hessence should have a conserved charge. However, we find that it is unclear to understand the physical meaning of the transformation equation (13) in terms of the traditional formalism of the complex scalar field, i.e. (ϕ_1, ϕ_2) or $\Phi = R\,{\rm e}^{{\rm i}\Theta}$. In addition, we find that the equations of the hessence are very involved in terms of (R, Θ) while it is convenient in the case of spintessence. We must find a new formalism to describe the new non-canonical complex scalar field, namely the hessence.

It is suggestive to note that (i) $\phi_1^2 - \phi_2^2 = {\rm const}$ is a hyperbola on the ϕ_1 versus ϕ_2 plane, and (ii) by the relations between angular functions and hyperbolic functions, one has

$$\sinh z = -{\rm i}\sin({\rm i}z), \qquad \sin z = -{\rm i}\sinh({\rm i}z),$$
$$\cosh z = \cos({\rm i}z), \qquad \cos z = \cosh({\rm i}z).$$

In terms of hyperbolic functions, transformation (13) can be rewritten as

$$\phi_1 \to \phi_1 \cosh(\mathrm{i}\alpha) - \phi_2 \sinh(\mathrm{i}\alpha), \qquad \phi_2 \to -\phi_1 \sinh(\mathrm{i}\alpha) + \phi_2 \cosh(\mathrm{i}\alpha). \tag{14}$$

Furthermore, we introduce two new variables (ϕ, θ) to describe the hessence, i.e.

$$\phi_1 = \phi \cosh\theta, \qquad \phi_2 = \phi \sinh\theta, \tag{15}$$

which are defined by

$$\phi^2 = \phi_1^2 - \phi_2^2, \qquad \coth\theta = \frac{\phi_1}{\phi_2}. \tag{16}$$

And then, the transformation equation (14) is equivalent to

$$\phi \to \phi, \qquad \theta \to \theta - \mathrm{i}\alpha, \tag{17}$$

which means an internal 'imaginary motion'. From now on, we will use the new formalism (ϕ, θ) to describe the new non-canonical complex scalar field. Here, one may see that the name 'hessence' arises from the prefix 'h-' standing for 'hyperbolic' and the traditional suffix '-essence' for dark energy.

Here, let us pause before discussing some physical aspects of the hessence. Strictly speaking, by the Lagrangian density equation (11), the hessence is identified with the quintom given by equation (6). However, the potential of the hessence is imposed to depend only on the quantity $\Phi^2 + \Phi^{*2}$ or $\phi_1^2 - \phi_2^2$ or the more convenient ϕ. In this sense, the hessence is not equivalent to the quintom model proposed by Feng, Wang and Zhang [12, 16]. The quintom model of dark energy can be viewed as a realization of dark energy which makes the transition from $w > -1$ to $w < -1$ or vice versa possible. To implement this, one may employ two independent real scalar fields as the case of the toy model proposed in [12, 16], while one may also employ a single field whose potential has some internal symmetry, e.g. the hessence proposed in this paper. Secondly, because ϕ_1 and ϕ_2 are independent in the quintom model proposed in [12, 16], one may worry about the possibility of $\phi^2 = \phi_1^2 - \phi_2^2$ becoming negative when ϕ_1^2 is less than ϕ_2^2. However, we stress once again that the hessence cannot be identified with the quintom model proposed in [12, 16]. The possibility of ϕ^2 becoming negative never occurs in the hessence model, since $\phi_1^2 \geqslant \phi_2^2$ is ensured by definition, see equations (15) and (16). On the other hand, we recall that the equation-of-state parameter $w > -1$ or $w < -1$ depends on $\dot\phi_1$ and $\dot\phi_2$ rather than ϕ_1 and ϕ_2 themselves. Thus, we should not worry that the definitions of equations (15) and (16) may ruin the possibility of w crossing the phantom divide $w = -1$. Thirdly, in the hessence model, the potential is imposed to be the form of $V(\phi)$, or equivalently, $V\left(\phi_1^2 - \phi_2^2\right)$. Except for the very special case of $V(\phi) \sim \phi^2$, the ϕ_1 and ϕ_2 are coupled in general. This is different from the quintom model studied in [12, 15, 16, 23]. Finally, we admit that the Lagrangian density of the hessence has not been proposed in ordinary particle physics or field theory before, to our knowledge. And one may feel that it is difficult to understand the so-called internal 'imaginary motion' mentioned above. However, we argue that they are not the reasons that prevent us from the possibility of using the novel non-canonical complex scalar field, i.e. hessence, to describe the dark energy. We think that it is not strange to use a new field to understand the new and unknown object: dark energy. On the other hand, old conceptions should not smother any new idea while it may open a new window to look into the unknown internal world of the dark energy.

2.2. Formalism

Assuming that we have been tolerated to continue, let us restart our discussion with the action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(-\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_h + \mathcal{L}_m \right), \tag{18}$$

where the Lagrangian density of the hessence is given by

$$\mathcal{L}_h = \tfrac{1}{4}[(\partial_\mu\Phi)^2 + (\partial_\mu\Phi^*)^2] - U(\Phi^2 + \Phi^{*2}) = \tfrac{1}{2}[(\partial_\mu\phi)^2 - \phi^2(\partial_\mu\theta)^2] - V(\phi). \tag{19}$$

Considering a spatially flat FRW universe with metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\,\mathrm{d}\mathbf{x}^2, \tag{20}$$

where $a(t)$ is the scale factor, from equations (18) and (19), we obtain the equations of motion for $\phi(\mathbf{x}, t)$ and $\theta(\mathbf{x}, t)$, namely

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2}{a^2}\phi + \phi(\partial_\mu\theta)^2 + V'(\phi) = 0, \tag{21}$$

$$\phi^2\ddot{\theta} + (3H\phi^2 + 2\phi\dot{\phi})\dot{\theta} - \phi^2\frac{\nabla^2}{a^2}\theta - \frac{2\phi}{a^2}\partial_i\phi\partial_i\theta = 0, \tag{22}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, $\nabla^2 \equiv \partial_i\partial_i$, an overdot and a prime denote the derivatives with respect to cosmic time t and ϕ, respectively. If ϕ and θ are homogeneous, the above equations become

$$\ddot{\phi} + 3H\dot{\phi} + \phi\dot{\theta}^2 + V'(\phi) = 0, \tag{23}$$

$$\phi^2\ddot{\theta} + (2\phi\dot{\phi} + 3H\phi^2)\dot{\theta} = 0. \tag{24}$$

The pressure and energy density of the hessence are

$$p_h = \tfrac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi), \tag{25}$$

$$\rho_h = \tfrac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + V(\phi), \tag{26}$$

respectively. The corresponding equation-of-state parameter is given by

$$w = \frac{p_h}{\rho_h} = \frac{\frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi)}{\frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + V(\phi)}. \tag{27}$$

It is easy to see that $w \geqslant -1$ when $\dot{\phi}^2 \geqslant \phi^2\dot{\theta}^2$ and $w < -1$ when $\dot{\phi}^2 < \phi^2\dot{\theta}^2$. The Friedmann equations read

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + V(\phi)\right], \tag{28}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\frac{\rho_m}{2} + (\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi)\right], \tag{29}$$

where ρ_m is the energy density of dust matter.

It is worth noting that if $\dot{\theta}^2 \sim 0$, the hessence reduces to an ordinary quintessence. From equations (25)–(29), we can see that the phantom-like role is played by the internal motion $\dot{\theta}$. In addition, equation (24) implies

$$Q = a^3\phi^2\dot{\theta} = \mathrm{const} \tag{30}$$

which is associated with the total conserved charge within the physical volume. It turns out

$$\dot{\theta} = \frac{Q}{a^3\phi^2}. \tag{31}$$

Substituting into equations (23) and (25)–(29), we can recast them as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{Q^2}{a^6\phi^3} + V'(\phi) = 0, \tag{32}$$

$$p_h = \frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6\phi^2} - V(\phi), \qquad \rho_h = \frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6\phi^2} + V(\phi),$$
$$w = \left[\frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6\phi^2} - V(\phi)\right] \Bigg/ \left[\frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6\phi^2} + V(\phi)\right], \tag{33}$$

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6\phi^2} + V(\phi)\right],$$
$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\frac{\rho_m}{2} + \dot{\phi}^2 - \frac{Q^2}{a^6\phi^2} - V(\phi)\right]. \tag{34}$$

From equation (30), one finds that the sign of the conserved charge Q is determined by the sign of $\dot{\theta}$. The conserved charge Q is positive for the case of $\dot{\theta} > 0$ while Q is negative for the case of $\dot{\theta} < 0$. On the other hand, it is easy to see that the governing equations, namely equations (23) and (25)–(29) (or equations (32)–(34)), are the same for the cases of $\dot{\theta} > 0$ and $\dot{\theta} < 0$, since they depend on $\dot{\theta}^2$ or Q^2 rather than $\dot{\theta}$ or Q themselves. It is possible that there are *dark energy and anti-dark energy* with opposite conserved charges Q in the universe, just like electrons and positrons. This new discovery may have some interesting implications for cosmology. For example, one may develop some cosmological observations attempting to find the signals coming from the annihilation of dark energy and anti-dark energy. On the other hand, if such observations cannot find anti-dark energy, it means an asymmetry between dark energy and anti-dark energy, just as in the case of baryons and anti-baryons. Putting these two asymmetries together, may give a novel solution to baryogenesis. Besides, if the conserved charge of the dark energy (hessence) corresponds to a kind of long-range force, just like the repulsive force between an assembly of electrons, they repel each other. Therefore, it is easy to understand why the dark energy is spatially homogeneous and not clumped to form structures. The novel features of the hessence are quite interesting for cosmology. We regard these as a new window to look into the internal world of the mysterious dark energy. A deeper understanding to dark energy might be possible through this new window.

2.3. *Dynamics*

Obviously, from equation (33), the equation-of-state parameter $w \geqslant -1$ when $\dot{\phi}^2 \geqslant Q^2/(a^6\phi^2)$ while $w < -1$ when $\dot{\phi}^2 < Q^2/(a^6\phi^2)$. The transition occurs when $\dot{\phi}^2 = Q^2/(a^6\phi^2)$.

In fact, it is difficult to obtain the analytic solutions for the equation of motion of hessence. To see the dynamics of the hessence, we have to adopt a numerical approach. To this end, we recast equation (32) and the first Friedmann equation (34) as the following first-order differential equations with respect to the scale factor a

$$\frac{\mathrm{d}\phi}{\mathrm{d}a} = \frac{\chi}{aH}, \qquad \frac{\mathrm{d}\chi}{\mathrm{d}a} = \frac{-1}{aH}\left[3H\chi + \frac{Q^2}{a^6\phi^3} + V'(\phi)\right],$$

and

$$H^2 = \frac{1}{3}\left[\rho_m + \frac{1}{2}\chi^2 - \frac{Q^2}{2a^6\phi^2} + V(\phi)\right],$$

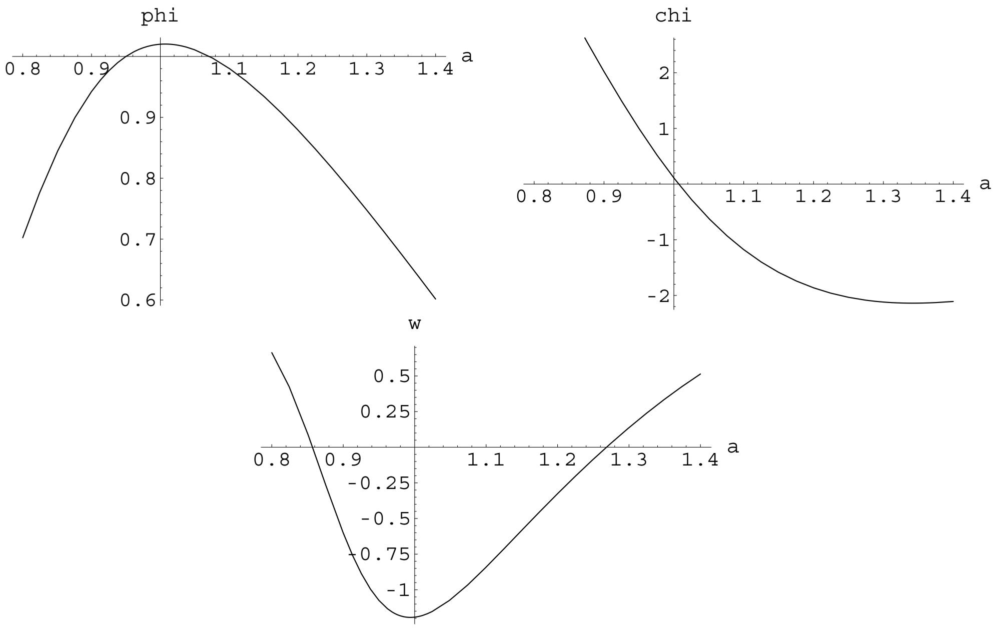

Figure 1. The numerical plots of ϕ, $\chi \equiv \dot{\phi}$ and the equation-of-state parameter w versus scale factor a for the $V(\phi) = \lambda\phi^4$ potential. We choose the demonstrative parameters as $\Omega_{m0} \equiv \rho_{m0}/(3H_0^2) = 0.3$, $\lambda = 5.0$, $Q = 1.0$. We set the scale factor $a_0 = 1$ and the unit $8\pi G = 1$. The equation-of-state parameter w goes beyond -1 at $a = 0.95$.

where $\chi \equiv \dot{\phi}$. For simplicity, we set the unit $8\pi G = 1$ in this subsection. The equation-of-state parameter is given by

$$w = \left[\frac{1}{2}\chi^2 - \frac{Q^2}{2a^6\phi^2} - V(\phi)\right] \Bigg/ \left[\frac{1}{2}\chi^2 - \frac{Q^2}{2a^6\phi^2} + V(\phi)\right].$$

We consider the case of minimal coupling between hessence and dust matter, thus

$$\rho_m = \rho_{m0}a^{-3},$$

where the subscript '0' indicates the present value of the corresponding quantity. To be definite, we take the potential

$$V(\phi) = \lambda\phi^4$$

for example, while the case of $V(\phi) \sim \phi^2$ is trival. We show the numerical result in figure 1.

It is obvious that the equation-of-state parameter w can indeed cross the phantom divide $w = -1$. Although one may find that $w < -1$ is transient in the case of the ϕ^4 potential, it is worth noting that the behaviour of w depends heavily on the form of the potential $V(\phi)$. The case of ϕ^4 potential presented here is only a naive demonstration. One can build a more realistic hessence dark energy model to fit the observation data by choosing a proper potential.

3. Free of the Q-ball formation

The formation of Q-balls is very generic for a complex field (see [22] for example). A Q-ball is a kind of nontopological soliton whose stability is guaranteed by some conserved charge Q [21]. In the case of spintessence [17–19], which is a canonical complex scalar field mentioned above, it is difficult to avoid Q-ball formation [17, 19]. Except in some special cases of spintessence with an unnatural potential, the fluctuations grow exponentially and go nonlinearly to form Q-balls. Once the Q-balls are formed, they will act as (dark) matter whose energy density decreases as a^{-3}. As for the late-time fate of the Q-balls, it depends on the shape of the potential, and they can be stable to be dark matter, or decay into other particles like radiations whose energy density decreases as a^{-4}. Therefore, the spintessence cannot be a viable candidate for the dark energy (as pointed out in [17], however, the spintessence may be a good candidate for cold dark matter). As a non-canonical complex scalar field, the hessence faces a similar situation. Fortunately, note that the terms $\dot{\theta}^2$ in the equations of spintessence correspond to $-\dot{\theta}^2$ in our hessence case, and that this term is crucial in the criterion for Q-ball formation [19]. Thus, we are optimistic in expecting that the hessence can avoid this difficulty by the help of the negative sign. It turns out that it is true.

We now consider the growth of perturbations in the hessence. Following [19], we assume that the gravity effect is weak, which is a good approximation here. Thus, we do not consider the metric perturbation arising from the fluctuations in the hessence and surrounding matter. Substituting $\phi(\mathbf{x},t) = \phi(t) + \delta\phi(\mathbf{x},t)$ and $\theta(\mathbf{x},t) = \theta(t) + \delta\theta(\mathbf{x},t)$ into the equations of motion of ϕ and θ, i.e. equations (21) and (22), and linearizing the resulting equations, we obtain

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + 2\phi\dot{\theta}\delta\dot{\theta} + \dot{\theta}^2\delta\phi + V''(\phi)\delta\phi - \frac{1}{a^2}\nabla^2\delta\phi = 0, \tag{35}$$

$$\phi^2\delta\ddot{\theta} + 3H\phi^2\delta\dot{\theta} + 2\phi(\dot{\phi}\delta\dot{\theta} + \dot{\theta}\delta\dot{\phi}) - 2\dot{\phi}\dot{\theta}\delta\phi - \frac{\phi^2}{a^2}\nabla^2\delta\theta = 0, \tag{36}$$

for fluctuations. We seek for the solutions in the form

$$\delta\phi = \delta\phi_0\, \mathrm{e}^{\omega t + \mathrm{i}\mathbf{k}\cdot\mathbf{x}}, \qquad \delta\theta = \delta\theta_0\, \mathrm{e}^{\omega t + \mathrm{i}\mathbf{k}\cdot\mathbf{x}}. \tag{37}$$

If ω is real and positive, these fluctuations grow exponentially and go nonlinearly to form Q-balls. Substituting equation (37) into equations (35) and (36), one has

$$\left[\omega^2 + 3H\omega + \dot{\theta}^2 + V''(\phi) + \frac{k^2}{a^2}\right]\delta\phi_0 + 2\omega\phi\dot{\theta}\delta\theta_0 = 0, \tag{38}$$

$$2\dot{\theta}(\phi\omega - \dot{\phi})\delta\phi_0 + \left(\phi^2\omega^2 + 3H\phi^2\omega + 2\phi\dot{\phi}\omega + \phi^2\frac{k^2}{a^2}\right)\delta\theta_0 = 0. \tag{39}$$

The condition for nontrivial $\delta\phi_0$ and $\delta\theta_0$ is given by

$$\left[\omega^2 + 3H\omega + \dot{\theta}^2 + V''(\phi) + \frac{k^2}{a^2}\right] \times \left(\phi^2\omega^2 + 3H\phi^2\omega + 2\phi\dot{\phi}\omega + \phi^2\frac{k^2}{a^2}\right) = 4\omega\phi\dot{\theta}^2(\phi\omega - \dot{\phi}). \tag{40}$$

Assuming that the cosmological expansion effect is negligible, we pay special attention to the case with a rapidly varying θ since the hessence reduces to the quintessence as $\dot{\theta}^2 \sim 0$ mentioned above, namely $H \sim 0$ and $\phi \sim$ const. In this case, condition (40) becomes

$$\omega^4 + \left(2\frac{k^2}{a^2} + V'' - 3\dot{\theta}^2\right)\omega^2 + \left(\frac{k^2}{a^2} + V'' + \dot{\theta}^2\right)\frac{k^2}{a^2} = 0. \tag{41}$$

We find that the Jeans wavenumber k_J at which $\omega^2 = 0$ is given by

$$\frac{k_J^2}{a^2} = -\dot{\theta}^2 - V''. \tag{42}$$

If the Jeans wavenumber exists, the instability band is

$$0 < \frac{k^2}{a^2} < \frac{k_J^2}{a^2}. \tag{43}$$

However, it is easy to see that if

$$\dot{\theta}^2 + V'' \geqslant 0, \tag{44}$$

the instability band does not exist. Then the Q-balls cannot be formed. Condition (44) is not difficult to satisfy for many potentials, such as $V(\phi) = V_0(\phi_0/\phi)^n$, $V(\phi) = V_0[\exp(\phi_0/\phi) - 1]$, $V(\phi) = V_0 \exp(-\lambda\phi)$ etc [20]. It is easy to see that the negative sign in front of $\dot{\theta}^2$ in equation (42) is crucial to prevent Q-ball formation [17, 19]. On the other hand, $w < -1/3$ does not restrict $V'' < 0$ for our case considered here, unlike the case of spintessence. After all, as illustrated in [17, 20], the essential information about the behaviour of perturbations for the hessence should still be valid in the full-blown relativistic analysis.

4. Hessence and Chaplygin gas

In [26], the so-called (generalized) Chaplygin gas was studied as an alternative to quintessence. Actually, the inhomogeneous (generalized) Chaplygin gas may be an unification of dark energy and dark matter. Furthermore, it was found that the (generalized) Chaplygin gas can arise from brane, quintessence, tachyon etc. In addition, the (generalized) Chaplygin gas can also be described by a formalism of canonical complex scalar field [26]. Thus, it is interesting to find the possible relation between the hessence and the Chaplygin gas. In this section, we will show that, by choosing a proper potential, the hessence can be described by a Chaplygin gas at late times.

The Chaplygin gas is an exotic fluid described by the equation of state

$$p = -\frac{A}{\rho}, \tag{45}$$

where A is a positive constant. It can be generalized to the so-called generalized Chaplygin gas whose equation of state is given by

$$p = -\frac{A}{\rho^\beta}, \tag{46}$$

where β is also a positive constant. Using the relativistic energy–momentum conservation equation

$$\dot{\rho} + 3H(p + \rho) = 0, \tag{47}$$

one has

$$\rho = \left[A - \frac{B}{a^{3(1+\beta)}}\right]^{1/(1+\beta)}, \tag{48}$$

where B is an integration constant. To find the possible relation between the hessence and the Chaplygin gas, we set

$$p_h = -\frac{A}{\rho_h^\beta}, \tag{49}$$

where ρ_h is assumed to have form (48). From equation (33), we get

$$2V(\phi) = \rho_h - p_h = \rho_h + \frac{A}{\rho_h^\beta}, \tag{50}$$

$$\dot{\phi}^2 - \frac{Q^2}{a^6\phi^2} = \rho_h + p_h = \rho_h - \frac{A}{\rho_h^\beta}. \tag{51}$$

Considering a hessence-dominated universe, the Friedmann equation reads

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_h. \tag{52}$$

Substituting $\dot{\phi} = \dot{a}(\mathrm{d}\phi/\mathrm{d}a)$ and equations (48) and (52) into equation (51), we can recast it as a differential equation of ϕ with respect to a. In principle, we can solve it and obtain $\phi(a)$. Then we get $V(\phi)$ from equation (50) by using $\phi(a)$ and equation (48). However, we find that it is difficult to solve $\phi(a)$ for the $Q \neq 0$ and/or $\beta \neq 1$ case. As mentioned above, the $Q = 0$ case is trivial since the hessence reduces to the quintessence. To find a sensible solution, we consider the case with a rapidly varying θ, namely,

$$\dot{\phi}^2 \ll \frac{Q^2}{a^6\phi^2}, \tag{53}$$

and $\beta = 1$ (see equation (45)). In this case, equations (48) and (51) become

$$\rho_h = \sqrt{A - \frac{B}{a^6}}, \tag{54}$$

$$-\frac{Q^2}{a^6\phi^2} = \rho_h - \frac{A}{\rho_h}. \tag{55}$$

It is easy to get $\rho_h(\phi)$ and $a(\phi)$ as

$$\rho_h = \frac{B\phi^2}{Q^2}, \qquad a = \left(\frac{BQ^4}{AQ^4 - B^2\phi^4}\right)^{1/6}. \tag{56}$$

It is worth noting that to get a real and positive scale factor a and positive energy density, the constant B must satisfy $B > 0$. In addition, one can see from equation (54) that it is valid only when $Aa^6 \geqslant B$. Further one can get $\phi(a)$ as

$$\phi = \pm\left(A - \frac{B}{a^6}\right)^{1/4} \frac{Q}{\sqrt{B}}. \tag{57}$$

From equation (50), we obtain the corresponding potential

$$V(\phi) = \frac{B\phi^2}{2Q^2} + \frac{AQ^2}{2B\phi^2}, \tag{58}$$

which is quite a simple form.

Finally, we will show the compatibility of the calculations above. By using $\dot{\phi} = \dot{a}(\mathrm{d}\phi/\mathrm{d}a)$ and equations (52), (54) and (57), we have

$$\dot{\phi}^2 = \frac{6\pi GBQ^2}{a^{12}(A - B/a^6)}. \tag{59}$$

On the other hand, from equation (57),

$$\frac{Q^2}{a^6\phi^2} = \frac{B}{a^6\sqrt{A - B/a^6}}. \tag{60}$$

Obviously, condition (53) is satisfied provided that the scale factor a is large. That is, it is at late times.

5. Summary and discussions

In summary, we propose a non-canonical complex scalar field, which we dub 'hessence', to implement the concept of quintom dark energy whose equation-of-state parameter w can cross the phantom divide $w = -1$. In the hessence model, the dark energy is described by a single field with an internal degree of freedom rather than two independent real scalar fields. Furthermore, the hessence has imposed an internal symmetry and then it has a conserved charge. We develop a new formalism to describe the new non-canonical complex scalar field, i.e. hessence. We find that in the hessence model, the phantom-like role is played by the internal motion. We regard this hessence model as a new window to look into the unknown internal world of the mysterious dark energy. In addition, we show that the hessence can avoid the difficulty of Q-ball formation which gives trouble for spintessence. Furthermore, we find that, by choosing a proper potential, the hessence can be described by a Chaplygin gas at late times.

Although the cosmological evolution of the quintom model proposed in [12] was studied by Guo *et al* [16], we find that it is still interesting to investigate the cosmological evolution of the hessence. In fact, the authors of [16] only considered the special case whose potential $V(\phi_1, \phi_2)$ can be decomposed into $V(\phi_1) + V(\phi_2)$, namely the case in which there is no direct coupling between ϕ_1 and ϕ_2. However, in the hessence model, the potential is imposed to be of the form of $V(\phi)$, or equivalently, $V\left(\phi_1^2 - \phi_2^2\right)$. Except for the very special case of $V(\phi) \sim \phi^2$, the ϕ_1 and ϕ_2 are coupled in general. Therefore it is of interest to further investigate the cosmological evolution of the hessence. Besides, there are some interesting open questions, such as

- Can the hessence arise from a more fundamental theory, such as string/M theory or a braneworld model?
- How to construct the quantum field theory for the hessence? As a non-canonical complex scalar field, its Lagrangian density never appeared in ordinary particle physics and quantum field theory, to our knowledge.
- What role may the conserved charge of the hessence play in cosmology? Can we imagine the novel possibility of 'dark energy and anti-dark energy' and so on?
- Is the hessence stable at the quantum level? While the phantom is not stable at this level [8, 34], one may worry about the hessence since it contains a phantom-like ingredient.

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Note added. After our paper was submitted, some papers concerning this issue appeared in the arXiv preprints [27–32, 36]. In particular, the data-fit of [32] shows that the SNe Ia Gold dataset favours the equation-of-state parameter w crossing the phantom divide $w = -1$. In [27], the quintom model with the special case of interaction $V_{int}(\phi_1, \phi_2) \sim \sqrt{V(\phi_1)V(\phi_2)}$ has been studied. On the other hand, a novel single real scalar field model with w crossing -1 has been proposed in [30], whose Lagrangian density contains a second-order differential term of the scalar field (to break the fourth condition of the no-go theorem [14] mentioned above). This kind of Lagrangian density can be used to drive the so-called B-inflation [33] also. Furthermore, it was shown in [36] that w crossing -1 is possible without introducing any phantom component in a Gauss–Bonnet braneworld with induced gravity.

References

[1] Riess A G *et al* (Supernova Search Team Collaboration) 2004 *Astrophys. J.* **607** 665 (*Preprint* astro-ph/0402512)
Knop R A *et al* (Supernova Cosmology Project Collaboration) 2003 *Astrophys. J.* **598** 102 (*Preprint* astro-ph/0309368)
Riess A G *et al* (Supernova Search Team Collaboration) 1998 *Astron. J.* **116** 1009 (*Preprint* astro-ph/9805201)
Perlmutter S *et al* (Supernova Cosmology Project Collaboration) 1999 *Astrophys. J.* **517** 565 (*Preprint* astro-ph/9812133)

[2] Bennett C L *et al* 2003 *Astrophys. J. Suppl.* **148** 1 (*Preprint* astro-ph/0302207)
Spergel D N *et al* 2003 *Astrophys. J. Suppl.* **148** 175 (*Preprint* astro-ph/0302209)

[3] Tegmark M *et al* (SDSS Collaboration) 2004 *Phys. Rev.* D **69** 103501 (*Preprint* astro-ph/0310723)
Abazajian K *et al* (SDSS Collaboration) 2004 *Preprint* astro-ph/0410239
Abazajian K *et al* (SDSS Collaboration) 2004 *Preprint* astro-ph/0403325
Abazajian K *et al* (SDSS Collaboration) 2003 *Preprint* astro-ph/0305492
Tegmark M *et al* (SDSS Collaboration) 2003 *Astrophys. J.* **606** 702 (*Preprint* astro-ph/0310725)

[4] Allen S W, Schmidt R W, Ebeling H, Fabian A C and van Speybroeck L 2004 *Mon. Not. R. Astron. Soc.* **353** 457 (*Preprint* astro-ph/0405340)

[5] Caldwell R R, Dave R and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582 (*Preprint* astro-ph/9708069)
Wetterich C 1988 *Nucl. Phys.* B **302** 668
Peebles P J E and Ratra B 1988 *Astrophys. J.* **325** L17
Steinhardt P J, Wang L M and Zlatev I 1999 *Phys. Rev.* D **59** 123504 (*Preprint* astro-ph/9812313)

[6] DeBenedictis A, Das A and Kloster S 2004 *Gen. Rel. Grav.* **36** 2481 (*Preprint* gr-qc/0402047)

[7] Caldwell R R 2002 *Phys. Lett.* B **545** 23 (*Preprint* astro-ph/9908168)
Caldwell R R, Kamionkowski M and Weinberg N N 2003 *Phys. Rev. Lett.* **91** 071301 (*Preprint* astro-ph/0302506)

[8] Carroll S M, Hoffman M and Trodden M 2003 *Phys. Rev.* D **68** 023509 (*Preprint* astro-ph/0301273)

[9] Onemli V K and Woodard R P 2002 *Class. Quantum Grav.* **19** 4607 (*Preprint* gr-qc/0204065)
Onemli V K and Woodard R P 2004 *Phys. Rev.* D **70** 107301 (*Preprint* gr-qc/0406098)
Brunier T, Onemli V K and Woodard R P 2005 *Class. Quantum Grav.* **22** 59 (*Preprint* gr-qc/0408080)

[10] Huterer D and Cooray A 2005 *Phys. Rev.* D **71** 023506 (*Preprint* astro-ph/0404062)
Alam U, Sahni V and Starobinsky A A 2004 *J. Cosmol. Astropart. Phys.* JCAP06(2004)008 (*Preprint* astro-ph/0403687)
Wang Y and Tegmark M 2005 *Phys. Rev.* D **71** 103513 (*Preprint* astro-ph/0501351)

[11] Upadhye A, Ishak M and Steinhardt P J 2004 *Preprint* astro-ph/0411803

[12] Feng B, Wang X L and Zhang X M 2005 *Phys. Lett.* B **607** 35 (*Preprint* astro-ph/0404224)

[13] Armendariz-Picon C, Mukhanov V and Steinhardt P J 2000 *Phys. Rev. Lett.* **85** 4438 (*Preprint* astro-ph/0004134)
Armendariz-Picon C, Mukhanov V and Steinhardt P J 2001 *Phys. Rev.* D **63** 103510 (*Preprint* astro-ph/0006373)
Chiba T, Okabe T and Yamaguchi M 2000 *Phys. Rev.* D **62** 023511 (*Preprint* astro-ph/9912463)

[14] Vikman A 2005 *Phys. Rev.* D **71** 023515 (*Preprint* astro-ph/0407107)

[15] Hu W 2005 *Phys. Rev.* D **71** 047301 (*Preprint* astro-ph/0410680)

[16] Guo Z K, Piao Y S, Zhang X M and Zhang Y Z 2005 *Phys. Lett.* B **608** 177 (*Preprint* astro-ph/0410654)

[17] Boyle L A, Caldwell R R and Kamionkowski M 2002 *Phys. Lett.* B **545** 17 (*Preprint* astro-ph/0105318)

[18] Gu J A and Hwang W Y 2001 *Phys. Lett.* B **517** 1 (*Preprint* astro-ph/0105099)

[19] Kasuya S 2001 *Phys. Lett.* B **515** 121 (*Preprint* astro-ph/0105408)

[20] Li X Z, Hao J G and Liu D J 2002 *Class. Quantum Grav.* **19** 6049 (*Preprint* astro-ph/0107171)

[21] Coleman S R 1985 *Nucl. Phys.* B **262** 263
Coleman S R 1986 *Nucl. Phys.* B **269** 744 (Erratum)

[22] Kasuya S and Kawasaki M 2000 *Phys. Rev.* D **61** 041301 (*Preprint* hep-ph/9909509)
Kasuya S and Kawasaki M 2000 *Phys. Rev.* D **62** 023512 (*Preprint* hep-ph/0002285)
Kasuya S and Kawasaki M 2000 *Phys. Rev. Lett.* **85** 2677 (*Preprint* hep-ph/0006128)
Kasuya S and Kawasaki M 2001 *Phys. Rev.* D **64** 123515 (*Preprint* hep-ph/0106119)

[23] Caldwell R R and Doran M 2005 *Preprint* astro-ph/0501104

[24] Nojiri S, Odintsov S D and Tsujikawa S 2005 *Preprint* hep-th/0501025

[25] Wei Y H and Zhang Y Z 2003 *Grav. Cosmol.* **9** 307 (*Preprint* astro-ph/0402515)
Wei Y H and Tian Y 2004 *Class. Quantum Grav.* **21** 5347 (*Preprint* gr-qc/0405038)
Wei Y H *Preprint* gr-qc/0410050

[26] Kamenshchik A Y, Moschella U and Pasquier V 2001 *Phys. Lett.* B **511** 265 (*Preprint* gr-qc/0103004)
Bilic N, Tupper G B and Viollier R D 2002 *Phys. Lett.* B **535** 17 (*Preprint* astro-ph/0111325)

Bento M C, Bertolami O and Sen A A 2002 *Phys. Rev.* D **66** 043507 (*Preprint* gr-qc/0202064)
Bertolami O 2004 *Preprint* astro-ph/0403310
Gorini V, Kamenshchik A, Moschella U and Pasquier V 2004 *Preprint* gr-qc/0403062)
[27] Zhang X F, Li H, Piao Y S and Zhang X M 2005 *Preprint* astro-ph/0501652
[28] Wei Y H 2005 *Preprint* gr-qc/0502077
[29] McInnes B 2005 *Preprint* hep-th/0502209
[30] Li M Z, Feng B and Zhang X M 2005 *Preprint* hep-ph/0503268
[31] Stefancic H 2005 *Preprint* astro-ph/0504518
Nojiri S and Odintsov S D 2005 *Preprint* hep-th/0505215
[32] Lazkoz R, Nesseris S and Perivolaropoulos L 2005 *Preprint* astro-ph/0503230
[33] Anisimov A, Babichev E and Vikman A 2005 *Preprint* astro-ph/0504560
[34] Doran M and Jaeckel J 2002 *Phys. Rev.* D **66** 043519 (*Preprint* astro-ph/0203018)
[35] Elizalde E, Nojiri S and Odintsov S D 2004 *Phys. Rev.* D **70** 043539 (*Preprint* hep-th/0405034)
[36] Cai R G, Zhang H S and Wang A 2005 *Preprint* hep-th/0505186