Published for SISSA by Springer

Received: May 23, 2012
Accepted: June 15, 2012
Published: June 27, 2012

JHEP06(2012)162

A note on poly-instanton effects in type IIB orientifolds on Calabi-Yau threefolds

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Abstract: The zero mode structure for the generation of poly-instanton corrections for Euclidian $D3$-branes wrapping complex surfaces in Type IIB orientifolds with $O7$- and $O3$-planes is analyzed. Working examples of such surfaces and explicit embeddings into compact Calabi-Yau threefolds are presented, with special emphasis on geometries capable of realizing the LARGE volume scenario.

Keywords: Intersecting branes models, D-branes, Superstring Vacua

ArXiv ePrint: 1205.2485

doi:10.1007/JHEP06(2012)162

Contents

1 Introduction

For connecting string theory to our four-dimensional world the understanding of moduli stabilization and its consequences for the effective four-dimensional theory is of utmost importance, both for particle physics and cosmology. During the last decade we have seen many advances in identifying mechanisms how moduli stabilization can be achieved, like non-trivial background fluxes [1–3], perturbative corrections [4, 5] and instanton effects [6]. A couple of principle mechanisms, which often invoke a combination of these effects, have been identified and extensively discussed in all its consequences in the literature. This includes the KKLT [7] and the LARGE volume scenario [8], which are best understood for Type IIB orientifolds with $O7$ and $O3$-planes. The rules of string model building for such backgrounds have been worked out in [9–12] and a couple of important steps toward honest fully fledged realistic string models have been done. Nevertheless, we think it is fair to say that from a top-down perspective no honest string compactification has been proven to give rise precisely to these effects. Clearly, this has to do with the large number of moduli and with the intricate relations and constraints for the various objects and structures present in genuine string models. Here we have in mind, constraints from tadpole cancellation, Freed-Witten anomalies [13] or correlations between the presence of fluxes/D-branes and the instanton zero mode structure [14].

Although the concept of inflation has been proposed quite some time ago as a solution to certain cosmological issues [15, 16], the embedding of inflationary scenarios into a semi-realistic model in string theory has been convincing only after all the moduli could be stabilized. Such an inflationary model has been initiated in [17] in which, following the idea of [18], an open string modulus appearing as a brane separation was argued to be an inflaton candidate. There has been a large amount of work dedicated to build sophisticated models of open string inflation (see the reviews [19–21] and references therein). However, in particular in the framework of the LARGE volume scenario, closed string moduli inflation has also been seriously considered [22–25]. In this respect, moduli are of interest which at leading order still have a flat potential and only by a, in the overall theory, subleading effect receive their dominant contribution. These can be either subleading perturbative or instanton effects.

Such a subleading instanton contribution is given by so-called poly-instanton effects, which can be briefly described as instanton corrections to instanton actions. These were introduced and studied in the framework of Type I string compactifications in [26]. The analogous poly-instanton effects will also appear in the aforementioned Type IIB orientifolds with $O7$ and $O3$ planes. Thus, these corrections seem to be quite promising from the point of view of constructing (semi-)realistic models for attempts to address several open issues in string cosmology as well as in string phenomenology.

Recently, utilizing these ideas of poly-instanton effects, moduli stabilization and inflationary aspects have been studied in a series of papers [27–31]. However the analysis had to be carried out from a rather heuristic point of view, as a clear understanding of the string theoretic conditions for the generation of these effects was lacking. It is the aim of this note to clarify the zero mode conditions for an Euclidian D3-brane instanton, wrapping a divisor of the threefold, to generate such a poly-instanton effect. We also provide concrete examples of such divisors and present a couple of Type IIB orientifolds on toric Calabi-Yau threefolds containing them. In this respect, we are heading for examples which also contain shrinkable del Pezzo surfaces so that the LARGE volume scenario can in principle be realized. We find that in the simplest class of Type IIB orientifold models, in which poly-instanton corrections are guaranteed to be present, the Kähler potential takes a very peculiar form which is different from those of the fibrations used in [11, 28, 29]. However, we would like to emphasize that the goal of our investigation is to formulate sufficient conditions for the generation of poly-instanton effects. After invoking additional effects, like gauge or closed string fluxes, it might be possible to soak up the additional fermionic zero modes [32, 33]. Clearly, in such cases a more detailed analysis of the precise intanton actions is necessary.

The paper is organized as follows: In section **2**, after recalling the structure of poly-instanton corrections to the superpotential and some basic notions of Type IIB orientifolds, we investigate the zero mode structure of the respective $E3$ instantons wrapping complex surface in Calabi-Yau threefolds. This leads to surfaces of a certain topology, whose embedding into concrete Calabi-Yau threefolds is studied in section **3**. After specifying the threefolds, we also identify admissible orientifold projections for the generation of poly-instanton corrections. Along with E3-instanton contributions, we also consider the possibility of non-perturbative contributions to the superpotential coming from gaugino condensation. Finally, the section **4** presents the conclusions.

In this paper, we rather focus on the model building and mathematical issues and postpone the interesting analysis of moduli stabilization and cosmological applications to up-coming work [34].

2 Poly-instanton corrections

The notion of poly-instantons was introduced in [26] (see [35–37] also for earlier related work) and means the correction of an Euclidian D-brane instanton action by other D-brane instantons. The configuration of interest in the following is that we have two instantons a and b. The zero mode structure of instanton a is such that it generates a correction to the holomorphic superpotential of the form

$$W = A_a \exp(-S_a)\,. \tag{2.1}$$

Here S_a denotes the classical D-brane instanton action and the prefactor A_a a moduli dependent one-loop determinant, which can be understood as the exponential of the holomorphic part of the one-loop correction to the classical instanton action. This is completely analogous to the classical holomorphic gauge kinetic function and its one-loop correction on a fictitious space-time filling D-brane. Due to a non-renormalization theorem, such a gauge kinetic function can be corrected at one-loop order and by instantons, so that one expects the same also for the classical instanton action. Thus, if the second instanton b has the right zero mode structure to generate such a correction, one gets

$$\begin{aligned} W &= \exp\left(-S_a + S_a^{1-\text{loop}} + A_b\, e^{-S_b}\right) \\ &= A_a \exp\left(-S_a\right) + A_a\, A_b \exp\left(-S_a - S_b\right) + \ldots\,. \end{aligned} \tag{2.2}$$

The contribution of the instanton b is clearly exponentially suppressed relative to the contribution of the instanton a. However, as we will review in a moment, the two zero mode structures are different so that the instanton actions S_a and S_b will generally depend on different moduli. Therefore, it can happen that the leading order occurrence of the moduli governing S_b is through this poly-instanton effect, which then has to be taken into account for moduli stabilization.

Type IIB orientifolds. In [26] the zero mode analysis was carried out for pure Ω orientifolds, i.e. compactications of the Type I superstring on Calabi-Yau manifolds. In this paper, we are interested in the case, where one compactifies the Type IIB superstring on a Calabi-Yau threefold $\mathcal{M}$ and performs an orientifold quotient $\Omega\sigma(-1)^{F_L}$. Here, σ is a holomorphic involution acting on the Kähler form J and the holomorphic $(3,0)$-form $\Omega_{3,0}$ as

$$\sigma(J) = J \qquad \sigma(\Omega_{3,0}) = -\Omega_{3,0}\,. \tag{2.3}$$

This leads to $O7$-and $O3$-planes as fixed point loci. Recall that it is for this class of compactification where moduli stabilization is understood best. Generically, the complex structure moduli and the dilaton get fixed by turning on a non-trivial closed string three-form flux $G_3 = F_3 + \tau H_3$. At tree-level this implies a no-scale structure leaving the Kähler moduli as flat directions. These can be stabilized by Euclidian D3-brane instantons wrapping four-cycles $E \subset \mathcal{M}$.

The presence of $O7$-planes wrapping a divisor $O7$ give rise to a tadpole for the R-R C_8-form which has to be canceled by stacks of $D7$-branes wrapping divisors D_a and their orientifold images D'_a so that

$$\sum_a N_a\,(D_a + D'_a) = 8\,O7\,. \tag{2.4}$$

There is an important subtlety which has to do with the Freed-Witten anomaly, appearing if the divisor D wrapped by a $D7$-brane is non-spin. The quantization condition for the gauge flux on the $D7$-brane reads

$$c_1(L) - i^*B + \frac{1}{2}c_1(K_D) \in H^2(D,\mathbb{Z})\,. \tag{2.5}$$

where i^* denotes the pull-back of forms from the Calabi-Yau threefold onto the divisor D. For a non-spin divisor $c_1(K_D)$ is not even so that one is forced to introduce a half-integer B-field or gauge flux $c_1(L) = \frac{1}{2\pi}\mathcal{F}$ with $\mathcal{F} = F + i^*B$. (see [9] for more details on building Type IIB orientifolds). For all the examples to be discussed later, we cancel the Freed-Witten anomaly by $\mathcal{F} = 0$ and appropriate half-integer B-field backgrounds.

For a brane wrapping a 4-cycle D which is invariant under the orientifold projection $\Omega\sigma(-1)^{F_L}$ one can get orthogonal and symplectic gauge groups. Turning on a gauge flux with $c_1(L) \in H^{11}_-(D)$ the fluxed brane is still invariant under the orientifold projection. This is in contrast to a flux $c_1(L) \in H^{11}_+(D)$ which breaks the gauge symmetry to a unitary group.

The $O7$-planes and the $D7$-branes also induce a $D3$-brane tadpole, which generally reads

$$N_{D3} + \frac{N_{\rm flux}}{2} + N_{\rm gauge} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{12} + \sum_a N_a\,\frac{\chi_o(D_a) + \chi_o(D'_a)}{48} \tag{2.6}$$

with $N_{\rm flux} = \frac{1}{(2\pi)^4\alpha'^2}\int H_3 \wedge F_3$, $N_{\rm gauge} = -\sum_a \frac{1}{8\pi^2}\int_{D_a} {\rm tr}\mathcal{F}_a^2$ and χ_o denoting the Euler characteristic of a smoothened divisor (as motivated by duality to F-theory [38]). In this paper we are not so much concerned with the physics on the $D7$-branes and for the concrete orientifolds we will always cancel the $D7$-brane tadpole by simply placing eight $D7$-branes on top of the $O7$-plane. Clearly, for concrete model building this simple assumption has to be relaxed. Now, the $D3$ tadpole condition simplifies to

$$N_{D3} + \frac{N_{\rm flux}}{2} + N_{\rm gauge} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{4}\,. \tag{2.7}$$

It will serve as a consistency check that this number is indeed an integer.

If $H^2_-(\mathcal{M}) \neq 0$ with some non-trivial gauge-flux turned on, one should also check whether the net $D5$-charge vanishes, i.e.

$$\sum_a \int_{\mathcal{M}} \omega \wedge ({\rm tr}\mathcal{F}_a \wedge D_a + {\rm tr}\mathcal{F}_{a'} \wedge D_{a'}) = 0 \tag{2.8}$$

for all $\omega \in H^2_-(\mathcal{M})$.

Instanton zero modes. Next, we discuss the zero mode structure of Euclidian $D3$-brane (short $E3$ instantons) with special emphasis on the case of a poly-instanton correction.[1] Thus, we describe the "T-dual" of the the zero mode analysis carried out in [26]. There, instanton a was an Euclidian $E1$-instanton wrapping a rigid curve of genus zero, i.e. an isolated $\mathbb{P}^1$. Instanton b, however, was an Euclidian $E1$-instanton wrapping a rigid curve of genus one, i.e. a torus. The single complex Wilson line Goldstinos were just the right zero modes to generate an instanton correction to the gauge kinetic function, respectively, the instanton action S_a.

Now, in the case of $\Omega\sigma(-1)^{F_L}$ orientifolds, a former $E1$ instanton becomes a Euclidian $E3$ instanton wrapping a divisor E on $\mathcal{M}$, i.e. a complex surface. In such a case, the instanton zero modes are still counted by certain cohomology classes on E, namely $H^{n,0}(E) = H^n(E, \mathcal{O})$, $n = 0, 1, 2$. However, one has to distinguish between σ-even and odd classes $H^{n,0}(E) = H^{n,0}_+(E) + H^{n,0}_-(E)$. Note that, for such equivariant cohomologies, one defines two chiral indices. First, there is the usual holomorphic Euler characteristic of the divisor E

$$\chi(E, \mathcal{O}_E) = \sum_{i=0}^{2} (-1)^i \, h^i(E, \mathcal{O}_E) = \sum_{i=0}^{2} (-1)^i \, h^{i,0}(E) \tag{2.9}$$

and second one can define an index taking the $\mathbb{Z}_2$ action of σ into account

$$\chi^\sigma(E, \mathcal{O}_E) = \sum_{i=0}^{2} (-1)^i \, \left(h^{i,0}_+(E) - h^{i,0}_-(E) \right) \, . \tag{2.10}$$

The Lefschetz fixed point theorem for our case of interest states that, if the fixed point set of the involution intersects the divisor E in an isolated curve $M^\sigma = O7 \cap E$, then

$$\chi^\sigma(E, \mathcal{O}_E) = -\frac{1}{4} \int_{M^\sigma} [E] \, , \tag{2.11}$$

where $[E] \in H^2(\mathcal{M})$ is Poincaré dual to the divisor E. For the equivariant Betti numbers a similar theorem applies. Under the same assumptions, one has

$$L^\sigma(E) = \sum_{i=0}^{4} (-1)^i \, (b^i_+ - b^i_-) = \chi(M^\sigma) \, . \tag{2.12}$$

In certain cases, these two index theorems are already sufficient to uniquely determine all non-vanishing equivariant cohomology classes. In all the other cases, we employ the algorithm presented in [40] which is based on the formalism presented in [41] for the computation of line bundle valued cohomology classes over toric varieties.[2]

The general zero mode structure for an $E3$-brane instanton wrapping a divisor E, which is invariant under the involution σ, was worked out in detail in [42] (see [43, 44] for

[1] For a general review on D-brane instantons see [39].

[2] Each cohomology class has a representative which is a rationom in the homogeneous coordinates. Once the action of the involution is fixed, one can determine its action on the relevant representatives and then run through the long exact sequences to finally determine the equivariant cohomology classes. We have used the **cohomCalg** implementation to carry out these computations and in particular to determine the relevant representatives.

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Zero modes	Statistics	Number
(X_μ, θ_α)	(bose, fermi)	$H^{0,0}_{+}(E) \times \square\!\square + H^{0,0}_{-}(E) \times \begin{smallmatrix}\square\\\square\end{smallmatrix}$
$\overline{\tau}_{\dot\alpha}$	fermi	$H^{0,0}_{-}(E) \times \square\!\square + H^{0,0}_{+}(E) \times \begin{smallmatrix}\square\\\square\end{smallmatrix}$
γ_α	fermi	$H^{1,0}_{+}(E) \times \square\!\square + H^{1,0}_{-}(E) \times \begin{smallmatrix}\square\\\square\end{smallmatrix}$
$(w, \overline{\gamma}_{\dot\alpha})$	(bose, fermi)	$H^{1,0}_{-}(E) \times \square\!\square + H^{1,0}_{+}(E) \times \begin{smallmatrix}\square\\\square\end{smallmatrix}$
χ_α	fermi	$H^{2,0}_{+}(E) \times \square\!\square + H^{2,0}_{-}(E) \times \begin{smallmatrix}\square\\\square\end{smallmatrix}$
$(c, \overline{\chi}_{\dot\alpha})$	(bose, fermi)	$H^{2,0}_{-}(E) \times \square\!\square + H^{2,0}_{+}(E) \times \begin{smallmatrix}\square\\\square\end{smallmatrix}$

Table 1. Zero modes for $O(N)$-instanton.

instantons with additional gauge flux). For a stack of N such Euclidian branes of type $O(N)$, the zero mode spectrum is summarized in table 1.

For an $SP(N)$ instanton, with necessarily N even, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ and $\square\!\square$ are exchanged. The zero modes X_μ, θ_α and $\overline{\tau}_{\dot\alpha}$ are also called universal zero modes, as they do not depend on the internal geometry of the four-cycle E. The remaining ones can be considered as Wilson line and deformation zero modes, i.e. as Goldstone bosons and Goldstinos of brane deformation moduli. In the second column we have indicated whether the orientifold projection leaves just a fermionic zero mode invariant or a bosonic and a fermionic zero mode.

Given a conformal field theory, the sign in the Möbius strip amplitudes uniquely distinguishes between O/SP instantons. From that we extract the following purely geometric characterization of O/SP instantons (in case we are only using $O7^-$-planes):

- Placing an $E3$ instanton right on top of an $O7$-plane gives an SP-instanton.
- If the divisor E intersects the $O7$-plane in a curve, we have four additional Neumann-Dirichlet type boundary conditions, which due to [45] changes the former SP- to an O-projection. Thus, we have an O-instanton.
- For a divisor E which is parallel to the $O7$ plane, in the sense $E \cap O7 = \emptyset$, the respective instanton is expected to be SP.

For contributions to the holomorphic superpotential respectively the gauge kinetic function, the anti-holomorphic $\overline{\tau}^{\dot\alpha}$ zero modes have to be removed. This happens for a single instanton being placed in an orientifold invariant position with an $O(1)$ projection, which corresponds to an SP-type projection for the fictitious space-time filling $D7$-branes. If an instanton wrapping a surface E satisfies this, we also say "it is $O(1)$". For instanton a there should not be any further zero modes, i.e. $H^{1,0}(E) = H^{2,0}(E) = 0$. For instanton b, the former (purely fermionic) Wilson line Goldstinos of the $E1$ instanton, for an $E3$ instanton, can arise from either Wilson line or deformation Goldstinos, counted by $H^{1,0}_{+}(E) + H^{2,0}_{+}(E)$. Therefore, we have the two possibilities listed in table 2 for the zero mode structure of the poly-instanton correction to the superpotential.

A well known example of a divisor having just one deformation zero mode is certainly a K3-surface. However, the question is whether for an $O(1)$-instanton the deformation Goldstino can be in $H^{2,0}_{+}(E)$. For the second class of b-instantons one first needs explicit

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Class	Instanton a	Instanton b
$H^{0,0}_{+}(E)$	1	1
$H^{1,0}_{+}(E)$	0	1 | 0
$H^{2,0}_{+}(E)$	0	0 | 1
$H^{n,0}_{-}(E)$	0	0

Table 2. Zero modes for poly-instantons a and b .

examples of surfaces with a single complex Wilson line. Second, as above the Wilson line need to be in $H^{1,0}_{+}(E)$. Even though a couple of brief arguments were already given in [28], let us analyze these two a priori options in more detail.

Analysis of $H^{2,0}_{+}(E)$. Recall that via contraction with the $\Omega_{3,0}$ form of the Calabi-Yau threefold, $H^2(E,\mathcal{O})$ is related to the sections of the normal bundle of $E \subset \mathcal{M}$ as

$$H^2_{\pm}(E,\mathcal{O}) = H^0_{\mp}(E,N_E)\,. \tag{2.13}$$

Note that, due to the adjunction formula, the normal bundle is equal to the canonical bundle of E, i.e. $N_E = K_E$.

Let us consider a K3 divisor E which intersects the $O7$-plane in a curve. Due to our former characterization, an $E3$-brane wrapping this K3 is $O(1)$. Applying the Lefschetz fixed point theorem in eq. (2.11), gives $\chi^\sigma(K3,\mathcal{O}) = 0$ and therefore $h^2_-(E,\mathcal{O}) = 1$. Thus, we see that for this $O(1)$ instanton we cannot have $h^2_+(E,\mathcal{O}) = 1$.

Now, let us assume that $H^2_+(E,\mathcal{O}) = 1$. This corresponds to a section of the normal bundle of E, which changes sign under σ. Intuitively this means that the surface E is "parallel" to the $O7$ plane and therefore supports an SP-instanton, which will not contribute to the superpotential (at least not without invoking new mechanism to soak up the extra zero modes).

To conclude, for an $O(1)$ instanton wrapping a K3-surface, the latter cannot be in $H^{2,0}_{+}(E)$. Thus, the Hodge-diamond splits as:

$$\begin{array}{ccccc} & & 1_+ & & \\ & 0 & & 0 & \\ 1_- & & h^{11}_+ + h^{11}_- & & 1_- \\ & 0 & & 0 & \\ & & 1_+ & & \end{array}$$

Analysis of $H^{1,0}_{+}(E)$. Now consider the case that the divisor admits a single complex Wilson line Goldstino so that the Hodge diamond of the divisor W has the form:

$$\begin{array}{ccccc} & & 1 & & \\ & 1 & & 1 & \\ 0 & & h^{11} & & 0 \\ & 1 & & 1 & \\ & & 1 & & \end{array}$$

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A class of examples of such surfaces W are $\mathbb{P}^1$ fibrations over tori,[3] which, in the following, we also call W-surfaces. A toric realization for a simple such fibration is

	x_1	x_2	x_3	x_4	x_5
0	1	1	0	0	0
3	0	-1	1	1	1

We define the divisor $F = \{x_5 = 0\} = \mathbb{P}^1 \sqcup \mathbb{P}^1 \sqcup \mathbb{P}^1$ and the base is given by $\Sigma = \{x_2 = 0\}$, which is the torus $\mathbb{P}_{111}[3]$. One indeed finds the Hodge diamond above with $h^{11} = 2$. Moreover $c_1(W) = 2\,\Sigma + F$ so that W is non-spin. In contrast to the deformations, there is no immediate argument why the Wilson line should not be in $H^1_+(W, \mathcal{O})$.

Let us make one comment motivated by the models discussed in [28, 29]. Such a W-divisor is not a Calabi-Yau twofold so that, opposed to $K3$ or T^4, it cannot be fibered over a base to give a Calabi-Yau threefold. However, the possibility of generating poly-instanton effects with the $K3$ or T^4 fibered geometries used in [28, 29] is not entirely ruled out in a more involved setup with fluxes.

3 Orientifolds with poly-instantons

As a proof of principle, we now present concrete examples of Type IIB orientifolds on compact Calabi-Yau threefolds which contain such W-surfaces as toric divisors. This includes that we explicitly identify orientifold projections so that the divisor W is $O(1)$ with $H^1_+(W, \mathcal{O}) = 1$. With future applications in the framework of the LARGE volume scenario in mind, we focus on threefolds which also have shrinkable del Pezzo surfaces, thus featuring a swiss-cheese type Kähler potential. We are here applying the rules for Type IIB orientifold model building laid out in [9–12], to which we refer for more information.

As a word of warning: we will employ the sufficient conditions for certain instanton corrections laid out in the last section. This means that the superpotentials we write down might not be complete.

3.1 Example A

In this subsection we present a concrete Type IIB orientifold on a Calabi-Yau manifold that not just admits a divisor W but in addition also two rigid and shrinkable del Pezzo divisors.

[3] It was proposed that a T^4-divisor is a potential candidate [28, 29]. Clearly, such a divisor has more zero modes as

$$\left(h^{00}(T^4), h^{10}(T^4), h^{20}(T^4), h^{11}(T^4)\right) = (1, 2, 1, 4)\,. \tag{2.14}$$

and one has to make the deformation and one Wilson line Goldstino massive via for instance turning on fluxes. For the Wilson line Goldstino a pure gauge flux does not work [43].

The threefold is defined by the following toric data

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
2	-1	0	1	1	0	0	0	1
4	-2	0	2	2	1	0	1	0
2	-3	0	2	1	1	1	0	0
2	1	1	0	0	0	0	0	0

and has Hodge numbers $(h^{21}, h^{11}) = (72, 4)$ with Euler number $\chi = -136$. The Stanley-Reisner ideal reads

$$\mathrm{SR} = \{x_1\, x_2, x_4\, x_7, x_5\, x_7, x_1\, x_4\, x_8, x_2\, x_5\, x_6, x_3\, x_4\, x_8, x_3\, x_5\, x_6, x_3\, x_6\, x_8\} \,. \tag{3.1}$$

The intersection form is most conveniently displayed choosing the basis of smooth divisors as $\{D_1, D_6, D_7, D_8\}$. Then, the triple intersections on the Calabi-Yau threefold have the form

$$\begin{aligned} I_3 = 9D_1^3 - 3D_1^2 D_6 - 4D_6 D_8^2 + D_1 D_6^2 - 3D_6^3 + 2D_7 D_8^2 + 2D_6 D_7 D_8 \\ + 2D_6^2 D_7 - 2D_7^2 D_8 - 2D_6 D_7^2 + 2D_7^3 \,. \end{aligned} \tag{3.2}$$

Writing the Kähler form in the above basis of divisors as $J = t_1 D_1 + t_6 D_6 + t_7 D_7 + t_8 D_8$, the resulting volume form in terms of two-cycle volumes t_i takes the form

$$\begin{aligned} \mathcal{V} \equiv \frac{1}{3!} \int_{\mathcal{M}} J \wedge J \wedge J = \frac{1}{6} \Big(9t_1^3 - 9t_1^2 t_6 + 3t_1 t_6^2 - 3t_6^3 + 6t_6^2 t_7 - 6t_6 t_7^2 + 2t_7^3 \\ + 12 t_6 t_7 t_8 - 6t_7^2 t_8 - 12 t_6 t_8^2 + 6 t_7 t_8^2 \Big) \,. \end{aligned} \tag{3.3}$$

Expanding the Kähler form as $J = r^i [K_i]$, for the following four divisors $\{K_i, i = 1, \ldots, 4\}$ the Kähler cone is given simply by $r^i > 0$:

$$\begin{aligned} K_1 &= 2D_6 + 2D_7 + D_8, \\ K_2 &= 2D_6 + 3D_7 + D_8, \\ K_3 &= D_1 + 4D_6 + 4D_7 + 2D_8, \\ K_4 &= D_1 + 5D_6 + 5D_7 + 2D_8 \,. \end{aligned} \tag{3.4}$$

For the Kähler parameters t_i this translates into

$$-t_6 + t_7 > 0,\ \ -2t_1 + t_6 - t_7 + t_8 > 0,\ \ t_1 - t_6 + 2t_8 > 0,\ \ t_6 - 2t_8 > 0 \,. \tag{3.5}$$

Defining the four-cycle volumes

$$\tau_i = \frac{1}{2} \int_{D_i} J \wedge J \,, \tag{3.6}$$

we find

$$\begin{aligned} \tau_1 &= \tfrac{1}{2}(-3t_1 + t_6)^2, \\ \tau_6 &= -\tfrac{3}{2} t_1^2 + t_1 t_6 - \tfrac{3}{2} t_6^2 + 2t_6 t_7 - t_7^2 + 2t_7 t_8 - 2t_8^2, \\ \tau_7 &= (t_6 - t_7 + t_8)^2, \\ \tau_8 &= (2t_6 - t_7)(t_7 - 2t_8) \,. \end{aligned} \tag{3.7}$$

Taking into account the Kähler cone constraints (3.5), the volume can be written in the strong swiss-cheese form

$$\mathcal{V} = \frac{1}{9}\Big(\frac{1}{\sqrt{2}}(\tau_1 + 3\tau_6 + 6\tau_7 + 3\tau_8)^{3/2} - \sqrt{2}\tau_1^{3/2} - 3\tau_7^{3/2} - 3(\tau_7 + \tau_8)^{3/2}\Big)\,. \tag{3.8}$$

The above volume form shows that the large volume limit is given by $\tau_6 \to \infty$ while keeping the other four-cycles small.

Computing the Hodge diamonds, one finds that the divisor D_1 is a $\mathbb{P}^2$ surface, D_7 is a dP_7 surface and the divisor D_8 is indeed the desired Wilson line divisor W. Moreover, the strong swiss-cheese form of the volume implies that both the $\mathbb{P}^2$ and the dP_7 divisor are shrinkable to a point in $\mathcal{M}$.

From the volume (3.8) it seems that the divisor $D_7 + D_8$ might also be a del Pezzo surface, but, as apparent from the toric data, the only monomial of this degree is $x_7\, x_8$. Therefore, this defines a singular surface which is just the intersection of the dP_7 and the W divisors. It can be shown that these two divisors intersect over a genus one curve in the Calabi-Yau threefold. One can also show that the intersection $D_5 \cap D_8$ in the Calabi-Yau threefold is also a T^2 curve, while D_1 intersects D_5 in a genus zero curve and does not intersect the divisors D_7 and W. See also table 3.

Orientifold projections. Next, we have to specify an orientifold projection so that W is $O(1)$ and that the Wilson line Goldstino is in $H^1_+(W, \mathcal{O})$. Restricting to the case that we just flip the sign of one homogeneous coordinate, we find two inequivalent involutions $\sigma : \{x_7 \leftrightarrow -x_7, x_4 \leftrightarrow -x_4\}$, which have $h^{11}_-(\mathcal{M}) = 0$.

Involution $\sigma : x_7 \leftrightarrow -x_7$. Let us discuss the involution $\sigma : x_7 \leftrightarrow -x_7$ in more detail. Taking also the constraints from the Stanley-Reisner ideal into account, the fixed point set of the toric four-fold is

$$\{\mathrm{Fixed}_{x_7 \leftrightarrow -x_7}\} = \{D_5\,,\, D_7\,,\, D_1 D_4 D_6\,,\, D_2 D_4 D_6\}\,. \tag{3.9}$$

This fixed point set intersects the respective σ-invariant hypersurface(see below) so that we get a number of $O7$ and $O3$-planes. Concretely, there are two $O7$-components

$$O7 = D_5 \sqcup D_7\,. \tag{3.10}$$

In order to determine the $O3$-planes, using the intersection form (3.2), we can compute the triple intersection numbers as

$$D_1\, D_4\, D_6 = 1, \qquad D_2\, D_4\, D_6 = 4 \tag{3.11}$$

so that in total there are five $O3$-planes. As mentioned, the $O7$-component D_7 is a dP_7 and the independent Hodge numbers of the D_5 are given by

$$(h^{00}(D_5), h^{10}(D_5), h^{20}(D_5), h^{11}(D_5)) = (1, 0, 1, 21) \tag{3.12}$$

with Euler number $\chi(D_5) = 25$.

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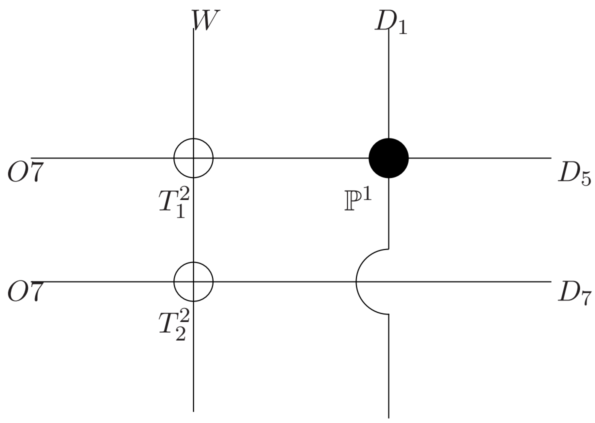

Figure 1. Relative positions of $O7$-planes and divisor W for example A with involution $\sigma : x_7 \leftrightarrow -x_7$.

Applying the Lefschetz theorems (2.11) and (2.12), we find that the Hodge numbers of the W-divisor D_8 are

$$(h^{00}(W), h^{10}(W), h^{20}(W), h^{11}(W)) = (1_+, 1_+, 0, 2_+)\,, \tag{3.13}$$

which is what we want for poly-instanton corrections. The divisor $D_1 = \mathbb{P}^2$ has the Hodge numbers

$$(h^{00}(D_1), h^{10}(D_1), h^{20}(D_1), h^{11}(D_1)) = (1_+, 0, 0, 1_+)\,. \tag{3.14}$$

Moreover, the relative positions of W, D_1 and the two components of the $O7$-plane are shown in figure 1.

The simplest solution to the $D7$-brane tadpole cancellation condition is that we place eight $D7$-branes right on top of the $O7$-plane. We cancel the Freed-Witten anomalies for branes on the divisors D_1, D_7 and D_8 by choosing $\mathcal{F}_1 = \mathcal{F}_7 = \mathcal{F}_8 = 0$ and turning on a global half-integer quantized B field with $c_1(B) = \frac{1}{2}(D_1 + D_7 + D_8)$. Using (2.7) the contribution to the $D3$-brane tadpole is

$$N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = \frac{N_{O3}}{4} + \frac{\chi(D_{O7})}{4} = \frac{5 + (10 + 25)}{4} = 10\,. \tag{3.15}$$

In this case, we will get two contributions to the non-perturbative superpotential. First, there will be an $E3$-instanton wrapping the divisor $D_1 = \mathbb{P}^2$ and second, the $\mathcal{N} = 1$ super Yang-Mills gauge theory on the dP_7 divisor develops a gaugino condensate. Thus, one gets a non-perturbative superpotential of the form

$$W = A_1 \exp(-2\pi T_1) + A_7 \exp(-a_7 T_7)\,, \tag{3.16}$$

where the T_i's are the complexified Kähler moduli appearing in the $\mathcal{N} = 1$ chiral supermultiplet. It is defined as $T_i = \tau_i + i\rho_i$ for a holomorphic, isometric involution such that $h^{1,1}_-(\mathcal{M}) = 0$, where the ρ_i denote the components of the C_4 axion. The divisor W seems to have the right zero modes to generate a poly-instanton correction to this. However, there is

Divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	Intersection curve
$D_7 = dP_7$	$(1_+, 0, 0, 8_+)$	$W : C_{g=1}$
D_5	$(1_+, 0, 1_+, 21_+)$	$W : C_{g=1}$
$D_8 = W$	$(1_+, 1_+, 0, 2_+)$	$D_5 : C_{g=1}, \quad D_7 : C_{g=1}$
$D_1 = \mathbb{P}^2$	$(1_+, 0, 0, 1_+)$	$D_5 : C_{g=0}$

Table 3. Divisors and their equivariant cohomology. The first two line are $O7$-plane components and the second two divisors can support $E3$ instantons. The D_7 divisor also supports gaugino condensation.

one subtlety with possible charged matter zero modes appearing on the intersection curve $C = E3 \cap D7$, i.e. localized at $D_1 \cap D_5 = \mathbb{P}^1$ and $D_{5,7} \cap D_8 = T^2$ in our case.

Since there is no non-trivial line bundle carried by the $D7$-branes, the number of such matter zero modes is counted by the cohomology groups $H^i(C, K_C^{1/2})$, where $i = \{0, 1\}$ and $K_C^{1/2}$ is the spin-bundle of C. Since $D_1 \cap D_5 = \mathbb{P}^1$ and $H^*(\mathbb{P}^1, \mathcal{O}(-1)) = (0, 0)$, there will be no extra matter zero modes. Since W intersects the $\mathrm{SO}(8) \times \mathrm{SO}(8)$ stacks of $D7$ branes over a T^2 and $H^*(T^2, \mathcal{O}) = (1, 1)$, there appear extra vector-like zero modes. As discussed in [9], these zero mode can pair up and become massive, if one has a non-trivial Wilson line on T^2. For this purpose, one must have the freedom to turn on an additional gauge bundle on the $D7$-brane divisor D, whose restriction on the curve $C = T^2$ is a non-trivial Wilson line. Therefore, an additional gauge bundle R which is supported only on two-cycles $C_i \subset D$ that are topological trivial in $\mathcal{M}$ but do intersect with the curve C, allows one to avoid these extra zero modes.

In our case, both D_5 and D_7 have more two-cycles than the ambient Calabi-Yau space so that there must exist such trivial two-cycles. Since the $D7$ branes lie right on top of the $O7$-planes, all two cycles in $H^{11}(D_7)$ and $H^{11}(D_5)$ are invariant. Therefore, turning on diagonal $\mathrm{U}(1)$ gauge flux along the trivial 2-cycles breaks the gauge symmetry from $\mathrm{SO}(8) \times \mathrm{SO}(8)$ to $\mathrm{U}(4) \times \mathrm{U}(4)$. This shows that the divisor W indeed generates a poly-instanton correction to (3.16) of the form

$$W = A_1 \exp(-2\pi T_1) + A_1 A_8 \exp(-2\pi T_1 - 2\pi T_8) + A_7 \exp(-a_7 T_7) + A_7 A_8 \exp(-a_7 T_7 - 2\pi T_8) + \ldots \, . \tag{3.17}$$

This simple example serves as a proof of principle that: a.) the divisor W can indeed by embedded into a compact Calabi-Yau threefold and b.) an orientifold projection can be identified so that it is $O(1)$ and $H^1_+(W, \mathcal{O}) = 1$.

To summarize, in our geometry we find divisors with the topological data listed in table 3.

Involution $\sigma : x_4 \leftrightarrow -x_4$. Let us briefly mention what happens for the second involution $x_4 \leftrightarrow -x_4$. Here, the fixed point set reads

$$\{\mathrm{Fixed}_{x_4 \leftrightarrow -x_4}\} = \{D_4,\, D_1 D_3 D_8\,,\, D_1 D_5 D_6\,, D_1 D_6 D_7\,,\, D_2 D_3 D_8\,,\, D_2 D_6 D_7\}\,. \tag{3.18}$$

Divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	Intersection curve
D_4	$(1_+, 0, 2_+, 29_+)$	$D_1 : C_{g=0}$
$D_7 = dP_7$	$(1_+, 0, 0, 5_+ + 3_-)$	$W : C_{g=1}$
$D_8 = W$	$(1_+, 1_+, 0, 2_+)$	$D_7 : C_{g=1}$
$D_1 = \mathbb{P}^2$	$(1_+, 0, 0, 1_+)$	$D_4 : C_{g=0}$

Table 4. Divisors and their equivariant cohomology. The first line is an $O7$-plane and the last three divisors support $E3$ instantons.

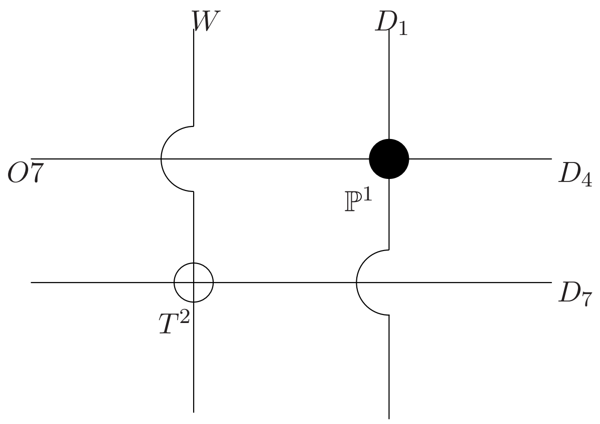

Figure 2. Relative position of $O7$-planes and divisor W for example A with involution $\sigma : x_4 \leftrightarrow -x_4$.

Therefore, we have a single $O7$-plane component on D_4 with $\chi(D_4) = 35$, while there are five $O3$-planes with one located at $D_1 \cap D_5 \cap D_6$ and four located at $D_2 \cap D_6 \cap D_7$. The contribution to the $D3$-brane tadpole in this case is $\frac{\chi(D_{O7})}{4} + \frac{N_{O3}}{4} = \frac{35+5}{4} = 10$.

We can employ **cohomCalg** in order to calculate the equivariant cohomology of the divisors. The relevant ones and their topological data are summarized in table 4. The relative positions of these divisors are shown in figure 2.

Since the $O7$-plane and the accompanying stack of eight $D7$-branes only intersects D_1 in a genus zero curve $\mathbb{P}^1$, there are no extra vector-like zero modes. Therefore, the gauge symmetry is SO(8).

For this model we can identify two kinds of non-perturbative contributions to the superpotential and their poly-instanton corrections. First, an $E3$ instanton wrapping the surface $D_1 = \mathbb{P}^2$ contributes to the superpotential and a second $E3$ instanton wrapping W generates a poly-instanton correction to that. Second, an $E3$-instanton on $D_7 = dP_7$ contributes to the superpotential. Since, $h^{11}_-(D_7) = 3$, there can be gauge fluxes in $F \in H^{11}_-(D_7)$ which, as mentioned in section 2, do not spoil the $O(1)$-property. Therefore, we have a sum over these fluxed instantons. Moreover, in general these fluxes will also make the vector-like zero modes on the $T^2 = W \cap D_7$ massive so that the fluxed instantons also receive a poly-instanton contribution.

From these considerations, for this model we expect a rather intricate total superpotential with leading order terms

$$\begin{aligned} W =& A_1 \exp\left(-2\pi T_1\right) + A_1\, A_8 \exp\left(-2\pi T_1 - 2\pi T_8\right) + A_7 \exp\left(-2\pi T_7\right) + \\ & \sum_{F_i \in H^{11}_{-}(D_7)} A_7^{(i)} \exp\left(-2\pi T_7 - f_i \tau\right) + A_7^{(i)}\, A_8 \exp\left(-2\pi T_7 - f_i\tau - 2\pi T_8\right) \ldots \end{aligned} \tag{3.19}$$

where τ denotes the axio-dilaton superfield and $f_i \simeq \int_{D_7} F^i \wedge F^i$ depends on the gauge flux.

3.2 Example B

In our scan for Calabi-threefolds having the desired divisors, we also found an example for which not all Kähler deformations are torically realized. The Calabi-Yau $\mathcal{M}$ is given by a hypersurface in a toric variety with defining data

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
3	1	0	0	0	0	0	0	1
4	1	1	1	0	0	0	1	0
12	4	3	3	0	1	1	0	0
6	2	2	1	1	0	0	0	0

with Hodge numbers $(h^{21}, h^{11}) = (89, 5)$ and Euler number $\chi = -168$. Since $h^{11} = 5$ exceeds the number of toric equivalence relations, one Kähler deformation is non-toric. The Stanley-Reisner ideal is

$$\begin{aligned} \mathrm{SR} = \big\{ & x_3\, x_4\,,\; x_3\, x_7\,,\; x_7\, x_8\,,\; x_1\, x_2\, x_4\,, x_1\, x_2\, x_8\,,\; x_1\, x_5\, x_6\,, \\ & x_2\, x_4\, x_7\,,\; x_3\, x_5\, x_6\,,\; x_5\, x_6\, x_8 \big\}\,. \end{aligned} \tag{3.20}$$

The triple intersection form in the basis of smooth divisors $\{D_4, D_6, D_7, D_8\}$ reads

$$\begin{aligned} I_3 = 9D_4^3 - 9D_4^2 D_8 + 9D_4 D_8^2 - 3D_4^2 D_6 + 3D_4 D_6 D_8 - 6D_6 D_8^2 \\ + D_4 D_6^2 - D_6^3 + 2D_6^2 D_7 - 6D_6 D_7^2 + 18D_7^3\,. \end{aligned} \tag{3.21}$$

Writing the Kähler form in the above basis of divisors as $J = t_4 D_4 + t_6 D_6 + t_7 D_7 + t_8 D_8$, the resulting volume form in terms of two-cycle volumes t_i is given as,

$$\begin{aligned} \mathcal{V} \equiv \frac{1}{3!} \int_{\mathcal{M}} J \wedge J \wedge J = \frac{1}{6} \Big(& 9t_4^3 - t_6^3 + 6t_6^2 t_7 + 18t_7^3 - 9t_4^2 (t_6 + 3t_8) \\ & + 3t_4 (t_6 + 3t_8)^2 - 18 t_6 (t_7^2 + t_8^2) \Big)\,. \end{aligned} \tag{3.22}$$

There are four generators $\{K_i, i = 1, 2, 3, 4\}$ for the toric Kähler cone. We can expand the Kähler form as $J = r^i [K_i]$ with $r^i > 0$ and

$$\begin{aligned} K_1 &= 2D_4 + 4D_6 + D_7 + D_8\,, \\ K_2 &= D_4 + 3D_6 + D_7\,, \\ K_3 &= 6D_4 + 12D_6 + 4D_7 + 3D_8\,, \\ K_4 &= 2D_4 + 6D_6 + 2D_7 + D_8\,. \end{aligned} \tag{3.23}$$

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In the basis $J = t_4 D_4 + t_6 D_6 + t_7 D_7 + t_8 D_8$ the Kähler cone is given by

$$t_6 - 3t_7 > 0,\ t_4 - 2t_8 > 0,\ t_4 - t_6 + 2t_7 > 0,\ -3t_4 + t_6 + 2t_8 > 0\,. \tag{3.24}$$

For the corresponding four-cycle volumes we find

$$\begin{aligned}
\tau_4 &= \frac{1}{2}(-3t_4 + t_6 + 3t_8)^2\,,\\
\tau_6 &= -\frac{3}{2}t_4^2 - \frac{1}{2}t_6^2 + 2t_6 t_7 + t_4(t_6 + 3t_8) - 3(t_7^2 + t_8^2)\,,\\
\tau_7 &= (t_6 - 3t_7)^2\,,\\
\tau_8 &= -\frac{3}{2}(3t_4 - 2t_6)(t_4 - 2t_8)\,.
\end{aligned} \tag{3.25}$$

Taking into account the Kähler cone constraints (3.24), the volume can be written again in the strong swiss-cheese form

$$\mathcal{V} = \frac{1}{9}\left(\sqrt{2}(2\tau_4 + 3\tau_6 + \tau_7 + \tau_8)^{3/2} - \sqrt{2}\tau_4^{3/2} - \tau_7^{3/2} - \sqrt{2}(\tau_4 + \tau_8)^{3/2}\right), \tag{3.26}$$

which shows that the large volume limit is defined as $\tau_6 \to \infty$ while keeping the other four-cycles small.

Computing the Hodge diamonds, one finds that the divisor D_4 is a $\mathbb{P}^2$ and the divisor D_8 a Wilson line divisor. For the Hodge diamond of the D_7 divisor **cohomCalg** gives the output $(h^{00}(D_7), h^{10}(D_7), h^{20}(D_7), h^{11}(D_7)) = (2, 0, 0, 2)$. Therefore, the locus $x_7 = 0$ seems to have two $\mathbb{P}^2$ components, D_7' and D_7'', of which one linear combination is toric and the other related to the fact that we have one non-toric element in $H^{11}(\mathcal{M})$. This suggests that the complete volume form is given by (3.26) with the simple substitution $\tau_7^{3/2} = (\tau_7')^{3/2} + (\tau_7'')^{3/2}$.

Again, the divisor $D_4 + D_8$ has no smooth surface representing it and the intersection $D_4 \cap D_8$ is a T^2 curve. For some purposes below, it can be shown that the intersection $D_3 \cap D_8$ on the Calabi-Yau threefold is also a T^2 curve. See also table 5.

Orientifold projections. We identified two orientifold projection $\sigma : \{x_4 \leftrightarrow -x_4, x_2 \leftrightarrow -x_2\}$ so that the Wilson line Goldstino is in $H^1_+(W, \mathcal{O})$ and W supports $\mathcal{O}(1)$ instanton. For both involutions, one can determine that $h^{11}(\mathcal{M}) = 4_+ + 1_-$. Hence the $\mathcal{N} = 1$ Kähler coordinates T_α will be modified by the presence of the single odd-modulus

$$G = \int_{D_-} B_2 + i \int_{D_-} C_2\,, \tag{3.27}$$

where $D_- \in H^{11}_-(\mathcal{M})$.

Involution $\sigma : x_4 \leftrightarrow -x_4$. Under this involution the fixed point set is

$$\{\text{Fixed}_{x_4 \leftrightarrow -x_4}\} = \{D_3\,,\ D_4\,,\ D_1 D_2 D_7\,,\ D_2 D_5 D_6\}\,. \tag{3.28}$$

Thus, there are the two $O7$-components

$$O7 = D_3 \sqcup D_4. \tag{3.29}$$

Divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	intersection curve
D_3	$(1_+, 0, 4_+, 50_+)$	$W : C_{g=1}$
$D_4 = \mathbb{P}^2$	$(1_+, 0, 0, 1_+)$	$W : C_{g=1}$
$D_7 = \mathbb{P}^2 \sqcup \mathbb{P}^2$	$(1_+ + 1_-, 0, 0, 1_+ + 1_-)$	null
$D_8 = W$	$(1_+, 1_+, 0, 2_+)$	$D_3 : C_{g=1}, \quad D_4 : C_{g=1}$

Table 5. Divisors and their equivariant cohomology. The first two line are $O7$-plane components and the last two divisors support $E3$ instantons. The D_4 divisor also supports gaugino condensation.

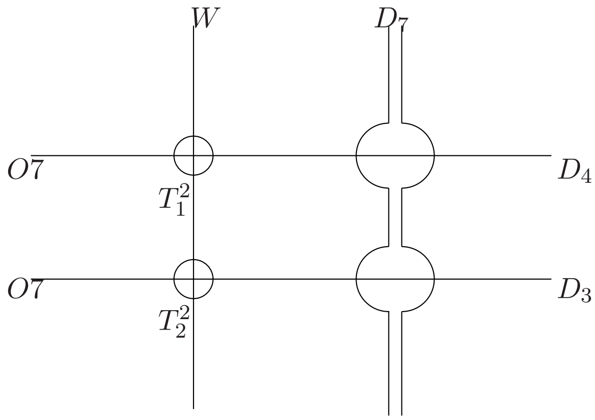

Figure 3. Relative position of $O7$-planes and divisor W for example B with involution $\sigma : x_4 \leftrightarrow -x_4$.

With the help of the intersection form (3.21), we find that $D_1 \cap D_2 \cap D_7$ does not intersect the hypersurface, while there is one $O3$-plane on $D_2 \cap D_5 \cap D_6$. For $D7$-brane tadpole cancellation, we again place eight $D7$-branes right on top of the $O7$-plane. The contribution to the $D3$-brane tadpole becomes

$$\frac{\chi(D_{O7})}{4} + \frac{N_{O3}}{4} = \frac{(60+3)+1}{4} = 16\,. \tag{3.30}$$

Since there is no non-trivial gauge field configuration on these $D7$-branes(see below), the net $D5$-brane charge vanishes too. For the relevant divisors, we find the topological data listed in table 5.

Note that the equivariant cohomology for D_7 suggests that its two $\mathbb{P}^2$ components are interchanged by the orientifold projection. For such a σ to be a symmetry one in particular needs $\tau_7' = \tau_7''$. The positions of these divisors are also shown in figure 3.

This model has some features which differ from example A. First, since $D_4 = \mathbb{P}^2$ one does not have the degree of freedom to give a mass to the vector-like matter zero modes on the $T^2 = W \cap D_4$ intersection. Second, under σ the two $\mathbb{P}^2$ components of D_7 are interchanged so that this is not an $O(1)$ instanton (but a U(1) instanton with a non-zero number of $\overline{\tau}_{\dot\alpha}$ zero modes).

Even though the superpotential receives a contribution from the gaugino condensation of the pure SO(8) super Yang-Mills theory on the divisor D_4, due to the vector-like matter zero modes, the W divisor has too many zero modes to generate an additional poly-instanton contribution. Although the superpotential has the simple form

Divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	intersection curve
D_2	$(1_+, 0, 5_+, 51_+)$	null
$D_4 = \mathbb{P}^2$	$(1_+, 0, 0, 1_+)$	$W : C_{g=1}$
$D_7 = \mathbb{P}^2 \sqcup \mathbb{P}^2$	$(1_+ + 1_-, 0, 0, 1_+ + 1_-)$	null
$D_8 = W$	$(1_+, 1_+, 0, 2_+)$	$D_4 : C_{g=1}$

Table 6. Divisors and their equivariant cohomology. The first line is a $O7$-plane component and the last three divisors support $E3$ instantons.

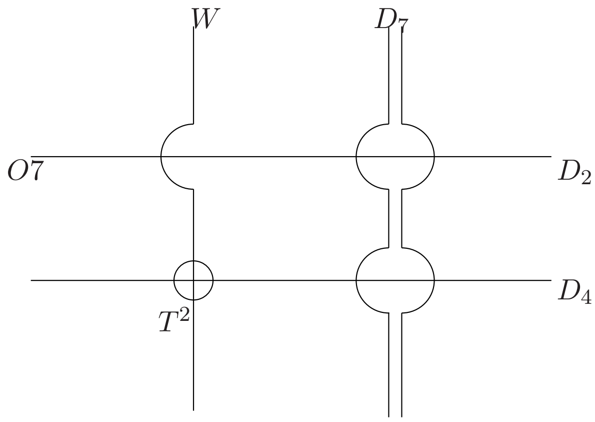

Figure 4. Relative position of $O7$-planes and divisor W for example B with involution $\sigma : x_2 \leftrightarrow -x_2$.

$W = A_4 \exp(-a_4 T_4)$, we think that the presentation of this model was nevertheless useful for illustrative purposes.

Involution $\sigma : x_2 \leftrightarrow -x_2$. For this involution the fixed point set consistent with the SR-ideal is

$$\{\mathrm{Fixed}_{x_2 \leftrightarrow -x_2}\} = \{D_2 \, , \; D_1 D_4 D_7 \, , \; D_4 D_5 D_6\} \, . \tag{3.31}$$

The $O7$-plane is located on the divisor D_2 with $\chi(D_2)$=63. In addition there is one $O3$-plane given by $D_4 \cap D_5 \cap D_6$. Placing eight $D7$-branes on top of the $O7$-plane, the contribution to the $D3$-brane tadpole is

$$\frac{\chi(D_{O7})}{4} + \frac{N_{O3}}{4} = \frac{63+1}{4} = 16 \, . \tag{3.32}$$

The topological data of the interesting divisors are summarized in table 6. Their relative position is depicted in figure 4.

In this case, still the pair of $\mathbb{P}^2$s wrapping D_7 support a U(1) instanton and the $E3$ instanton wrapping D_4 does not receive a poly-instanton correction, as the vector-like zero modes on $W \cap D_4$ cannot be made massive. Therefore, we have only the simple superpotential $W = A_4 \exp(-2\pi T_4)$.

3.3 Example C

The last example provides a model where components of the orientifold $O7$-plane are non-generic. The Calabi-Yau $\mathcal{M}$ is given by a hypersurface in a toric variety with defining data

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
2	1	0	0	0	0	0	0	1
3	1	1	0	0	0	0	1	0
9	3	3	0	1	1	1	0	0
4	2	1	1	0	0	0	0	0

and has Hodge numbers $(h_{21}, h_{11}) = (112, 4)$ and Euler characteristic $\chi(\mathcal{M}) = -216$. The resulting Stanley-Reisner ideal is

$$\mathrm{SR} = \{x_1\, x_3, x_1\, x_8, x_2\, x_3, x_2\, x_7, x_7\, x_8, x_4\, x_5\, x_6\}\ . \tag{3.33}$$

The intersection form is most conveniently displayed choosing the basis of smooth divisors as $\{D_1, D_3, D_7, D_8\}$. Then, the triple intersections on the Calabi-Yau threefold have the form

$$I_3 = 27D_1^3 + 9\, D_7^3 + 9\, D_3^3 - 9\, D_3^2\, D_8 + 9\, D_3\, D_8^2\,. \tag{3.34}$$

Expanding the Kähler form as $J = t_1\, D_1 + t_3\, D_3 + t_7\, D_7 + t_8\, D_8$, the volume of the Calabi-Yau is given by

$$\mathcal{V} \equiv \frac{1}{3!}\int_{\mathcal{M}} J\wedge J\wedge J = \tfrac{1}{2}\left(9\, t_1^3 + 3\, t_7^3 + 3\, t_3^3 - 9\, t_3^2\, t_8 + 9\, t_3\, t_8^2\right)\ . \tag{3.35}$$

There are four generators $\{K_i, i = 1, 2, 3, 4\}$ for the toric Kähler cone. We can expand the Kähler form as $J = r^i[K_i]$ with $r^i > 0$ and

$$\begin{aligned} K_1 &= D_1 - D_3 - D_8\,,\\ K_2 &= D_1 - 2D_3 - D_7 - D_8\,,\\ K_3 &= D_1\,,\\ K_4 &= 2D_1 - 2D_3 - D_8 \end{aligned} \tag{3.36}$$

and the Kähler cone is

$$t_7 < 0, \quad t_3 - 2\, t_8 > 0, \quad t_1 + t_3 - t_7 > 0, \quad -t_3 + t_7 + t_8 > 0\,. \tag{3.37}$$

The corresponding 4-cycle volumes read

$$\tau_1 = \tfrac{27}{2}\, t_1^2, \quad \tau_7 = \tfrac{9}{2}\, t_7^2, \quad \tau_3 = \tfrac{9}{2}\,(t_3 - t_8)^2, \qquad \tau_8 = \tfrac{9}{2}\,(2\, t_3\, t_8 - t_3^2)\,. \tag{3.38}$$

Taking into account that the Kähler cone constraints (3.37) imply $t_1 > 0$, $t_3 - t_8 < 0$ and $t_8 < 0$, the volume can be written in the strong swiss-cheese form

$$\mathcal{V} = \tfrac{\sqrt{2}}{9}\left(\tfrac{1}{\sqrt{3}}\,(\tau_1)^{\frac{3}{2}} - (\tau_7)^{\frac{3}{2}} - (\tau_3)^{\frac{3}{2}} - (\tau_3 + \tau_8)^{\frac{3}{2}}\right)\ . \tag{3.39}$$

Computing the Hodge diamond, one finds that the divisors D_3 and D_7 are $\mathbb{P}^2$ surfaces and that the divisor D_8 is a W-surface. The strong swiss-cheese form might motivate to introduce the divisor $D_3 + D_8$, but as can easily be seen from the toric data the only monomial of this degree is $x_3 \cdot x_8$, so that there is no smooth surface representing it.

Divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	intersection curve
D_2	$(1_+, 0, 10_+, 92_+)$	$W : C_{g=1}$
$D_3 = \mathbb{P}^2$	$(1_+, 0, 0, 1_+)$	$W : C_{g=1}$
$D_7 = \mathbb{P}^2$	$(1_+, 0, 0, 1_+)$	null
$D_8 = W$	$(1_+, 1_+, 0, 2_+)$	$D_2 : C_{g=1}, \quad D_3 : C_{g=1}$

Table 7. Divisors and their equivariant cohomology. The first three lines are $O7$-plane components and the last one supports an $E3$ instanton.

Orientifold projection. There exist two inequivalent orientifold projections $\sigma : \{x_1 \leftrightarrow -x_1, x_3 \leftrightarrow -x_3\}$ featuring that the Wilson line goldstino is in $H^1_+(W, \mathcal{O})$ and $h^{11}_-(\mathcal{M}) = 0$. However, only under the second involution $x_3 \leftrightarrow -x_3$ the W divisor is $\mathcal{O}(1)$ while for the first involution W is of SP-type.

Involution $\sigma : x_3 \leftrightarrow -x_3$. The fixed point locus in this case is a bit intricate to find. First, there is again a component given by $O7 = D_2 \sqcup D_3$. However, looking more closely one finds that the intersection of the two divisors $D_1 \cap D_7$ gives a $\mathbb{P}^2$ surface in the toric ambient space. By the equivalence relations it is fixed under σ. The hypersurface, in the orientifold containing only polynomials invariant under σ, intersects this $\mathbb{P}^2$ non-generically.[4] In fact, it lies already completely on the hypersurface. Since D_7 intersects the hypersurface also in a $\mathbb{P}^2$, it means that these two $\mathbb{P}^2$ become identical for the σ-invariant restricted hypersurface. Therefore, in total we have three $O7$-components of the fixed point locus on $\mathcal{M}$

$$O7 = D_2 \sqcup D_3 \sqcup D_7 \,, \tag{3.40}$$

and no $O3$-planes. Therefore, the $\mathbb{P}^2$ divisors D_3, D_7 are occupied by components of the $O7$-plane. Therefore, the contribution to the $D3$-brane tadpole is $\frac{\chi(O7)}{4} = \frac{114+3+3}{4} = 30$. The topological data of relevant divisors is shown in table 7. The relative position of W to the three components of the $O7$-plane is shown in figure 5.

Now, we face the same situation as in Example B, namely that the vector-like zero modes coming from the $T^2 = W \cap D_3$ intersection cannot be made massive. Thus, we just have the superpotential for the gaugino condensation on the pure SO(8) super Yang-Mills theories localized on the stacks of $D7$ branes wrapping D_3 and D_7

$$W = A_3 \exp(-a_3 T_3) + A_7 \exp(-a_7 T_7) \,. \tag{3.41}$$

4 Conclusions

In this paper we have investigated under what circumstances there can be poly-instanton corrections for Type IIB orientifolds of the type $\Omega\sigma(-1)^{F_L}$ with $O7$- and $O3$-planes. We worked out the zero mode structure for an $E3$-instanton wrapping a surface in the Calabi-Yau threefold. In principle, the required single additional fermionic zero mode could arise

[4]We are grateful to Christoph Mayrhofer to pointing this out to us.

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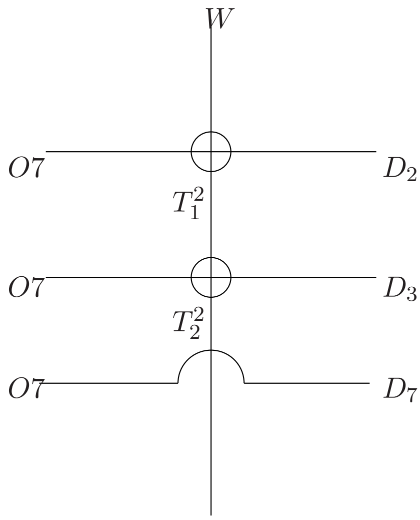

Figure 5. Relative position of $O7$-planes and divisor W for example C with involution $\sigma : x_3 \leftrightarrow -x_3$.

from a deformation or a Wilson line modulino of the surface. However, we found that holomorphicity, i.e. the requirement the instanton to be $O(1)$, rules out the possibility of a deformation zero mode. Therefore, one needs precisely one Wilson line modulino in $H^1_+(E, \mathcal{O})$.

We proposed that examples of such surfaces are given by $\mathbb{P}^1$ fibrations over two-tori and presented a couple of concrete Calabi-Yau threefolds where such divisors appear. Here we were concentrating on threefolds which also contained a couple of swiss-cheese type del Pezzo surfaces. Moreover, we were also specifying a couple of admissible orientifold projections and worked out the relevant equivariant cohomology. These models are still quite simple but they can serve both as a proof of principle that poly-instanton corrections are possible and as proto-type examples for further studies on moduli stabilization and inflation. Clearly, our derivation of the contributions to the superpotential did sensitively depend on the chosen configuration of tadpole canceling $D7$-branes. As emphasized in [14], the instanton zero mode structure crucially depends on the positions of all $D7$-branes.

As an intriguing observation, we found that the volume form took a very peculiar strong swiss-cheese like form. The implied tree-level Kähler potential is of the schematic form[5]

$$K = -2\log\left(a(\tau_b)^{\frac{3}{2}} - b(\tau_s)^{\frac{3}{2}} - c(\tau_s+\tau_w)^{\frac{3}{2}}\right)\,, \tag{4.1}$$

and the poly-instanton generated superpotential

$$W = A\,\exp(-a_s T_s) + B\,\;\exp(-a_s\, T_s - a_w\, T_w)\,. \tag{4.2}$$

can serve as the starting point of a discussion of moduli stabilization and inflation for this class of models [34].

[5]Apart from the difference in the fibration part, a similar volume form has been used in [30].

Acknowledgments

We would like to thank Christoph Mayrhofer and Timo Weigand for helpful discussions and Michele Cicoli also for useful comments about the manuscript. R.B. is grateful to the Simons Center for Physics and Geometry at Stony Brook University for hospitality. XG is supported by the MPG-CAS Joint Doctoral Promotion Programme. PS is supported by a postdoctoral research fellowship from the Alexander von Humboldt Foundation.

A K3-fibration

In this appendix, we analyze one of the $K3$ fibrations found in [11] from the new perspective developed in this paper.

As discussed in section **2**, without invoking additonal mechanisms, an instanton supported on the $K3$ fiber does not generate a poly-instanton correction to the superpotential. However, it is still possible to have such a correction as long as the underlying (fibred) Calabi-Yau threefold contains a Wilson line divisor being of type $O(1)$ with $H^1_+(W, \mathcal{O}) = 1$ under an appropriate holomorphic involution.

As an example, consider the Calabi-Yau threefold $\mathcal{M}$ defined by the following toric data

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
12	0	1	0	0	2	2	6	1
6	0	0	0	1	1	1	3	0
4	0	0	1	0	0	1	2	0
6	1	1	0	0	1	0	3	0

with SR-ideal:

$$\mathrm{SR} = \{x_1\, x_2, x_2\, x_8, x_1\, x_3, x_1\, x_4, x_4\, x_5, x_3\, x_6\, x_7, x_5\, x_6\, x_7\, x_8\} \,. \tag{A.1}$$

This Calabi-Yau has been also presented in appendix of [11], from where we recall some of the relevant information. In the divisor basis $\{D_1, D_2, D_3, D_4\}$ the triple intersection form is given as,

$$I_3 = D_1^3 - 2D_2D_3^2 + 4D_3^2 + 4D_3^3 + 2D_2D_3D_4 - 4D_3D_4^2. \tag{A.2}$$

Expanding the Kähler form as $J = t_1\, D_1 + t_2\, D_2 + t_3\, D_3 + t_4\, D_4$, the volume of the Calabi-Yau is given by

$$\mathcal{V} \equiv \frac{1}{3!}\int_{\mathcal{M}} J \wedge J \wedge J = \frac{t_1^3}{6} - t_2 t_3^2 + \frac{2t_3^3}{3} + 2t_2t_3t_4 - 2t_3t_4^2\,. \tag{A.3}$$

D_1 is a shrinkable del Pezzo dP_8, D_2 is a $K3$, D_3 is a non-shrinkable dP_5 and the D_4 is the relevant Wilson line divisor W. There are four generators $\{K_i, i = 1, 2, 3, 4\}$ for the toric Kähler cone and expanding the Kähler form as $J = r^i[K_i]$, the Kähler cone is given

Divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	Intersection curve
$D_2 = K3$	$(1_+, 0, 1_+, 20_+)$	$W : C_{g=1}$
$D_8 = dP_{10}$	$(1_+, 0, 0, 11_+)$	$W : C_{g=1}, \quad D_1 : C_{g=1}$
$D_1 = dP_8$	$(1_+, 0, 0, 5_+ + 4_-)$	$D_8 : C_{g=1}$
$D_4 = W$	$(1_+, 1_+, 0, 2_+)$	$D_2 : C_{g=1}, \quad D_8 : C_{g=1}$

Table 8. Divisors and their equivariant cohomology. Under $x_2 \leftrightarrow -x_2$. The first two lines are $O7$-plane components and the D_1, D_4 divisors support $E3$ instanton.

as $r^i > 0$ for

$$\begin{aligned} K_1 &= -3D_1 + 6D_2 + 2D_3 + 3D_4\,, \\ K_2 &= D_2\,, \\ K_3 &= 2D_2 + D_4\,, \\ K_4 &= 6D_2 + 2D_3 + 3D_4\,. \end{aligned} \tag{A.4}$$

The corresponding 4-cycle volumes read

$$\begin{aligned} &\tau_1 = \frac{t_1^2}{2}\,, \qquad \tau_2 = -t_3(t_3 - 2t_4)\,, \\ &\tau_3 = 2(t_3 - t_4)(-t_2 + t_3 + t_4)\,, \qquad \tau_4 = 2t_3(t_2 - 2t_4)\,. \end{aligned} \tag{A.5}$$

Taking into account the Kähler cone constraints (A.4), the volume of the Calabi-Yau can be written in terms of four-cycle volumes as

$$\begin{aligned} \mathcal{V} = -\frac{\sqrt{2}}{3}\tau_1^{3/2} + \frac{1}{6\sqrt{2}}\Big(2(2\tau_2 + 2\tau_3 + \tau_4) - \sqrt{8\tau_2\tau_3 + (2\tau_3 + \tau_4)^2}\Big) \\ \times \sqrt{(2\tau_2 + 2\tau_3 + \tau_4) + \sqrt{8\tau_2\tau_3 + (2\tau_3 + \tau_4)^2}}\,. \end{aligned} \tag{A.6}$$

The above volume form is quite complicated and one way to define the large volume limit is taking the large fiber limit $\tau_2 \to \infty$ while keeping the other divisor volumes to be relatively small. In this limit, the above volume expression (A.6) reduces to

$$\mathcal{V}|_{\tau_2 \to \infty} = \tfrac{2}{3}\tau_2^{3/2} + \tfrac{1}{2}(\tau_3 + \tau_4)\sqrt{\tau_2} - \tfrac{\sqrt{2}}{3}\tau_1^{3/2} + \tfrac{1}{3\sqrt{2}}\tau_3^{3/2} + \mathcal{O}({\tau_2}^{-1/2}). \tag{A.7}$$

We observe that the typical volume factor for a $K3$ fibration $(\tau_3 + \tau_4)\sqrt{\tau_2}$ indeed appears and that it now also contains the volume of the Wilson line divisor τ_4.

There exist two inequivalent orientifold projections $\sigma : \{x_2 \leftrightarrow -x_2, x_5 \leftrightarrow -x_5\}$ featuring that the Wilson line Goldstino is in $H^1_+(W, \mathcal{O})$ and $h^{11}_-(\mathcal{M}) = 0$. It can also be checked that the $D3$ and $D7$-branes tadpoles can be canceled. The topological data of the relevant divisors are shown in table 8 and table 9. Finally, it can be shown that for both involutions there are no vector-like zero modes on the intersection $E3 \cap D7$ and one gets the following form of the superpotential,

$$W = A_1 \exp(-2\pi T_1) + A_1 A_4 \exp(-2\pi\, T_1 - 2\pi\, T_4)\,. \tag{A.8}$$

Divisor	$(h^{00}, h^{10}, h^{20}, h^{11})$	Intersection curve
D_5	$(1_+, 0, 2_+, 31_+)$	$D_1 : C_{g=1}$
$D_1 = dP_8$	$(1_+, 0, 0, 5_+ + 4_-)$	$D_5 : C_{g=1}$
$D_4 = W$	$(1_+, 1_+, 0, 2_+)$	$D_2 : C_{g=1}$
$D_2 = K3$	$(1_+, 0, 1_-, 10_+ + 10_-)$	$W : C_{g=1}$

Table 9. Divisors and their equivariant cohomology. Under $x_5 \leftrightarrow -x_5$. The first line is an $O7$-plane component and the D_1, D_4 divisors support $E3$ instanton.

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