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Inflation on a warped Dvali–Gabadadze–Porrati brane

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Abstract. We discuss an inflation model, in which the inflation is driven by a single scalar field with exponential potential on a warped DGP brane. In contrast to the power law inflation in the standard model, we find that the inflationary phase can exit spontaneously without any mechanism. The running of the index of the scalar perturbation spectrum can take a large enough value to match the observation data, while other parameters are in a reasonable region.

Keywords: cosmological constant experiments, inflation, physics of the early universe

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1. Introduction

Astronomical observations in recent years, especially by Wilkinson microwave anisotropy probe (WMAP) [1], lead to a high precision era of cosmology. All the results imply that there exists an accelerating moment (inflationary phase) in the very early universe. The angle power spectrum of the cosmological microwave background (CMB) provides some evidence that the universe is almost spatially flat and that large scale structure is formed from a primordial spectrum of adiabatic, normal and nearly Harrison–Zeldovich (scale invariant) density perturbations. This can be explained by the simplest model of inflation [2].

Despite the great success of the big bang standard model of cosmology together with inflation, there are still several serious problems in our present scenario of cosmology. The cosmological constant (dark energy), unexpected low power spectrum at large scales, and egregious running (to the common inflationary models) of power spectrum index are distinctive ones. The latter two might relate to very high energy physics. Although many approaches have been made in the literature, to be fair, these problems still persist.

In view of the achievements and shortcomings of the inflationary scenario in Einstein gravity, it is necessary to further deepen our understanding of the inflationary scenario from a theoretical perspective. In nonperturbative string/ M theories there are topological solitons, called branes, in ten- or 11-dimensional spacetime. In the Horava–Witten model [3], gauge fields of the standard model are confined on two nine-branes located at the end points of an S^1/Z_2 orbifold, i.e., a circle folded on itself across a diameter. Inspired by the Horava–Witten model, the idea that our universe is a three-brane embedded in a higher dimensional spacetime has received a great deal of attention in recent years [4]. In this brane world scenario, the standard model particles are confined on the three-brane, while the gravitation can propagate in the whole space. In this picture, the Friedmann equation of the brane universe is modified [5], compared to the one for the standard model. The chaotic inflation model on the RSII brane has been studied in [6]. The inflation model in the brane world scenario with a Gauss–Bonnet term in the bulk has also been discussed recently by Lidsey and Nunes [7].

In the RSII model [8], a brane with positive tension is embedded in five-dimensional anti-de Sitter space. Due to the warped effect of bulk geometry, general relativity on the brane can be recovered in low energies, while gravity on the brane is five dimensional in the high energy limit. Compared to the RSII model, the brane world model proposed by Dvali, Gabadadze and Porrati (DGP) [9] is also very interesting. In the DGP model, the bulk is a flat Minkowski spacetime, but a reduced gravity term appears on the brane without tension. In this model, gravity appears four dimensional at short distances but is altered at distances large compared to some freely adjustable crossover scale r_0 through the slow evaporation of the graviton off our four-dimensional brane world universe into an unseen, yet large, fifth dimension. The late time acceleration is driven by the manifestation of the excruciatingly slow leakage of gravity off our four-dimensional world into an extra dimension. This offers an alternative explanation for the current acceleration of the universe.

In the DGP model, the gravitational behaviours on the brane are commanded by the competition between the five-dimensional curvature scalar ${}^{(5)}R$ in the bulk and the four-dimensional curvature scalar R on the brane. At short distances the four-dimensional curvature scalar R dominates and ensures that gravity looks four dimensional. At large distances the five-dimensional curvature scalar ${}^{(5)}R$ takes over and gravity spreads into the extra dimension. As a result, the Newton-like force law becomes a five-dimensional one. Thus, gravity begins weaker at cosmic distances. So it is natural that such a dramatic modification affects the expansion velocity of the Universe. In fact the DGP model was applied to cosmology immediately after DGP put forward their model [10].

In this paper, we will study an inflation model in a brane world scenario, which combines the RSII model and the DGP model. That is, an induced curvature term will also appear on the brane in the RSII model. We call the brane a warped DGP brane. In this model, inflation of the universe is driven by a single scalar field with exponential potential on the brane. In contrast to the power law inflation in the standard model, we find that the inflationary phase exits spontaneously. The running of the index of the scalar perturbation spectrum can take negative values, which contain the high precision observations by WMAP. At the same time other parameters are in a reasonable region.

The organization of this paper is as follows. In the next section, we present the Friedmann equation on the warped DGP brane. In sections 3 and 4 the inflation model is introduced. We analyse the parameter space allowed by the observation data in section 5. The paper ends in section 6 with conclusions and discussions.

2. The model and Friedmann equation

We start from the action of the generalized DGP model given in [11]

$$S = S_{\text{bulk}} + S_{\text{brane}}, \quad (1)$$

where

$$S_{\text{bulk}} = \int_{\mathcal{M}} d^5 X \sqrt{-{}^{(5)}g} \left[\frac{1}{2\kappa_5^2} {}^{(5)}R + {}^{(5)}L_m \right], \quad (2)$$

and

$$S_{\text{brane}} = \int_M d^4 x \sqrt{-g} \left[\frac{1}{\kappa_5^2} K^\pm + L_{\text{brane}}(g_{\alpha\beta}, \psi) \right]. \quad (3)$$

Here κ_5^2 is the five-dimensional gravitational constant and ${}^{(5)}R$ and ${}^{(5)}L_m$ are the five-dimensional curvature scalar and the matter Lagrangian in the bulk, respectively. x^μ ($\mu = 0, 1, 2, 3$) are the induced four-dimensional coordinates on the brane, K^\pm is the trace of extrinsic curvature on either side of the brane and $L_{\text{brane}}(g_{\alpha\beta}, \psi)$ is the effective four-dimensional Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields ψ on the brane.

Consider the brane Lagrangian consisting of the following terms:

$$L_{\text{brane}} = \frac{\mu^2}{2}R - \lambda + L_m, \quad (4)$$

where μ is a parameter with dimension of $[mass]$, R denotes the curvature scalar on the brane, λ is the tension of the brane, and L_m stands for the Lagrangian of other matters on the brane. We assume that there is only a cosmological constant ${}^{(5)}\Lambda$ in the bulk. Therefore the action (1) describes a generalized DGP model or a generalized RSII model, since it goes to the DGP model if $\lambda = 0$ and ${}^{(5)}\Lambda = 0$, or to the RSII model if $\mu = 0$.

From the field equations induced on the brane given in [11], one can define an effective four-dimensional cosmological constant on the brane

$$\Lambda = \frac{1}{2}({}^{(5)}\Lambda + \frac{1}{6}\kappa_5^4\lambda^2), \quad (5)$$

which is the same as that in the RSII model. Considering a Friedmann–Robertson–Walker (FRW) metric on the brane, the dynamical equation of the brane, namely the Friedmann equation, is found to be [11]

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[\rho + \rho_0(1 + \epsilon\mathcal{A}(\rho, a)) \right], \quad (6)$$

where as usual k is the constant curvature of the maximal symmetric space of the FRW metric and ϵ denotes either $+1$ or -1 . \mathcal{A} is defined by

$$\mathcal{A} = \left[\mathcal{A}_0^2 + \frac{2\eta}{\rho_0} \left(\rho - \mu^2 \frac{\mathcal{E}_0}{a^4} \right) \right]^{1/2}, \quad (7)$$

where

$$\mathcal{A}_0 = \sqrt{1 - 2\eta \frac{\mu^2\Lambda}{\rho_0}}, \quad \eta = \frac{6m_5^6}{\rho_0\mu^2} \quad (0 < \eta \leq 1), \quad (8)$$

$$\rho_0 = m_\lambda^4 + 6 \frac{m_5^6}{\mu^2}. \quad (9)$$

Note that here there are three mass scales, $m_4 = \mu$, $m_\lambda = \lambda^{1/4}$, and $m_5 = \kappa_5^{-2/3}$. Further, due to the appearance of ϵ , there are two branches in this model, as in the original DGP model. \mathcal{E}_0 is a constant related to dark radiation [5]. Since we are interested in the inflation dynamics of the model, as usual, we neglect the curvature term and dark radiation term in what follows. Then the Friedmann equation (6) can be rewritten as

$$H^2 = \frac{1}{3\mu^2} \left[\rho + \rho_0 + \epsilon\rho_0 \left(\mathcal{A}_0^2 + \frac{2\eta\rho}{\rho_0} \right)^{1/2} \right]. \quad (10)$$

For the DGP model, one has $\eta = 1$ and $\mathcal{A}_0 = 1$. In the high energy limit $\rho/\rho_0 \gg 1$, the Friedmann equation reduces to

$$H^2 \approx \frac{1}{3\mu^2} (\rho + \epsilon\sqrt{2\rho\rho_0}). \quad (11)$$

This describes a four-dimensional gravity with a minor correction, which implies that the parameter μ must have an energy scale as the Planck scale: $\mu \sim 10^{18}$ GeV in the DGP model. On the other hand, in the low energy limit $\rho/\rho_0 \ll 1$, the Friedmann equation (10) becomes

$$H^2 \approx \frac{1}{3\mu^2} \left((1 + \epsilon)\rho + (1 + \epsilon)\rho_0 - \frac{\epsilon}{4} \frac{\rho^2}{\rho_0} \right). \quad (12)$$

When $\epsilon = 1$, the equation describes a four-dimensional gravity and the term ρ_0 gives an effective cosmological constant. To be consistent with the current observation, ρ_0 must be in the order of the current critical energy density of the universe, namely, $\rho_0 \sim (10^{-3} \text{ eV})^4$. When $\epsilon = -1$, however, the equation tells us that the gravity on the brane is five dimensional. This requires that at least $\rho_0 \sim (10^{-3} \text{ eV})^4$, once again.

For the warped DGP model with $\lambda \neq 0$ and ${}^5\Lambda \neq 0$, a remarkable point is that the two conditions $\mu \sim 10^{18}$ GeV and $\rho_0 \sim (10^{-3} \text{ eV})^4$ are not necessary. To see this, let us consider the limit of $\mu \rightarrow 0$ of equation (10). For simplicity, as in the RSII model, we set $\Lambda = 0$ in (5). In this case, we have $\mathcal{A}_0 = 1$ and $\eta \sim 1 - \mathcal{O}(\mu^2 m_\lambda^4 / m_5^6)$. In the ultra-high energy limit where $\rho \gg \rho_0 \gg m_\lambda^4$, the Friedmann equation (10) is

$$H^2 \approx \frac{1}{3\mu^2} (\rho + \epsilon \sqrt{2\rho\rho_0}). \quad (13)$$

This describes a four-dimensional gravity on the brane. In the intermediate energy region where $\rho \ll \rho_0$ but $\rho \gg m_\lambda^4$, for the branch with $\epsilon = -1$, the Friedmann equation changes to

$$H^2 \approx \frac{m_\lambda^4}{18m_5^6} \left(\rho + \frac{\rho^2}{2m_\lambda^4} - \frac{\mu^2 m_\lambda^4}{6m_5^6} \rho - \frac{\mu^2}{4m_5^6} \rho^2 \right). \quad (14)$$

This is just the Friedmann equation in the RSII model with some corrections proportional to μ^2 . When $\mu^2 = 0$, the Friedmann equation in the RSII model is restored. Finally, in the low energy limit where $\rho \ll \rho_0$ and $\rho \ll m_\lambda^4$, we know from the above equation that the four-dimensional general relativity with the Planck energy $m_p^2 = 6m_5^6/m_\lambda^4$ is recovered on the brane.

In this paper, we do not restrict ourselves to the small μ limit; instead, we take μ in the almost same order as the Planck scale, as in the original DGP model. Then in the high energy limit with $\rho \gg \rho_0$ we have the Friedmann equation

$$H^2 \approx \frac{1}{3\mu^2} (\rho + \epsilon \sqrt{2\rho\rho_0}), \quad (15)$$

while in the low energy limit with $\rho \ll \rho_0$, for the branch with $\epsilon = -1$, the Friedmann equation is

$$H^2 = \frac{1}{3\mu_p^2} \left(\rho + \mathcal{O} \left(\frac{\rho}{\rho_0} \right)^2 \right), \quad (16)$$

where $\mu_p^2 = \mu^2/(1 - \eta)$ is the effective four-dimensional Planck scale. As a result, due to the appearance of the tension λ , one need not take ρ_0 as a very low energy scale $\rho_0 \sim (10^{-3} \text{ eV})^4$ as it is in the original DGP model; instead, one can see from (15) and (16) that in the warped DGP model ρ_0 can be taken as at least $\rho_0 \sim (1 \text{ MeV})^4$, the BBN energy scale. Note that here the four-dimensional Planck energy scales change from the high energy limit (15) to the low energy limit (16).

In the following discussions, we will not take equation (15) in the high energy limit or equation (16) in the low energy limit as our starting point; instead, we will start from the complete equation (10). This equation (10) shows some new interesting features. A term proportional to $\rho^{1/2}$, in contrast to ρ^2 in the RSII model, appears. In particular, a noticeable point is that this term can be negative. We may expect that the inflationary behaviour on the brane will be significantly different from that in the standard model or RSII model. It turns out that this is true.

3. Inflationary phase

As a simple inflation model, we assume that the inflation is driven by a single scalar field ϕ with a potential $V(\phi)$ on the brane. As usual, the scale field is minimally coupled to the gravity field. The action of the scalar field then can be written down as

$$S_m = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \right). \quad (17)$$

Varying the action yields the equation of motion of the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \quad (18)$$

The slow roll parameters defined by

$$\begin{aligned} f &\triangleq -\frac{\dot{H}}{H^2}, \\ \alpha &\triangleq \frac{d^2V}{d\phi^2} \bigg/ (3H^2), \\ \xi^2 &\triangleq \frac{d^3V}{d\phi^3} \frac{dV}{d\phi} \bigg/ (3H^2)^2, \end{aligned} \quad (19)$$

can be expressed as

$$f = \frac{\mu^2 (dV/d\phi)^2}{2 V^2} \left(\frac{1 + \epsilon\eta(\mathcal{A}_0^2 + (2\eta\rho/\rho_0))^{1/2}}{\{1 + (\rho_0/\rho)[1 + \epsilon(\mathcal{A}_0^2 + (2\eta\rho/\rho_0))^{1/2}]\}^2} \right) \quad (20)$$

in the slow roll approximation, $(\dot{\phi})^2 \ll V$ and $\ddot{\phi} \ll |3H\dot{\phi}|$. The other two parameters can also be calculated similarly. The term in brackets is the correction to the standard model. It is easy to see that this term loosens the condition for inflation when $\epsilon = 1$ and tightens the condition when $\epsilon = -1$, in contrast to the RSII model where the correction term always loosens the inflation condition. The number of e-folds $N \triangleq \ln(a_{\text{end}}/a_0)$ can be expressed by

$$N = - \int_{\phi_i}^{\phi_{\text{end}}} 3H^2 \frac{d\phi}{dV} d\phi, \quad (21)$$

in the slow roll approximation, where ϕ_{end} is the value of ϕ when the universe exits from the inflationary phase and ϕ_i denotes the value of ϕ when the universe scale observed today crosses the Hubble horizon during inflation.

Next we investigate the scalar curvature perturbation of the metric. The perturbations generated quantum mechanically from a single field during inflation are

adiabatic. The curvature perturbation on a uniform density hypersurface is conserved on large scales as a result of energy–momentum conservation on the brane [12]. So we expect that, as in the RSII model, the scalar curvature perturbation amplitude of a given mode when re-entering the Hubble radius is still given by $A_S^2 = H^4/(25\pi^2\dot{\phi}^2)$ [2] (see also [7]). Substituting (18) with the slow roll approximation, we obtain

$$A_S^2 = \frac{1}{25\mu^6\pi^2} \frac{9H^6}{(dV/d\phi)^2} \Big|_{k=aH}. \quad (22)$$

The COBE normalization gives us $A_S^2 = 4 \times 10^{-10}$ [13]. With the definition of the scalar spectrum index

$$n_S = 1 + d \ln A_S^2 / d \ln k,$$

we have

$$n_S = 1 - 6f + 2\alpha, \quad (23)$$

and the running of the scalar spectrum index

$$\frac{dn_S}{d \ln k} = 16f\alpha - 24f^2 - 2\xi^2. \quad (24)$$

4. Exponential potential

In inflation models, the exponential potential is an important example which can be solved exactly in the standard model. In addition, we know that such exponential potentials of scalar fields occur naturally in some fundamental theories such as string/ M theories. On the other hand, we find that the inflation phase in the warped DGP model even with an exponential potential on the brane can exit naturally. It is therefore quite interesting to investigate such a potential in the warped DGP model, and to compare its predictions with observational data.

We write down the potential as

$$V = \tilde{V} e^{-\sqrt{2/p}(\phi/\mu)}, \quad (25)$$

where \tilde{V} and p are two constants. Calculating the slow roll parameters, we have

$$f = \frac{1}{p} \frac{1 + \epsilon\eta(\mathcal{A}_0^2 + 2\eta x)^{-1/2}}{\{1 + (1/x)[1 + \epsilon(\mathcal{A}_0^2 + 2\eta x)^{1/2}]\}^2}, \quad (26)$$

$$\alpha = \frac{2}{p} \frac{x}{x + 1 + \epsilon(\mathcal{A}_0^2 + 2\eta x)^{1/2}}, \quad (27)$$

$$\xi^2 = \alpha^2, \quad (28)$$

where $x = V/\rho_0$. We know that when $f = 1$, the inflation ends. If the universe can enter the inflationary phase, i.e., $p > 2$, it cannot exit in the branch of $\epsilon = 1$. But we find that the f in (26) can increase to one from a value which is less than one in the branch of $\epsilon = -1$. It implies that the inflationary phase can exit naturally without any other mechanism in the branch of $\epsilon = -1$. So in what follows, we only consider the branch of $\epsilon = -1$. However, the equation $f = 1$, namely,

$$\frac{1}{p} \frac{1 - \eta(\mathcal{A}_0^2 + 2\eta x)^{-1/2}}{\{1 + (1/x)[1 - (\mathcal{A}_0^2 + 2\eta x)^{1/2}]\}^2} = 1 \quad (29)$$

is a quintic equation, whose roots cannot be written down in a finite algebraic way. Thus we will give some numerical results in parameter space. To show that equation (29) could indeed hold, let us first consider a special case, namely the original DGP model where $\eta = \mathcal{A}_0 = 1$. In this case, we have

$$\lim_{x \rightarrow 0} f \rightarrow \infty, \quad (30)$$

and

$$\lim_{x \rightarrow \infty} f = \frac{1}{p}. \quad (31)$$

Because f is a continuous function of x , there must exist an $x \in (0, \infty)$ which satisfies $f = 1$ for $p > 1$. So indeed in the original DGP model the inflation phase can exit spontaneously. But we find that this happens around $x \sim 1$. Since $\rho_0 \sim (10^{-3} \text{ eV})^4$ in the original DGP model, the inflation would last to such a low energy scale. This is obviously unreasonable. In the warped DGP model, however, we find that this difficulty can be avoided.

Integrating equation (21), we obtain

$$N = -\frac{p}{2} \left(\ln x - x^{-1} + \left(Wx^{-1} + \frac{2\eta}{\mathcal{A}_0} \tanh^{-1} \left(\frac{W}{\mathcal{A}_0} \right) \right) \right) \Big|_{x_i}^{x_{\text{end}}}, \quad (32)$$

where $W = (\mathcal{A}_0^2 + 2\eta x)^{1/2}$, x_i stands for the value of x when the cosmic scale observed today crosses the Hubble horizon during inflation. Denote

$$h(x) = - \left(\ln x - x^{-1} + Wx^{-1} + \frac{2\eta}{\mathcal{A}_0} \left[\tanh^{-1} \left(\frac{W}{\mathcal{A}_0} \right) + i \frac{\pi}{2} \right] \right). \quad (33)$$

Equation (32) can be expressed as

$$h(x_{\text{end}}) = h(x_i) + \frac{2N}{p}. \quad (34)$$

Next from the scalar curvature perturbation amplitude equation (22), we can get a constraint among V , ρ_0 and p

$$\frac{p}{150\pi^2 V^2 \mu^4} \left(V + \rho_0 + \epsilon \rho_0 \left(\mathcal{A}_0^2 + \frac{2\eta V}{\rho_0} \right)^{1/2} \right)^3 \Big|_{k=aH} = 4 \times 10^{-10}. \quad (35)$$

Substituting (20) into equations (23) and (24), we arrive at

$$n_s = 1 - \frac{2 \mathcal{A}_0^2 + \eta x + (-2 + x)W}{p W(1 + x - W)}, \quad (36)$$

$$\frac{dn_s}{d \ln k} = \frac{8 x^2 \{ -\mathcal{A}_0^4 + \mathcal{A}_0^2 [-1 + 2x - 2(-1 + x)W] + \eta x [-2 + x(4 + \eta - 2W)] \}}{p^2 W^2 (1 + x - W)^4}, \quad (37)$$

respectively. Note that when $x \rightarrow \infty$, the results reduce to those of the standard model, as we expected.

5. Analysis of the parameter space

For the sake of demonstration, we give an concrete example to show some interesting properties of this inflationary model on the warped DGP brane. Because equation (34) of the number of e-folds is very complicated, we add an additional assumption that we take $\eta = 0.99$, although our method can be used for any value of η . As we mentioned above, the inflation phase can exit naturally in this model. For example, numerical analysis shows that the equation $f = 1$ has a positive real root of x if $\eta \in (0.96, 1]$ as $p \in [20, 25]$, while if $\eta \in (0.97, 1]$ as $p \in [25, 30]$. Below we present comprehensively studies on the relations of several constraint equations and parameter space. For simplicity, we take $\Lambda = 0$ in (5), which implies $\mathcal{A}_0 = 1$. From (29) we can find a reasonable finite algebraic root on x , but it is quite involved. We do not present its expression here; instead, we only express it formally

$$x_{\text{end}} = x_{\text{end}}(p). \quad (38)$$

From the scalar perturbation amplitude constraint equation (35) we get

$$x_i = \frac{2(-1 + c + \eta)}{(-1 + c)^2}, \quad (39)$$

where $c = y^{-1/3}(6 \times 10^{-7}/p)^{1/3}$, and $y = V_i/\mu^4$. Here the subscript i means to take the value at $k = aH$. Then equation (34) of the number of e-folds can be re-expressed as

$$h(x_{\text{end}}(p)) - h(x_i(p, y)) = \frac{2N}{p}. \quad (40)$$

We note that there is a singularity in the function of $h(y, p)$,

$$\lim_{y \rightarrow y_s} h(y, p) \rightarrow -\infty, \quad (41)$$

where

$$y_s = 6 \times 10^{-7}/p. \quad (42)$$

This singularity appears when $x_i = \infty$ in (39). The singularity equation (41) imposes a restriction on the energy scale of inflation: $y < y_s$. Otherwise, one may get a non-physical result: a negative number of e-folds if one takes $y > y_s$. For a given p and a required N , we always can get a y (which satisfies $y < y_s$), so that equation (40) holds.

In figure 1 we draw some contours of the number N of e-folds through the relation (40) with respect to p and x_i . Note that here x_i is determined by y and p through (39). From the figure we can read the direct relation of x_i and p for a given N . For instance the curve just below the notation ‘ $N = 60$ ’ gives the relation of p to x_i for the case of $N = 60$. In figure 2 we plot the relation among the parameters y , x_i and p through (39), which shows us the energy scale of inflation. Note that in these figures $\eta = 0.99$ is taken. In this case, one has $\mu^2 = 0.01\mu_p^2 \sim (10^{17} \text{ GeV})^2$. So the energy scale of inflation in this setting is $\rho_i \sim (10^{15} \text{ GeV})^4$, a reasonable inflation scale. In figure 3 we plot the energy scale of inflation with respect to N and p .

Now we consider a special case with fixed $N = 60$ and $p = 50$. In this case, we have $x_{\text{end}} = 0.05$, $x_i = 36$, and $y = 2.4 \times 10^{-8}$. Furthermore we find $\lambda/\mu^4 = 6.7 \times 10^{-12}$, $m_5/\mu = 0.02$, and ${}^{(5)}\Lambda/\mu^2 = 6.7 \times 10^{-14}$. In the Planck unit $\mu_p \sim 10^{18} \text{ GeV}$, they change to $\lambda/\mu_p^4 = 6.7 \times 10^{-16}$, $m_5/\mu_p = 0.002$, and ${}^{(5)}\Lambda/\mu_p^2 = 6.7 \times 10^{-16}$. For the concrete model,

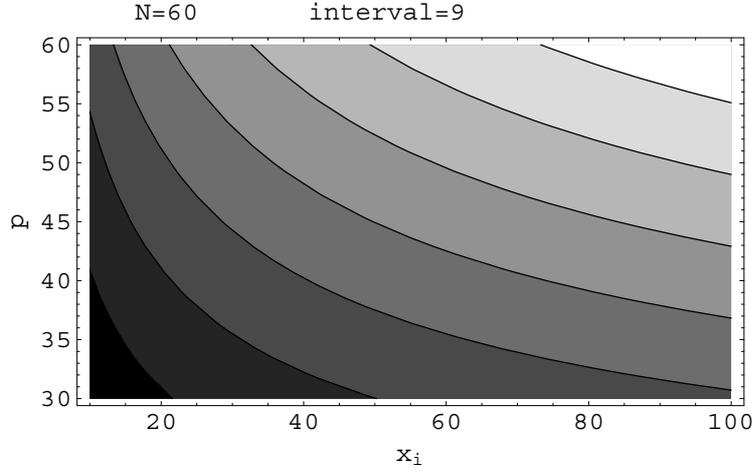


Figure 1. The contours of the number N of e-folds. The curve just below the notation ‘ $N = 60$ ’ is the one for the case of $N = 60$. The curves on the right hand side of that one represent cases with $N > 60$, and the interval is $\Delta N = 9$.

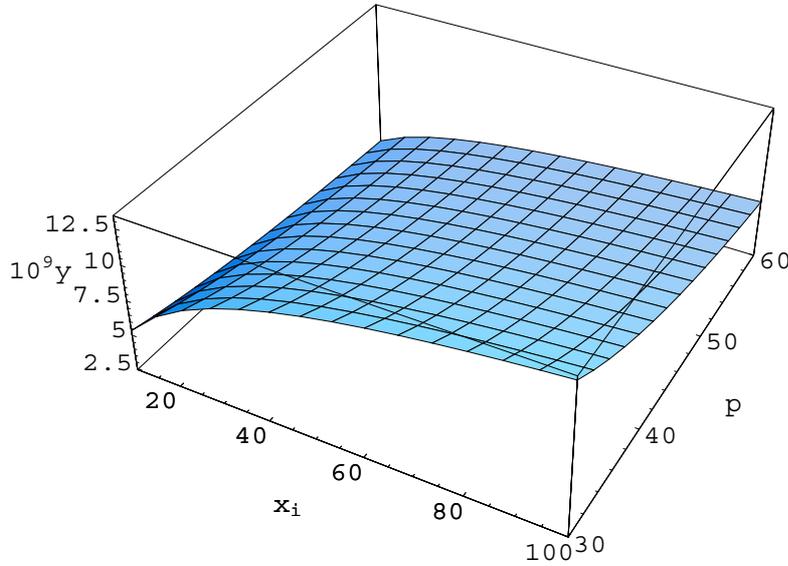


Figure 2. The inflation energy scale V_i/μ^4 versus the parameters x_i and p .

in figures 4–6 we display the slow roll parameters, scalar spectrum index and the running of scalar spectrum index, respectively. Indeed, these quantities are in good agreement with the observation data [1].

6. Conclusion and discussion

Various inflationary models have been proposed since Guth’s seminal work [14]. Going with the observations of high precision, we realize that a successful inflation model must at least possess the following properties: (1) a sufficient large number of e-folds, (2) a near

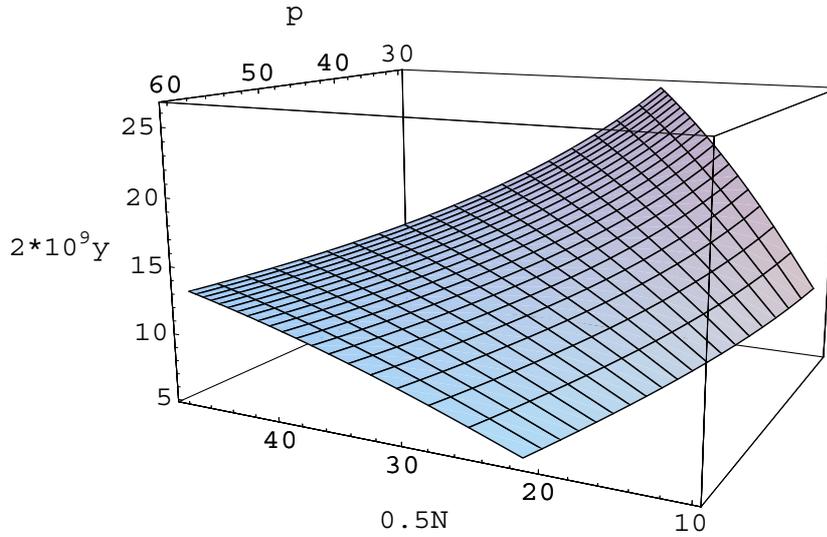


Figure 3. The inflation energy scale y versus the parameters N and p .

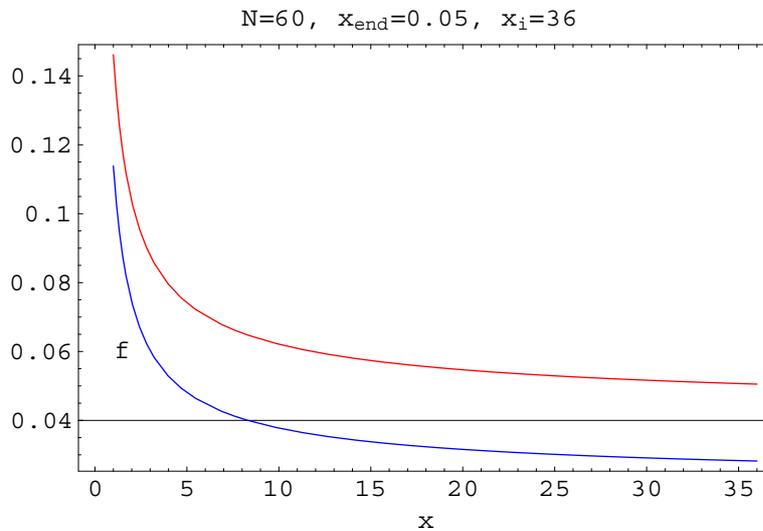


Figure 4. The slow roll parameters f and α versus x for the case of $N = 60$ and $p = 50$. The upper red curve represents α and the bottom blue one stands for f .

Harrison–Zeldovich (scale invariant) spectrum, (3) a negative running of spectrum index. The model we discussed in this paper contains these. In particular, it is worth noting that in this inflation model based on the warped DGP brane world scenario, even for an exponential potential, the inflationary phase can exit naturally, and within a reasonable parameter region the model can give us a negative running of the scalar spectrum index. By choosing more appropriate parameters in the model, we may obtain much better consistency with the current observation data. These features are quite attractive. Other properties of this model deserve further study. For example, it would be interesting to

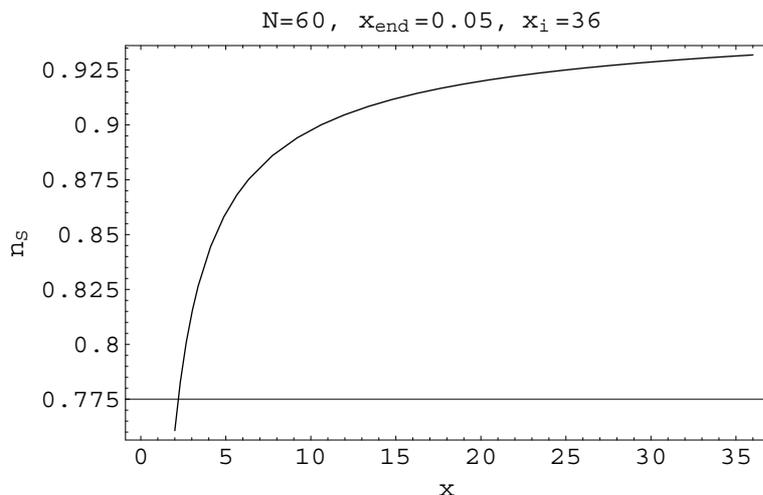


Figure 5. The scalar spectrum index n_s versus x for the case of $N = 60$ and $p = 50$.

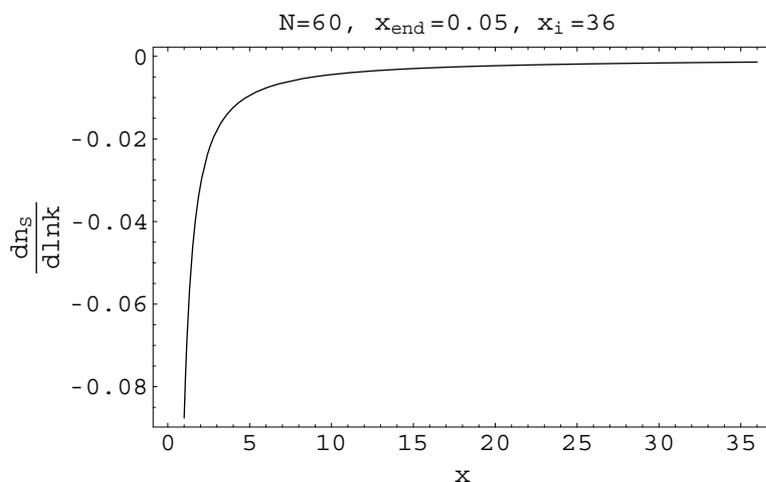


Figure 6. The running of the scalar spectrum index $dn_s/d \ln k$ versus x for the case of $N = 60$ and $p = 50$.

investigate carefully the relations of the spectrum index and its running to the comoving wavenumber k . In addition, in the warped DGP brane model, it is certainly of interest to further construct inflation models with scalar spectrum index larger than one at larger scales.

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Note added. While we were finishing this paper, a paper [15] appeared on the net, which also discusses an chaotic inflation model on a brane with induced gravity. More recently, two related and interesting papers have occurred on the net. In [16] the authors calculate the amplitude of gravitational waves from brane-world inflation with induced gravity, while the authors of [17] study the induced gravity with a non-minimally coupled scalar field on the brane.

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