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# Understanding for flavor physics in the lepton sector

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In this paper, we give a model for understanding flavor physics in the lepton sector mass hierarchy among different generations and neutrino mixing patterns. The model is constructed in the framework of supersymmetry, with a family symmetry $S4 * U(1)$. There are two right-handed neutrinos introduced for a seesaw mechanism, while some standard model gauge group singlet fields are included, which transforms nontrivially under family symmetry. In the model, each order of contributions are suppressed by $\delta \sim 0.1$ compared to the previous one. In order to reproduce the mass hierarchy, $m_\tau$ and $\sqrt{\Delta m^2_{\rm atm}}$, and $m_\mu$ and $\sqrt{\Delta m^2_{\rm sol}}$ are obtained at leading order and next-to-leading order, respectively, while the electron can only get its mass through next-to-next-to-next-to-leading order contributions. For neutrino mixing angels, $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ are 45°, 45°, 0, i.e., the bimaximal mixing pattern as a first approximation, while higher order contributions can make them consistent with experimental results. As corrections for $\theta_{12}$ and $\theta_{13}$ originate from the same contribution, there is a relation predicted for them: $\sin\theta_{13} = \frac{1-\tan\theta_{12}}{1+\tan\theta_{12}}$. Besides, the deviation from $\frac{\pi}{4}$ for $\theta_{23}$ should have been as large as the deviation from 0 for $\theta_{13;}$ if not, the former is suppressed by a factor of 4 compared to the latter.

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## I. INTRODUCTION

Before, many believed that $\theta_{13}$ is very small due to the fact that experiments could only give an upper bound for it for a long time. In this situation, the scenario with the so-called tri-bimaximal mixing pattern [1] which gives a vanishing $\theta_{13}$ as a leading order (LO) approximation was very popular. Remarkably, it is found that many models with a discrete flavor symmetry can realize this mixing pattern from an underlying theory [2,3]. However, considering the latest experimental results for $\theta_{13}$ [4–8], which turned out to be much larger than many expected, the tri-bimaximal model needs substantial modification. The main problem with it lies in that it is difficult to understand how and why next-to-leading order (NLO) corrections take $\theta_{13}$ from 0 to a rather large angle while preserving $\theta_{12}$ and $\theta_{23}$ close to their values given at LO.

In this paper, we would like to give a simple alternative to the tri-bimaximal model. For convenience of expressing our ansatz for leptons' mass matrices, a model realizing our ansatz is given. In the following, this realistic model is given directly, while observations about lepton flavor physics are given implicitly under discussions for the model. In the model, the mass hierarchy in the lepton sector as well as the neutrino mixing pattern are natural results. To reproduce the mass hierarchy, fermion masses are produced in different orders of contributions. Because of the special forms for mass matrices, a realistic neutrino mixing pattern will also be obtained.

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## II. THE MODEL

The model is built in the framework of supersymmetry with family symmetry $S4 * U(1)$; field contents and their transformation properties under $S4 * U(1) * U(1)_R$ are given in Table I. The $S4$ group has 2 singlet representations, 1 doublet representations, and 2 triplet representations which are denoted as 1, 1′, 2, 3, and 3′, in order. For its presentation, we will adopt the presentation reported in the Appendix of Ref. [9] where readers can find details about multiplication rules and Clebsch-Gordan coefficients. (For a recent review about models for flavor physics with $S4$ family symmetry, please see Ref. [10].) As shown in Table I, we require that three $SU(2)_L$ doublets combine to be in representation 3 under $S4$, $e^c$ and ($\mu^c$, $\tau^c$) transform as representation 1 and 2, respectively, while right-handed neutrinos $N_1$ and $N_2$ are in representation 1 and 1′. Higgs fields $H_{u,d}$ are trivial representations under family symmetry, while some standard model (SM) singlet flavon fields which fall in nontrivial representations of $S4$ are included.

Due to the transformation properties distributed above, LO terms that can generate masses for leptons are of dimension 5. Besides, only when flavon fields get vacuum expectation values (VEVs) can fermion masses be produced. In this situation, terms contributing to fermion masses are characterized by $(\frac{v}{M})^n$ where $v$ denotes flavon fields' VEVs, $M$ is the cutoff scale for flavon physics, and $n$ indicates the number of flavon fields. In the following discussions, we assume that $\delta = \frac{v}{M} \sim 0.1$. As a result, the order a term belongs to can be classified by the number of flavon fields.

 

TABLE I. Transformation properties of all the fields under $S4 * U(1) * U(1)_R$.

| | $L$ | $e^c$ | $(\mu^c,\tau^c)$ | $N_1$ | $N_2$ | $H_{u,d}$ | $\Psi$ | $\Omega$ | $\Sigma$ | $\Phi$ | $\Pi^0$ | $\Theta^0$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $S4$ | 3 | 1 | 2 | 1 | $1'$ | 1 | $3'$ | $3'$ | 2 | $3'$ | 2 | 3 |
| $U(1)$ | 2 | 16 | 3 | 0 | 0 | 0 | $-1$ | $-2$ | $-3$ | $-5$ | 3 | 10 |
| $U(1)_R$ | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 2 |

A $U(1)$ symmetry is also introduced, which plays a similar role as Froggate-Nelson symmetry [11]. With family symmetry $U(1)$, appropriate flavon fields can be picked out for different leptons to produce mass matrices that are needed. Besides, $R$ symmetry is included, which plays a key part in discussing flavon fields' VEVs. As flavon fields have 0 charge under $R$ symmetry, they have to appear in combination with a driving field which is marked with a suffix 0 in Table I. Consequently, the supersymmetric condition that $F$ components of driving fields cannot have VEVs provides constrains on flavon fields' VEVs.

For the time being, we just assume flavon fields' VEVs have the following form and are stable against higher order contributions:

$$\langle\Psi\rangle = \begin{pmatrix} v_1 \\ 2v_1 \\ 0 \end{pmatrix}, \qquad \langle\Omega\rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix},$$
$$\langle\Sigma\rangle = \begin{pmatrix} 0 \\ v_3 \end{pmatrix}, \qquad \langle\Phi\rangle = \begin{pmatrix} 0 \\ v_4 \\ v_4 \end{pmatrix}, \tag{1}$$

where all the VEVs are assumed to be close to each other $v_1 \sim v_2 \sim v_3 \sim v_4 \sim v$. In the end of this section, we will justify these VEV alignments.

### A. Physics at leading order

At LO, the superpotential includes the following terms that contribute to lepton masses:

$$\frac{1}{\Lambda} y_1[(\mu^c, \tau^c)L]_{3'}\Phi H_d + \frac{1}{\Lambda} y_2 N_2 L\Omega H_u + M_1 N_1 N_1 + M_2 N_2 N_2, \tag{2}$$

where we use $[]_{3'}$ to indicate that $(\mu^c, \tau^c)L$ combines to become representation $3'$ and so on. In this work, all dimensionless coupling such as $y_1$ and $y_2$ in Eq. (2) are assumed to be order 1 and close to each other. With VEVs in Eq. (1), mass matrices for charged leptons and light neutrinos are as follows:

$$M_e = \frac{y_1 v_d v_4}{\Lambda}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix},$$
$$M_\nu = \frac{(y_1 v_u v_2)^2}{M_2\Lambda^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

$M_e^\dagger M_e$ have the following form:

$$M_e^\dagger M_e = \left(\frac{y_1 v_d v_4}{\Lambda}\right)^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \tag{4}$$

Thus, only tau and one neutrino have nonzero masses $m_\tau = \sqrt{2}\frac{y_1 v_d v_4}{\Lambda}$ and $m_3 = \frac{(y_1 v_u v_2)^2}{M_2\Lambda^2}$.

### B. Physics at next to leading order

After taking NLO contributions into consideration, there are new terms contributing to lepton masses,

$$\frac{1}{\Lambda^2} y_3[(\mu^c, \tau^c)L]_3[\Omega\Sigma]_3 H_d + \frac{1}{\Lambda^2} y_4[(\mu^c, \tau^c)L]_{3'}[\Omega\Sigma]_{3'} H_d + \frac{1}{\Lambda^2} y_5 N_1 L[\Psi\Psi]_3 H_u. \tag{5}$$

As a result, charged lepton and light neutrino mass matrices become

$$M_e' = \frac{y_1 v_d v_4}{\Lambda}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} - \frac{\delta_1}{4} + \frac{3\delta_2}{4} \\ 0 & \frac{1}{2} + \frac{\sqrt{3}\delta_1}{4} + \frac{\sqrt{3}\delta_2}{4} & \frac{1}{2} \end{pmatrix}, \tag{6}$$

where $\delta_1 = \frac{y_3 v_2 v_3}{y_1 v_4 \Lambda} \sim \delta_2 = \frac{y_4 v_2 v_3}{y_1 v_4 \Lambda} \sim \delta$;

$$M_\nu' = \frac{(y_1 v_u v_2)^2}{M_2\Lambda^2}\begin{pmatrix} 16\delta_3^2 & 16\delta_3^2 & 0 \\ 16\delta_3^2 & 16\delta_3^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{7}$$

where $\delta_3^2 = \frac{M_2}{M_1}(\frac{y_5 v_1 v_1}{y_2 v_2 \Lambda})^2 \sim \delta^2$.

There will be no higher orders of contributions to neutrino mass, so Eq. (7) is the final result. The following matrix diagonalizes Eq. (7) to obtain three mass eigenvalues $m_1 = 0$, $m_2 = 32\delta_3^2 \frac{(y_1 v_u v_2)^2}{M_2\Lambda^2}$, $m_3 = \frac{(y_1 v_u v_2)^2}{M_2\Lambda^2}$:

$$U_\nu' = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

With experimental results for neutrino oscillations [12], $\frac{m_2}{m_3} = 32\delta_3^2 = \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} = 0.18$. Noteworthy, in the mass matrix for light neutrinos, NLO contributions should have been 2 orders smaller than LO ones after the seesaw mechanism. The factor 16 arising from Clebsch-Gordan coefficients in Eq. (7) plays a crucial role in making $\frac{m_2}{m_3}$ consistent with the experimental result without fine-tuning.

Intuitively, the smaller eigenvalue of $M_e^{\prime\dagger} M_e^{\prime}$ should be about $\delta(\frac{y_1 v_d v_4}{\Lambda})^2$. However, it can be proved that the smaller eigenvalue of $M_e^{\prime\dagger} M_e^{\prime}$ is about $\delta^2(\frac{y_1 v_d v_4}{\Lambda})^2$ while the larger one remains about $2(\frac{y_1 v_d v_4}{\Lambda})^2$. Thus, $\frac{m_\mu}{m_\tau} \approx \sqrt{\frac{\delta^2}{2}}$, which is compatible with the experimental result, 0.06. As for $U_e^{\prime}$, only $\theta_{23}$ is nonzero and there is an estimate for it:

$$\tan 2\theta_{23} \approx \frac{2}{\delta}. \tag{9}$$

If we parametrize the deviation from $\frac{\pi}{4}$ for $\theta_{23}$ with a small quantity $\epsilon_1$, it is about $\frac{\delta}{4}$ and $U_e^{\prime}$ can be described in the following form:

$$U_e^{\prime} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}(1+\epsilon_1) & \frac{1}{\sqrt{2}}(1-\epsilon_1) \\ 0 & -\frac{1}{\sqrt{2}}(1-\epsilon_1) & \frac{1}{\sqrt{2}}(1+\epsilon_1) \end{pmatrix}. \tag{10}$$

### C. Physics at next-to-next-to-leading order

New terms that contribute to lepton masses are listed below:

$$\begin{aligned} &\frac{1}{\Lambda^3} y_6[(\mu^c, \tau^c)L]_3[(\Omega\Omega)_2\Psi]_3 H_d \\ &\quad + \frac{1}{\Lambda^3} y_7[(\mu^c, \tau^c)L]_3[(\Omega\Omega)_3\Psi]_3 H_d \\ &\quad + \frac{1}{\Lambda^3} y_8[(\mu^c, \tau^c)L]_{3'}[(\Omega\Omega)_2\Psi]_{3'} H_d \\ &\quad + \frac{1}{\Lambda^3} y_9[(\mu^c, \tau^c)L]_3[\Sigma(\Psi\Psi)_3]_3 H_d. \end{aligned} \tag{11}$$

At this stage, the mass matrix for charged leptons becomes

$$M_e^{\prime\prime} = \frac{y_1 v_d v_4}{\Lambda} \times \begin{pmatrix} 0 & 0 & 0 \\ -\sqrt{3}\delta_4^2 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} - \frac{\delta_1}{4} + \frac{3\delta_2}{4} \\ 0 & \frac{1}{2} + \frac{\sqrt{3}\delta_1}{4} + \frac{\sqrt{3}\delta_2}{4} & \frac{1}{2} \end{pmatrix}, \tag{12}$$

where $\delta_4^2 = \frac{y_6 v_1 v_2 v_2}{y_1 v_4 \Lambda\Lambda} \sim \delta^2$. In Eq. (12), we have neglected contributions to those matrix elements which are nonzero at NLO.

In order to make physics clear, we do a qualitative analysis for $U_e^{\prime\prime}$ up to matrix elements' orders. First of all, we effect transformation $U_e^{\prime}$ on $M_e^{\prime\prime\dagger} M_e^{\prime\prime}$,

$$U_e^{\prime\dagger} M_e^{\prime\prime\dagger} M_e^{\prime\prime} U_e^{\prime} \approx \frac{y_1 v_d v_4}{\Lambda} \begin{pmatrix} \delta^4 & \delta^3 & \delta^2 \\ \delta^3 & \delta^2 & 0 \\ \delta^2 & 0 & 2 \end{pmatrix}. \tag{13}$$

To diagonalize Eq. (13), we just need to effect a transformation with $\sin\theta_{13} \sim \delta^2$ and a transformation with $\sin\theta_{12} \sim \delta$ successively. If we ignore the negligibly small $\theta_{13}$ and parametrize $\theta_{12}$ with another small quantity $\epsilon_2$, $U_e^{\prime\prime}$ have the following form:

$$U_e^{\prime\prime} = \begin{pmatrix} 1 & \epsilon_2 & 0 \\ -\frac{\epsilon_2}{\sqrt{2}} & \frac{1}{\sqrt{2}}(1+\epsilon_1) & \frac{1}{\sqrt{2}}(1-\epsilon_1) \\ \frac{\epsilon_2}{\sqrt{2}} & -\frac{1}{\sqrt{2}}(1-\epsilon_1) & \frac{1}{\sqrt{2}}(1+\epsilon_1) \end{pmatrix}. \tag{14}$$

At present, we can discuss neutrino mixing angles. $U_{\rm PMNS}$ [13] is obtained by $U_e^{\prime\prime\dagger} U_\nu^{\prime}$,

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\epsilon_2\right) & \frac{1}{\sqrt{2}}\left(1-\frac{1}{\sqrt{2}}\epsilon_2\right) & \frac{\epsilon_2}{\sqrt{2}} \\ \cdots & \cdots & \frac{1}{\sqrt{2}}(1-\epsilon_1) \\ \cdots & \cdots & \frac{1}{\sqrt{2}}(1+\epsilon_1) \end{pmatrix}, \tag{15}$$

where we just list the matrix elements involved in fixing neutrino mixing angles. In this case, $\tan\theta_{12} = \frac{1-\frac{1}{\sqrt{2}}\epsilon_2}{1+\frac{1}{\sqrt{2}}\epsilon_2}$, $\sin\theta_{13} = \frac{\epsilon_2}{\sqrt{2}}$, and $\tan\theta_{23} = \frac{1-\epsilon_1}{1+\epsilon_1}$. The larger the deviation from $\frac{\pi}{4}$ for $\theta_{12}$, the larger $\theta_{13}$ is, resulting from the relation $\sin\theta_{13} = \frac{1-\tan\theta_{12}}{1+\tan\theta_{12}}$. With the result for $\theta_{12}$ [12], $\sin\theta_{13}$ is obtained as $0.188^{+0.015}_{-0.017}$, which is a little larger than Daya Bay's result $0.153^{+0.018}_{-0.019}$ and consistent with RENO's result $0.171^{+0.023}_{-0.027}$ and Double Chooz's result $0.167^{+0.040}_{-0.050}$. Noteworthy, the order $\delta^2$ contribution, which is neglected in obtaining Eq. (14), can lead to additional $O(0.01)$ contribution to $\sin\theta_{13}$, making the model consistent with experimental results. As far as $\theta_{23}$ is concerned, its deviation from $\frac{\pi}{4}$ should have been as large as $\theta_{12}$'s deviation from $\frac{\pi}{4}$ and $\theta_{13}$'s deviation from 0, if it were not for the fact that the former is suppressed by additional factor of 4. Therefore, $\theta_{23}$'s deviation from $\frac{\pi}{4}$ is a little smaller than $\theta_{12}$'s deviation from $\frac{\pi}{4}$ but is too large for $\theta_{23}$ to be taken as maximal.

### D. Physics at next-to-next-to-next-to-leading order

The electron cannot obtain its mass until this order where the interacting terms between $e^c$ and $L$ appears for the first time,

$$\begin{aligned} &\frac{1}{\Lambda^4}[e^c L]_3 H_d\{y_{10}[(\Phi\Phi)_1(\Sigma\Phi)_3]_3 + y_{11}[(\Phi\Phi)_2(\Sigma\Phi)_3]_3 \\ &\quad + y_{12}[(\Phi\Phi)_2(\Sigma\Phi)_{3'}]_3 + y_{13}[(\Phi\Phi)_3(\Sigma\Phi)_3]_3 \\ &\quad + y_{14}[(\Phi\Phi)_3(\Sigma\Phi)_3]_{3'} + y_{15}[(\Phi\Phi)_{3'}(\Sigma\Phi)_3]_3 \\ &\quad + y_{16}[(\Phi\Phi)_{3'}(\Sigma\Phi)_{3'}]_{3'}\}. \end{aligned} \tag{16}$$

The charged leptons' mass matrix now becomes

$$M_e''' = \frac{y_1 v_d v_4}{\Lambda} \times \begin{pmatrix} 0 & \delta_5^3 + \delta_6^3 & \delta_5^3 + \delta_6^3 \\ -\sqrt{3}\delta_4^2 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} - \frac{\delta_1}{4} + \frac{3\delta_2}{4} \\ 0 & \frac{1}{2} + \frac{\sqrt{3}\delta_1}{4} + \frac{\sqrt{3}\delta_2}{4} & \frac{1}{2} \end{pmatrix}, \tag{17}$$

where $\delta_5^3 = \frac{y_{10} v_3 v_4 v_4}{y_1 \Lambda\Lambda\Lambda} \sim \delta_6^3 = \frac{y_{11} v_3 v_4 v_4}{y_1 \Lambda\Lambda\Lambda} \sim \delta^3$. Equation (17) leads to a naturally small electron mass, without interfering the above discussions.

### E. Flavon fields' VEVs

Finally, we would like to address issues concerning VEV alignments in Eq. (1), which need to be a reasonable result in order to make this model convincing. We just need to show that these VEV alignments are valid up to next-to-next-to-leading order (NNLO), because our physical results, except for the electron mass, have been achieved by this order while the production of the electron mass does not rely on those specific VEV alignments. As we have said, supersymmetric requirements result in a constraint on $F$ components of driving fields $\Pi^0$ and $\Theta^0$ so that $\langle F_i \rangle = 0$ with $i$ representing $\Pi_1^0$, $\Pi_2^0$, $\Theta_1^0$, $\Theta_2^0$, $\Theta_3^0$. Relevant terms are given below:

$$\begin{aligned}
&\text{LO,}\ \ \Pi^0\{m\Sigma + c_1(\Omega\Psi)_2\} + \Theta^0\{c_4(\Phi\Phi)_3\},\\
&\text{NLO,}\ \ \frac{1}{\Lambda}\Pi^0\{c_2[(\Psi\Psi)_3\Psi]_2 + c_3[(\Psi\Psi)_{3'}\Psi]_2\} + \frac{1}{\Lambda}\Theta^0\{c_5[(\Sigma\Phi)_3\Omega]_3 + c_6[(\Sigma\Phi)_{3'}\Omega]_3\},\\
&\text{NNLO,}\ \ \frac{1}{\Lambda^2}\Theta^0\{c_7\{[(\Sigma\Phi)_3\Psi]_{1'}\Psi\}_3 + c_8\{[(\Sigma\Phi)_3\Psi]_2\Psi\}_3 + c_9\{[(\Sigma\Phi)_3\Psi]_3\Psi\}_3 + c_{10}\{[(\Sigma\Phi)_3\Psi]_{3'}\Psi\}_3\\
&\quad + c_{11}\{[(\Sigma\Phi)_{3'}\Psi]_2\Psi\}_3 + c_{12}\{[(\Sigma\Phi)_{3'}\Psi]_3\Psi\}_3 + c_{13}\{[(\Sigma\Phi)_{3'}\Psi]_{3'}\Psi\}_3 c_{14}\{[(\Omega\Omega)_1\Phi]_{3'}\Psi\}_3\\
&\quad + c_{15}\{[(\Omega\Omega)_2\Phi]_3\Psi\}_3 + c_{16}\{[(\Omega\Omega)_2\Phi]_{3'}\Psi\}_3 + c_{17}\{[(\Omega\Omega)_3\Phi]_{1'}\Psi\}_3 + c_{18}\{[(\Omega\Omega)_3\Phi]_2\Psi\}_3\\
&\quad + c_{19}\{[(\Omega\Omega)_3\Phi]_3\Psi\}_3 + c_{20}\{[(\Omega\Omega)_3\Phi]_{3'}\Psi\}_3 + c_{21}\{[(\Omega\Omega)_{3'}\Phi]_2\Psi\}_3 + c_{22}\{[(\Omega\Omega)_{3'}\Phi]_3\Psi\}_3\\
&\quad + c_{23}\{[(\Omega\Omega)_{3'}\Phi]_{3'}\Psi\}_3 + c_{24}\{[(\Sigma\Sigma)_1\Sigma]_2\Psi\}_3 + c_{25}\{[(\Sigma\Sigma)_{1'}\Sigma]_2\Psi\}_3 + c_{26}\{[(\Sigma\Sigma)_2\Sigma]_{1'}\Psi\}_3 + c_{27}\{[(\Sigma\Sigma)_2\Sigma]_2\Psi\}_3\\
&\quad + c_{28}\{[(\Sigma\Sigma)_1\Omega]_{3'}\Omega\}_3 + c_{29}\{[(\Sigma\Sigma)_{1'}\Omega]_3\Omega\}_3 + c_{30}\{[(\Sigma\Sigma)_2\Omega]_3\Omega\}_3 + c_{31}\{[(\Sigma\Sigma)_2\Omega]_{3'}\Omega\}_3.
\end{aligned}$$

There are, in total, 13 equations for $\langle F_i \rangle = 0$ at LO, NLO, and NNLO, respectively, among which 8 equations are automatically satisfied when flavon fields take VEVs as shown in Eq. (1). The other 5 equations are as follows:

$$\begin{cases} m v_3 + \sqrt{3} c_1 v_1 v_2 & = 0 \\ (c_5 - \sqrt{3} c_6) v_2 v_3 v_4 & = 0 \\ \lambda_1 v_1 v_1 v_3 v_4 + \lambda_2 v_1 v_2 v_2 v_4 + \lambda_3 v_1 v_3 v_3 v_3 + \lambda_4 v_2 v_2 v_3 v_3 & = 0 \\ \lambda_5 v_1 v_1 v_3 v_4 + \lambda_6 v_1 v_2 v_2 v_4 + \lambda_7 v_1 v_3 v_3 v_3 & = 0 \\ \lambda_8 v_1 v_1 v_3 v_4 + \lambda_9 v_1 v_2 v_2 v_4 & = 0, \end{cases} \tag{18}$$

where

$$\begin{aligned}
&\lambda_1 = c_7 - \frac{1}{2}c_8 - c_9 - c_{10} - \frac{\sqrt{3}}{2}c_{11} - \sqrt{3}c_{12} - \sqrt{3}c_{13}, \quad \lambda_2 = \sqrt{3}c_{15} - 3c_{16} - 2c_{19} + 2c_{20},\\
&\lambda_3 = -c_{24} + c_{26} - c_{27}, \quad \lambda_4 = -c_{28} + \frac{\sqrt{3}}{2}c_{30} + \frac{1}{2}c_{31},\\
&\lambda_5 = 2c_7 + \frac{1}{2}c_8 - \frac{3}{2}c_9 - \frac{1}{2}c_{10} + \frac{\sqrt{3}}{2}c_{11} + \frac{5\sqrt{3}}{2}c_{12} - \frac{\sqrt{3}}{2}c_{13}, \quad \lambda_6 = \frac{\sqrt{3}}{2}c_{15} + \frac{3}{2}c_{16} + c_{19} + c_{20},\\
&\lambda_7 = c_{24} + 2c_{26} + c_{27}, \quad \lambda_8 = \frac{3}{2}c_8 + \frac{1}{2}c_9 + \frac{3}{2}c_{10} - \frac{3}{2}c_{11} + \frac{\sqrt{3}}{2}c_{12} + \frac{3\sqrt{3}}{2}c_{13}, \quad \lambda_9 = -\frac{\sqrt{3}}{2}c_{15} - \frac{3}{2}c_{16} + c_{19} + c_{20}.
\end{aligned}$$

For the second equation in Eq. (18), we have to assume an accidental relation $c_5 - \sqrt{3}c_6 = 0$. In this situation, values of $v_1 - v_4$ are determined from the other 4 equations and should be in the order of $m$, which is the only coefficient that has a dimension. Thus, we can say that the assumption $v_1 \sim v_2 \sim v_3 \sim v_4 \sim v$ is reasonable.

## III. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have built a model for understanding flavor physics in the lepton sector, mainly mass spectrum and mixing pattern. The model is constructed under family symmetry $S4 * U(1)$. With the assumption that higher order contribution is suppressed by $\delta \sim 0.1$ compared to the previous one, the mass hierarchy $\frac{m_e}{m_\mu}$, $\frac{m_\mu}{m_\tau}$, and $\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$ are natural results of this model. This is realized by producing $m_\tau$, $m_3$ at LO, $m_\mu$, $m_2$ at NLO, and $m_e$ at next-to-next-to-next-to-leading order while $m_1 = 0$ to all orders. In fact, this realization is to some extent inspired by Chun Liu's works [14] where electron, muon, and tau get their mass from the breaking of different symmetries. In the same process of reproducing the mass spectrum, a realistic mixing pattern is obtained (in Ref. [15], the authors also attempted to connect mixing angles with mass hierarchy). As a matter of fact, this model's mixing pattern for the first approximation is actually bimaximal [16,17]. There are some works [18–20] obtaining a bimaximal mixing pattern with the same starting point as this model. (After finishing this work, we have received some works [21] which are related to ours.) As far as mixing angels are concerned, this work not only provides a realistic model where higher order contributions change $\theta_{13}$ and $\theta_{12}$ considerably without interfering $\theta_{23}$ much, but also predicts a relation $\sin\theta_{13} = \frac{1-\tan\theta_{12}}{1+\tan\theta_{12}}$. However, there is still one problem in this model, i.e., the unnatural relation $c_5 - \sqrt{3}c_6 = 0$. Nevertheless, this problem is dependent on our choice of family symmetry and field contents. It is possible that another model where field contents or even family symmetry are different from those used here can realize our ansatz for leptons' mass matrices as described in this paper, without getting in trouble with unnatural relation such as when flavon fields' VEVs are discussed here.

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