



# Boosted domain wall and charged Kaigorodov space

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## Abstract

The Kaigorodov space is a homogeneous Einstein space and it describes a pp-wave propagating in anti-de Sitter space. It is conjectured in the literature that M-theory or string theory on the Kaigorodov space times a compact manifold is dual to a conformal field theory in an infinitely-boosted frame with constant momentum density. In this Letter we present a charged generalization of the Kaigorodov space by boosting a non-extremal charged domain wall to the ultrarelativity limit where the boost velocity approaches the speed of light. The finite boost of the domain wall solution gives the charged generalization of the Carter–Novotný–Horský metric. We study the thermodynamics associated with the charged Carter–Novotný–Horský space and discuss its relation to that of the static black domain walls and its implications in the domain wall/QFT (quantum field theory) correspondence.

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## 1. Introduction

In recent years there has been considerable interest in anti-de Sitter (AdS) space and asymptotically AdS space. This should be mainly attributed to the celebrated AdS/CFT (conformal field theory) correspondence [1], which says that there is a duality between M-theory or string theory on the AdS space times a compact space and a strongly coupling conformal field theory residing on the boundary of the AdS space. In the spirit of the AdS/CFT correspondence, it was convincingly argued by Witten [2] that the thermodynamics of black holes in AdS space can be identified with that of the dual CFT at high temperature. Further it

was proposed by de Boer et al. [3] that there is a correspondence between the classical evolution equations of bulk supergravity and the renormalization group equations of the dual boundary conformal field theory.

The Kaigorodov space [4] is a homogeneous Einstein space and it describes a pp-wave propagating in a four-dimensional AdS space [5]. Its higher-dimensional generalization was given in [6]. In that reference, it was found that the Kaigorodov space naturally appears as the near-horizon geometry of non-dilatonic  $p$ -branes superimposed by a pp-wave propagating along a direction in the worldvolume of the  $p$ -branes. For example, five-, four- and seven-dimensional Kaigorodov spaces arise as the near-horizon geometries of D3-branes in type IIB string theory, M2-branes and M5-branes in M-theory, respectively. Following the AdS/CFT correspondence, naturally it can be conjectured that M-theory or string

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theory on the Kaigorodov space times a compact space is dual to a certain conformal field theory in an infinitely-boosted frame with finite momentum density [6] (see also [7]). Further, it was found that the Carter–Novotný–Horský (CNH) metric [8], which is a solution of the Einstein equations with a negative cosmological constant, occurs as the near-horizon geometry of non-extremal non-dilatonic  $p$ -branes superimposed by a pp-wave propagating along a direction of the worldvolume of the  $p$ -branes.<sup>1</sup> Having considered possible application in the AdS/CFT correspondence, it is worthwhile to further discuss the Kaigorodov space and CNH space. On the other hand, they are of interest in their own right since they are exact solutions of clearly physical meanings to the Einstein equations with a negative cosmological constant.

In this short Letter we present a charged generalizations of the Kaigorodov space and the CNH space by boosting a class of charged domain wall solution. They are exact solutions to the Einstein–Maxwell equations with a negative cosmological constant. The charged Kaigorodov space has an explanation as a pp-wave propagating in the domain wall background imposed a null gauge field and the charged CNH space comes from the finite boost of the domain wall space. It is argued that gauged supergravity on the Kaigorodov space is dual to a certain strongly coupling CFT with R-charge in an infinitely boosted frame with finite momentum density. We will also discuss the thermodynamics of the charged CNH space, its relation to that of static domain wall and its implications in the generalized AdS/CFT correspondence.

## 2. Charged Kaigorodov space

Let us start with an  $(n + 2)$ -dimensional ( $n \geq 2$ ) charged topological black hole solution [9,10]

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Sigma_{n,k}^2, \\ A = -\frac{1}{c} \frac{\tilde{q}}{r^{n-1}} dt, \quad c = \sqrt{\frac{2(n-1)}{n}}, \quad (2.1)$$

<sup>1</sup> In the case of non-extremal  $p$ -branes it seems more suitable to say that there is a finite momentum, instead of a pp-wave, imposed on the  $p$ -branes. A pp-wave propagates with the speed of light, but the boost velocity is less than the speed of light in the case of non-extremal  $p$ -branes.

where

$$f(r) = k - \frac{m}{r^{n-1}} + \frac{\tilde{q}^2}{r^{2(n-1)}} + \frac{r^2}{l^2}. \quad (2.2)$$

Here  $m$  and  $\tilde{q}$  are two integration constants,  $d\Sigma_{n,k}^2$  denotes the line element of an  $n$ -dimensional Einstein space with constant curvature,  $n(n+1)k$ . Without loss of generality, one can take the value of  $k$  as  $\pm 1$  and 0. It is easy to check that the solution (2.1) is an exact solution of the following Einstein–Maxwell equations with a negative cosmological constant,  $-n(n+1)/2l^2$ ,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{n(n+1)}{2l^2}g_{\mu\nu} = 2F_{\mu\lambda}F_{\nu}^{\lambda} - \frac{1}{2}g_{\mu\nu}F^2, \\ \partial_{\mu}(\sqrt{-g}F^{\mu\nu}) = 0. \quad (2.3)$$

When  $k = 1$ , the hypersurface  $\Sigma_{n,1}$  could be an  $n$ -dimensional sphere. In that case, the solution (2.1) represents an  $(n + 2)$ -dimensional spherically symmetric charged black hole in AdS space. When  $k = -1$ ,  $\Sigma_{n,-1}$  is an  $n$ -dimensional negative constant curvature space and it could be a closed hypersurface with arbitrary high genus under appropriate identification. In this Letter we are interested in the case of  $k = 0$  with  $\Sigma_{n,0}$  being an  $n$ -dimensional Euclidean space

$$dE_n^2 \equiv d\Sigma_{n,0}^2 = (dx_1^2 + dx_2^2 + \dots + dx_n^2)/l^2, \quad (2.4)$$

where we have rescaled the coordinates  $x_i$ :  $x_i \rightarrow x_i/l$ . Clearly, in this case, the solution describes a black domain wall spacetime.<sup>2</sup> To see this, let us take the limit  $r \rightarrow \infty$ , with which one can find that the Poincaré symmetry is recovered on the worldvolume  $(t, x_i)$  of the domain wall.  $r$  is the transverse coordinate of the domain wall. To more clearly see this, let us rewrite the solution (2.1) in terms of “isotropic” coordinates. Defining [12]

$$m = \mu + 2q, \quad \tilde{q}^2 = q(\mu + q), \\ r^{n-1} \rightarrow r^{n-1} + q, \quad (2.5)$$

we can change (2.1) to the following form

$$ds^2 = \frac{r^2 H^{2/(n-1)}}{l^2} \\ \times \left[ -\left(1 - \frac{\mu l^2}{r^{n+1}} H^{-2n/(n-1)}\right) dt^2 + dE_n^2 \right]$$

<sup>2</sup> In [11] such type of solutions is called black plane solutions.

$$A = -\frac{1}{c} \frac{\tilde{q}}{r^{n-1} + q} dt, \quad (2.6)$$

where

$$\begin{aligned} \tilde{f}(r) &= -\frac{\mu}{r^{n-1}} + \frac{r^2}{l^2} H^{2n/(n-1)}, \\ H &= 1 + \frac{q}{r^{n-1}}. \end{aligned} \quad (2.7)$$

When the Schwarzschild parameter  $\mu = 0$  and keep  $q$  as a constant, the solution is supersymmetric, preserving 1/2 supersymmetries of gauged supergravity. However,  $r = 0$  is a naked singularity in this case. In these coordinates, the Poincaré symmetry on the worldvolume is evident once one takes  $\mu = 0$ . This makes it possible to impose a pp-wave propagating on the domain wall [13].

Now we make a boost transformation along one of worldvolume coordinates, say  $x_1$ , as follows,

$$\begin{aligned} t &\rightarrow t \cosh \alpha - x_1 \sinh \alpha, \\ x_1 &\rightarrow -t \sinh \alpha + x_1 \cosh \alpha, \end{aligned} \quad (2.8)$$

where  $\alpha$  is the boost parameter. The solution (2.1) is then changed to

$$\begin{aligned} ds^2 &= \frac{r^2}{l^2} \left( -dt^2 + dx_1^2 \right. \\ &\quad \left. + \left( \frac{ml^2}{r^{n+1}} - \frac{\tilde{q}^2 l^2}{r^{2n}} \right) \right. \\ &\quad \left. \times d(\cosh \alpha dt - \sinh \alpha dx_1)^2 \right. \\ &\quad \left. + dE_{n-1}^2 \right) + f^{-1}(r) dr^2, \\ A &= -\frac{1}{c} \frac{\tilde{q}}{r^{n-1}} (\cosh \alpha dt - \sinh \alpha dx_1), \end{aligned} \quad (2.9)$$

where  $dE_{n-1}^2 = dx_2^2 + \dots + dx_n^2$ . Next we take the limit  $\alpha \rightarrow \infty$ . In that case, the transformation (2.8) is singular. To get a well-defined new solution to the Einstein–Maxwell equations with a negative cosmological constant, we take  $m \rightarrow 0$  and  $\tilde{q} \rightarrow 0$  and keep  $m \cosh^2 \alpha = P$  and  $\tilde{q} \cosh \alpha = P_e$  as two constants while  $\alpha \rightarrow \infty$ . With this approach we obtain

$$\begin{aligned} ds^2 &= \frac{r^2}{l^2} \left( du dv + \left( \frac{Pl^2}{r^{n+1}} - \frac{P_e^2 l^2}{r^{2n}} \right) du^2 + dE_{n-1}^2 \right) \\ &\quad + \frac{l^2}{r^2} dr^2, \end{aligned}$$

$$A = -\frac{1}{c} \frac{P_e}{r^{n-1}} du, \quad (2.10)$$

where  $u, v = x_1 \mp t$  are two null coordinates. It is easy to check that the new solution (2.10) satisfies the Einstein–Maxwell equations (2.3) with a negative cosmological constant. When  $P_e = 0$ , the solution (2.10) reduces to the higher-dimensional generalization of the Kaigorodov space obtained in [6]. Evidently the solution (2.10) represents a pp-wave propagating along the direction  $v$ . The constant  $P$  has the physical interpretation as the momentum density of the pp-wave. The term involving  $P_e$  comes from the contribution of the null Maxwell field. The solution (2.10) is just the charged generalization of the Kaigorodov space. As the Kaigorodov space [6], we expect the charged generalization (2.10) also preserves 1/4 supersymmetries of gauged supergravity. Note that the solution (2.10) is asymptotically AdS. When  $P = P_e = 0$ , the solution is just the AdS space in the Poincaré coordinates, it can be viewed as a special domain wall space. As a result, the charged Kaigorodov space can also be explained as a pp-wave propagating on the domain wall with a null Maxwell field.

In the AdS/CFT correspondence, the bulk Maxwell field could be dual to the gauge field of R-symmetry of boundary conformal field theory and the bulk gauge charge could be identified to the R-charge of boundary field theory [10,14]. Following the proposal that M-theory or string theory on the Kaigorodov space times a compact space is dual to a certain strongly coupling conformal field theory in an infinitely boosted frame with a finite momentum density [6], and the domain wall/QFT (quantum field theory) correspondence which says that gauge supergravity on a domain wall could be equivalent to a certain quantum field theory living on the domain wall [15], we have the following generalization: gauge supergravity on the charged Kaigorodov space (2.10) is dual to a conformal field theory with R-charge in an infinitely boosted frame with finite momentum density.

### 3. Charged Carter–Novotný–Horský space and associated thermodynamics

From the Lorentz transformation (2.8) we know the boost velocity  $v$  is

$$v = \tanh \alpha. \quad (3.1)$$

When  $\alpha \rightarrow \infty$ , the boost velocity approaches to the speed of light. Therefore, the charged Kaigorodov metric (2.10) is obtained by boosting the charged domain wall solution to the ultrarelativity limit  $v = 1$ . Note that in this limit the Lorentz transformation (2.8) is singular. However, the resulting new solution (2.10) is well-defined.

Now we discuss the case for a finite boost, namely the case when the boost parameter  $\alpha$  is a finite constant. In this case we notice that when  $\tilde{q} = 0$ , the solution (2.9) gives us the Carter–Novotný–Horský (CNH) metric [6,8], describing a non-extremal black domain wall with a pp-wave propagating on the worldvolume, or say, a boosted black domain wall with a finite momentum density. Therefore, our solution (2.9) gives a charged generalization of the CNH space. Clearly the solution (2.9) is locally equivalent to the static solution (2.1) since the former comes from the latter via the coordinate transformation (2.8). However, this transformation is valid only if the coordinate  $x_1$  is not periodic [6]. Therefore, if the coordinate  $x_1$  is periodic, the solution (2.9) is not equivalent to the static one (2.1) globally.

In [2] it is argued that the thermodynamics of black holes in AdS space can be identified with that of the dual CFT residing on the boundary of the AdS space. To check the gravity/field theory duality that gauged supergravity on the charged Kaigorodov space (2.10) is dual to a dual CFT in an infinitely boosted frame with a finite momentum density, we now show that the thermodynamics associated with the static solution (2.1) and boosted solution (2.9) can indeed be connected via a Lorentz transformation.

For the static black domain wall solution (2.1), the mass density and the electric charge density on the domain wall are

$$\begin{aligned} \mathbf{m} &= \frac{M}{L_1 V_{n-1}} = \frac{nm}{16\pi G l^n}, \\ \mathbf{q} &= \frac{Q}{L_1 V_{n-1}} = \frac{\sqrt{2n(n-1)}}{8\pi G l^n} \tilde{q}, \end{aligned} \quad (3.2)$$

and the chemical potential conjugate to the electric charge is

$$\Phi = \frac{\tilde{q}}{c r_+^{n-1}}. \quad (3.3)$$

Here  $L_1$  is the length of the coordinate  $x_1$  in (2.4),  $V_{n-1}$  denotes the volume of the hypersurface spanned by coordinates  $x_2, \dots, x_n$ , and  $r_+$  stands for the horizon location of the black domain wall (2.1), the large root of the equation  $f(r_+) = 0$ . Note that the coordinates  $x_i$  are dimensionless so that here the length  $L_1$  and the volume  $V_{n-1}$  are also dimensionless. The Hawking temperature  $T$  and entropy density of the static solution are found to be

$$\begin{aligned} T &= \frac{1}{4\pi r_+} \left( (n+1) \frac{r_+^2}{l^2} - (n-1) \frac{\tilde{q}^2}{r_+^{2(n-1)}} \right), \\ \mathbf{s} &= \frac{S}{L_1 V_{n-1}} = \frac{r_+^n}{4G l^n}. \end{aligned} \quad (3.4)$$

These thermodynamic quantities obey the first law of black hole thermodynamics,

$$d\mathbf{m} = T d\mathbf{s} + \Phi d\mathbf{q}. \quad (3.5)$$

On the other hand, for the boosted solution, the horizon location  $r_+$  is still determined by the equation  $f(r_+) = 0$ . From the solution (2.9), we obtain the mass density and electric charge density

$$\begin{aligned} \mathbf{m}' &= \frac{M'}{L'_1 V_{n-1}} = \frac{nm}{16\pi G l^n} \cosh^2 \alpha, \\ \mathbf{q}' &= \frac{Q'}{L'_1 V_{n-1}} = \frac{\sqrt{2n(n-1)}}{8\pi G l^n} \tilde{q} \cosh \alpha, \end{aligned} \quad (3.6)$$

and the chemical potential associated with the charge is

$$\Phi' = \frac{\tilde{q}}{c r_+^{n-1}} \cosh \alpha. \quad (3.7)$$

Besides, the solution (2.9) has the momentum density, which can be read off directly from the solution,

$$\mathbf{p}' = \frac{P'_v}{L'_1 V_{n-1}} = \frac{nm}{16\pi G l^n} \sinh \alpha \cosh \alpha. \quad (3.8)$$

The conjugate quantity to the momentum is the boost velocity,  $v = \tanh \alpha$ . Here  $L'_1$  denotes the length of the coordinate  $x_1$  in the solution (2.9) and the volume of the hypersurface described by the line element  $dE_{n-1}^2$  has been assumed the same as the one for the static solution (2.1). In addition, we find that the Hawking temperature and entropy density are changed to

$$T' = \frac{1}{4\pi r_+ \cosh \alpha} \left( (n+1) \frac{r_+^2}{l^2} - (n-1) \frac{\tilde{q}^2}{r_+^{2(n-1)}} \right)$$

$$= T / \cosh \alpha, \quad (3.9)$$

$$s' = \frac{S'}{L'_1 V_{n-1}} = \frac{r_+^n}{4G l^n} \cosh \alpha.$$

Contrast to the first law (3.5), due to the appearance of the momentum density, one might think that the boosted black domain wall solution (2.9) has the following first law

$$d\mathbf{m}' = T' ds' + \Phi' d\mathbf{q}' + v d\mathbf{p}'. \quad (3.10)$$

Substituting those quantities into (3.10), however, it is found that this expression does not hold at all. Have a look again at the solution (2.9), we find that due to the boost, there is an electric current along the direction  $x_1$ . The current density is found to be

$$\mathbf{j}' = \frac{J'}{L'_1 V_{n-1}} = \frac{\sqrt{2n(n-1)}}{8\pi G l^n} \tilde{q} \sinh \alpha = v \mathbf{q}', \quad (3.11)$$

and the conjugate chemical potential is

$$\Psi' = -\frac{\tilde{q}}{c r_+^{n-1}} \sinh \alpha. \quad (3.12)$$

With this component, we obtain the correct first law of thermodynamics associated with the boosted charged domain wall

$$d\mathbf{m}' = T' ds' + \Phi' d\mathbf{q}' + \Psi' d\mathbf{j}' + v d\mathbf{p}'. \quad (3.13)$$

Now it is an easy job to check that those thermodynamic quantities indeed satisfy the first law (3.13).

According to the generalized AdS/CFT correspondence, the thermodynamics of the static charged domain wall (2.1) can be identified with that of the dual CFT with a R-charge in a static frame, while the thermodynamics associated with the boosted domain wall (2.9) can be mapped to that of the same CFT in a frame with a finite boost with velocity  $v = \tanh \alpha$ . To check this correspondence, it is interesting to compare the thermodynamics of the boosted and static domain walls. We know that entropy accounts for the number of independent quantum states of a system. Therefore, the entropy should remain unchanged within the Lorentz boost transformation. We see from (3.4) and (3.9) that indeed the total entropy remain invariant  $S = S'$  once taking into account the relation

$$L'_1 = L_1 / \cosh \alpha, \quad (3.14)$$

which is just the Lorentz relation between these two different frames. The electric charge  $Q$  (corresponding

to the R-charge in the dual CFTs) is a conserved quantity, which should be also invariant within the Lorentz transformation. From (3.2) and (3.6) we have indeed  $Q = Q'$  once considering the relation (3.14). Further because the Hawking temperature of event horizon is related to what coordinates one used are, one can see from (3.4) and (3.9) they are different. However, they are related to each other via the relation  $T' = T / \cosh \alpha$ , which is just the manifest of the Lorentz transformation. In addition, in the static coordinates the thermodynamic system has energy  $M$  and vanishing momentum, while in the boosted frame the energy is changed to  $M' = M \cosh \alpha$ , and the momentum is non-vanishing,  $P'_v = vM / \sqrt{1 - v^2}$ . These are just the Lorentz transformation relation for the four-momentum. In a word, we verify that those thermodynamic quantities of CFTs dual to the boosted and static black domain walls are indeed related via the Lorentz transformation. Therefore, this result provides support of the conjecture that gauged supergravity on the Kaigorodov space is dual to a boundary CFT in an infinitely-boosted frame with a finite momentum density.

#### 4. Conclusion

In summary we have presented a charged generalization of the Kaigorodov metric by boosting a charged black domain wall solution to the ultrarelativity limit where the boost velocity approaches the speed of light. Although the limit is singular, the resulting new solution, the charged Kaigorodov space, is well-defined. The solution is an exact solution to the Einstein–Maxwell equations with a negative cosmological constant and describes a pp-wave propagating on an AdS domain wall with a null Maxwell field.

It is conjectured that gauge supergravity on the charged Kaigorodov space is dual to a strongly coupling CFT with R-charge in an infinitely boosted frame with a finite momentum density. To show this, we have discussed the thermodynamics of the charged Carter–Novotný–Horský (CNH) solution, which is a result of a finite boost for the charged black domain wall metric, and have compared the thermodynamics of booted and static black domain walls. According to the generalized AdS/CFT correspondence, the thermodynamics of the charged black domain wall can

be identified with that of dual CFT with R-charge. We have shown that the thermodynamic quantities associated with the charged CNH solution indeed can be related to those of static domain wall solution through a Lorentz transformation. Note that both the charged Kaigorodov space and charged CHN space discussed in this note are asymptotically AdS. It is therefore of interest to further discuss this correspondence to the case of asymptotically non-AdS domain wall spaces in the sense of the domain wall/QFT correspondence [15].

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