Phenomenological aspects of *R*-parity violating supersymmetry with a vectorlike extra generation

Xue Chang,* Chun Liu,† and Yi-Lei Tang‡

State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

(Received 29 January 2013; published 16 April 2013)

Phenomenological analysis to the *R*-parity violating supersymmetry with a vectorlike extra generation is performed in detail. It is found that, via the trilinear couplings, the correct neutrino spectrum can be obtained. The Higgs mass rises to 125 GeV by new up-type Yukawa couplings of vectorlike quarks with no need of very heavy superpartners. Phenomena of new heavy fermions at LHC are predicted.

DOI: 10.1103/PhysRevD.87.075012 PACS numbers: 12.60.Jv, 14.60.Hi, 14.60.St, 14.65.Jk

I. INTRODUCTION

Recently, a standard model (SM) Higgs-like particle with a mass of 125–126 GeV was discovered [1]. In the paradigm of the weak scale supersymmetry (SUSY) which aims at the naturalness of the electroweak scale, however, such a Higgs mass brings in tensions, especially the minimal SUSY SM (MSSM). Nonminimal and still natural scenarios of SUSY are thus motivated. One of them is the MSSM with a vectorlike generation [2–5]. It gives the right Higgs mass naturally, is consistent with precision electroweak measurements, and has a rich phenomenology [2–6]. In the framework of SUSY, vectorlike fermions can also be motivated by other theories beyond the SM, such as a SUSY extension with extra dimensions or with composite states [7]. So it is worth asking the question whether such a scenario also provides explanations to other problems such as neutrino masses.

Neutrino oscillations are the undoubted new physics beyond the SM. Daya Bay [8] and RENO [9] experiments recently discovered a relatively large $\theta_{13} \simeq 8.8° \pm 0.8°$. Within the framework of SUSY, in the absence of *R*-parity conservation, neutrino masses and mixings can be generated from lepton number violating (LPV) couplings [10]. This approach was extensively studied before [11]. It is known that all the neutrino experimental results, including that of oscillation phenomena like the large atmospheric mixing angle θ_{23}, the hierarchy of oscillation frequencies $\Delta m^2_{21} \ll \Delta m^2_{32}$, and the smallness of θ_{13}, can be understood in three-generation LPV MSSM. However, this needs some special requirements for relevant coupling constants and mass parameters.

Combining both considerations above, we will work in the LPV MSSM with a vectorlike extra generation [4]. While this model takes the vectorlike slepton doublets as the two Higgs doublets needed for the electroweak symmetry breaking (EWSB), the SM-like Higgs mass can be naturally 125 GeV [5]. Extra trilinear LPV couplings between ordinary fermions and vectorlike fermions provide a much larger parameter space to explain neutrino phenomena right.

In this paper, phenomenological aspects of the model will be analyzed. In Sec. II, we make a brief review of the model. In Sec. III, neutrino masses are calculated. For the neutrino physics, noting the enlarged parameter space, we consider trilinear LPV couplings carefully. One-loop contribution to neutrino masses due to new trilinear LPV couplings is calculated, theoretical analyses are performed, and numerical results are shown in detail. Besides, we analyze the SM-like Higgs mass and explicitly show that it can be increased to 125 GeV by two new Yukawa couplings of the up-type Higgs with vectorlike quarks in Sec. IV. The LHC phenomenology of the new fermions is analyzed in Sec. VI. The summary and discussions are given in the last section.

II. A BRIEF REVIEW OF THE MODEL

This model [4] is SUSY and SM gauge invariant, and *R*-parity violation with baryon number conservation is assumed. For the matter content, in addition to the ordinary three generations (3G), a vectorlike generation is introduced. Without *R*-parity conservation, this can be also thought of as that there are 4 + 1 chiral generations, where "4" stands for four chiral generations with SM quantum numbers and "1" for another chiral generation with opposite quantum numbers. The four chiral generations with same quantum numbers mix. The 1 has Dirac masses with only one combination of the 4; thus, there are always SM required three massless chiral generations and one massive vectorlike generation.

In terms of mass eigenstates (before electroweak symmetry breaking), the massive sleptons in the vectorlike generation are taken as the two Higgs doublets. New particles beyond the MSSM are the following with quantum numbers under $SU(3)_c \times SU(2)_L \times U(1)_Y$:

*chxue@itp.ac.cn
†liuc@mail.itp.ac.cn
‡tangyilei@itp.ac.cn

 © 2013 American Physical Society

$$E_4^c(1,1,2),\quad E_H^c(1,1,-2),\quad Q_4\left(3,2,\frac{1}{3}\right),\quad Q_H\left(\bar{3},2,-\frac{1}{3}\right),$$
$$U_4^c\left(\bar{3},1,-\frac{4}{3}\right),\quad U_H^c\left(3,1,\frac{4}{3}\right),\quad D_4^c\left(\bar{3},1,\frac{2}{3}\right),\quad D_H^c\left(3,1,-\frac{2}{3}\right).$$

The superpotential is conveniently written as

$$\mathcal{W} = \mathcal{W}_0 + \mathcal{W}_{\not{L}}, \tag{1}$$

where $\mathcal{W}_0$ and $\mathcal{W}_{\not{L}}$ stand for that with lepton number conservation and LPV, respectively,

$$\begin{aligned}\mathcal{W}_0 = {}& \mu H_u H_d + \mu^e E_4^c E_H^c + \mu^Q Q_4 Q_H + \mu^U U_4^c U_H^c + \mu^D D_4^c D_H^c + y_{ij}^l L_i H_d E_j^c + y_{ij}^d Q_i H_d D_j^c \\ & + y_{ij}^u Q_i H_u U_j^c + y_i^E L_i H_d E_4^c + y_i^{Q'} Q_4 H_d D_i^c + y_i^D Q_i H_d D_4^c + y^{QD} Q_4 H_d D_4^c + y_i^U Q_i H_u U_4^c \\ & + y_i^Q Q_4 H_u U_i^c + y^{QU} Q_4 H_u U_4^c + y^H Q_H H_d U_H^c + y^{H'} Q_H H_u D_H^c\end{aligned}$$

and

$$\mathcal{W}_{\not{L}} \supset \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} Q_i L_j D_k^c + \lambda_{ij}^E L_i L_j E_4^c + \lambda_{ij}^Q Q_4 L_i D_j^c + \lambda_{ij}^D Q_i L_j D_4^c + \lambda_i^{QD} Q_4 L_i D_4^c + \lambda_i^H Q_H L_i U_H^c, \tag{2}$$

where L_i, Q_i, E_i^c, D_i^c, and U_i^c, $i = 1$–3, are the first three-generation $SU(2)_L$ doublet leptons, doublet quarks, singlet charged leptons, singlet down-type quarks, and singlet up-type quarks, respectively. H_u and H_d are the up-type and down-type Higgs, respectively. Note that the term $Q_H H_u D_H^c$ in $\mathcal{W}_0$ was missed in Ref. [4].[1] And in $\mathcal{W}_{\not{L}}$ interactions of purely singlets are omitted, which are irrelevant to our study.

By assuming universality of the mass-squared terms, the alignment of the B terms the soft mass terms and the trilinear soft terms of all fermion superpartners in the model are

$$\begin{aligned}-\mathcal{L} \supset {}& M^2 \tilde{L}_i^\dagger \tilde{L}_i + M^2 H_d^\dagger H_d + M_h^2 H_u^\dagger H_u + M_E^2 \tilde{E}_m^{c\dagger} \tilde{E}_m^c + M_Q^2 \tilde{Q}_m^\dagger \tilde{Q}_m + M_U^2 \tilde{U}_m \tilde{U}_m^c + M_D^2 \tilde{D}_m^{c\dagger} \tilde{D}_m^c + M_{EH}^2 \tilde{E}_H^{c*} \tilde{E}_H^c \\ & + M_{QH}^2 \tilde{Q}_H^\dagger \tilde{Q}_H + M_{UH}^2 \tilde{U}_H^{c*} \tilde{U}_H^c + M_{DH}^2 \tilde{D}_H^{c*} \tilde{D}_H^c + (B\mu H_d H_u + B^e \mu^e \tilde{E}_4^c \tilde{E}_H^c + B^Q \mu^Q \tilde{Q}_4 \tilde{Q}_H + B^U \mu^U \tilde{U}_4^c \tilde{U}_H^c \\ & + B^D \mu^D \tilde{D}_4^c \tilde{D}_H^c + \text{H.c.}).\end{aligned} \tag{3}$$

Proper values of the new $B^{Q,U,D}\mu^{Q,U,D}$ terms are set to avoid unwanted color symmetry and purely $U(1)_Y$ symmetry breaking—see Eqs. (11) and (12) in Ref. [4]—therefore EWSB in our model is just the same as in MSSM. After EWSB, the specific fermion mass matrixes and sfermion mass-squared matrixes are given in Appendix A.

III. NEUTRINO MASSES AND MIXINGS

LPV results in nonvanishing neutrino masses. In this model, in addition to traditional R-parity violation in the MSSM, a lot more bilinear and trilinear LPV interactions are brought in through the vectorlike generation. In this work, the trilinear R-parity violating interactions will be studied. To avoid complication due to too many LPV sources, sneutrino vacuum expectation values (VEVs) will not be considered. There are several reasons for this. First, we can phenomenologically assume the universality of the soft SUSY-breaking mass terms at the weak scale, to avoid dangerously large flavor changing neutral currents, without considering any UV completion of the model. In that case, because of the alignment in bilinear terms of the superpotential and that of soft terms, R-parity violating bilinear terms can be rotated away via field redefinition, and sneutrino VEVs vanish in the physical basis. The second reason is from consideration of underlying models. SUSY breaking is introduced effectively in our model; it can result from gauge mediated SUSY breaking. Then the messenger scale can be as low as 100 TeV, even if the universality scale is at the SUSY-breaking messenger scale, the running effect is small, and the bilinear LPV is not important compared to the trilinear ones. Finally, small sneutrino VEVs can be included in the analysis nevertheless in future works, after the role of new trilinear LPV interactions gets a thorough understanding.

The trilinear LPV Lagrangian relevant to neutrino masses is from $\mathcal{W}_{\not{L}}$:

[1] It modifies the down-type fermion mass matrix and scalar mass-squared matrix. Correct ones, as well as the resulting mixing matrix, are given in Appendix A.

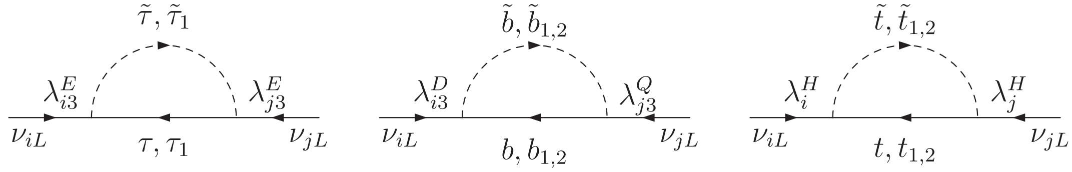

FIG. 1. New one-loop contributions to the neutrino masses and mixings from $\lambda^E\lambda^E$, $\lambda^Q\lambda^D$, and $\lambda^H\lambda^H$ type couplings. All particles stay in mass eigenstates.

$$\begin{aligned}\mathcal{L} \supset &-\lambda_{ijk}(\tilde{l}^*_{kR}\bar{\nu}^c_{iR}l_{jL} + \tilde{l}_{jL}\bar{l}_{kR}\nu_{iL}) - \lambda'_{ijk}(\tilde{d}^*_{kR}\bar{\nu}^c_{iR}d_{jL} + \tilde{d}_{jL}\bar{d}_{kR}\nu_{iL}) - \lambda^E_{ij}(\tilde{E}^c_4\bar{\nu}^c_{iR}l_{jL} + \tilde{l}_{jL}E^{cT}_4\nu_{iL}) - \lambda^Q_{ij}(\tilde{d}^*_{kR}\bar{\nu}^c_{iR}Q_4 + \tilde{Q}_4\bar{d}_{kR}\nu_{iL})\\ &- \lambda^D_{ij}(\tilde{D}^c_4\bar{\nu}^c_{iR}d_{jL} + \tilde{d}_{jL}\bar{D}^{cT}_4\nu_{iL}) - \lambda^{QD}_i(\tilde{D}^c_4\bar{\nu}^c_{iR}Q_4 + \tilde{Q}_4\bar{D}^{cT}_4\nu_{iL}) - \lambda^H_i(\tilde{U}^c_H\bar{\nu}^c_{iR}Q_H + \tilde{Q}_H\bar{U}^{cT}_H\nu_{iL}) + \text{H.c.},\end{aligned} \quad (4)$$

where $\bar{\nu}^c_{iR}$ stands for the left-hand neutrino.

The seven types of trilinear LPV interactions in the above equation induce 14 types of one-loop diagrams contributing to the neutrino spectrum, which are proportional to $\lambda\lambda$, $\lambda'\lambda'$, $\lambda^E\lambda^E$, $\lambda\lambda^E$, $\lambda^Q\lambda^Q$, $\lambda^Q\lambda^D$, $\lambda^D\lambda^D$, $\lambda^H\lambda^H$, $\lambda^{QD}\lambda^{QD}$, $\lambda'\lambda^Q$, $\lambda'\lambda^D$, $\lambda'\lambda^{QD}$,$\lambda^Q\lambda^{QD}$, and $\lambda^D\lambda^{QD}$, respectively. The Feynman diagrams and the corresponding analytical results are shown in Fig. 7 in Appendix B. For simplicity and without losing our purpose, in the Yukawa interactions of $\mathcal{W}_0$ we assume that only y^E, $y^{Q'}$, $y^{Q'}$, y^D, y^U, y^H, and $y^{H'}$ are nonvanishing; that is, vectorlike particles have Yukawa interactions only with the third generation. Thus, the vectorlike generation has little constraints from the collider phenomenology.

Before starting to analyze the neutrino mass spectrum, some assumptions are introduced in order to control the parameter space and get a relatively simple analytical result. Since four new up-type Higgs Yukawa couplings y^U, y^Q, y^{QU}, and $y^{H'}$ and five new down-type Higgs Yukawa couplings y^E, y^D, $y^{Q'}$, y^{QD}, and y^H appear in our model, and among which y^{QD}, y^{QU}, y^H, and $y^{H'}$ provide the mass mixings between vectorlike generations, and furthermore, they have an infrared quasifixed point [5], we assume $y^{QD} = y^H = 0$ and $y^{QU} \sim y^{H'} \equiv y^t_V \le 1$. We also set $y^D = y^{Q'} = 0$, $y^E < 0.04$, and $y^U \sim y^Q \equiv y^t_{34} \le 0.08$. In other words, we neglect all new down-type Higgs Yukawa couplings in quark sectors while we consider all of the new up-type Higgs Yukawa couplings only and take $y^t_{34} \ll y^t_V$, which is a reasonable assumption.

Basing on the above assumptions, contributions from $\lambda\lambda$, $\lambda'\lambda'$ type diagrams can be simplified to the familiar forms [12–14]

$$\begin{aligned} M^\nu_{ij}|_{\lambda\lambda} &\simeq \frac{1}{8\pi^2}\lambda_{i33}\lambda_{j33}m_\tau \sin\alpha_{\tilde{\tau}}\cos\alpha_{\tilde{\tau}}\ln\frac{\tilde{\tau}_R}{\tilde{\tau}_L},\\ M^\nu_{ij}|_{\lambda'\lambda'} &\simeq \frac{3}{8\pi^2}\bigg(\lambda'_{i33}\lambda'_{j33}m_b\sin\alpha_{\tilde{b}}\cos\alpha_{\tilde{b}}\ln\frac{\tilde{b}_R}{\tilde{b}_L}\\ &\quad + \lambda'_{i23}\lambda'_{j32}m_s\sin\alpha_{\tilde{b}}\cos\alpha_{\tilde{b}}\ln\frac{\tilde{b}_R}{\tilde{b}_L}\\ &\quad + \lambda'_{i32}\lambda'_{j23}m_b\sin\alpha_{\tilde{s}}\cos\alpha_{\tilde{s}}\ln\frac{\tilde{s}_R}{\tilde{s}_L}\bigg), \end{aligned} \quad (5)$$

where, in the first equation, we keep only the dominant contributions and, in the second equation, we keep the dominant and subdominant ones. α_τ, α_b, α_s, and α_t are the angles of the corresponding $2\times 2\tilde{\tau}_{L(R)}$, $\tilde{b}_{L(R)}$, $\tilde{s}_{L(R)}$, and $\tilde{t}_{L(R)}$ unitary matrices. Unfortunately, the other equations, (B3)–(B14) in Appendix B, cannot be simplified by following a similar process, because there are mixings between different vectorlike generations. So these can be analyzed only numerically and will be discussed later.

At last, without loss of generality, among all seven types of LPV trilinear couplings, we take four of them, λ^E, λ^D, λ^Q, and λ^H, for consideration while assuming the rest of them, λ, λ', and λ^{QD}, are negligible. The realization through different LPV trilinear coupling combinations can be derived straightforwardly. The method to calculate the neutrino mass matrix we use is given in Appendix C.

Here we list the parameters of neutrino oscillation given by experiments: $\Delta m^2_{21} = (7.59 \pm 0.21)\times 10^{-5}\,\text{eV}^2$, $\Delta m^2_{32} = (2.43 \pm 0.13)\times 10^{-3}\,\text{eV}^2$ and $\sin^2 2\theta_{12} = 0.861^{+0.026}_{-0.022}$, $\sin^2 2\theta_{23} > 0.92$, $\sin^2 2\theta_{13} = 0.088 \pm 0.008$. Scanning the parameter space with proper EWSB, we find that, by adjusting the ratios and values of the LPV trilinear couplings we choose, the correct neutrino spectrum can be generated through the $\lambda^E\lambda^E$, $\lambda^D\lambda^Q$, and $\lambda^H\lambda^H$ type one-loop diagrams. A numerical illustration is shown in Table I; in set I we take the mass mixings assumptions mentioned before, and in set II we take different mass mixings and bigger vectorlike masses for comparison. For the specific parameters settings, see Appendix C.

That is, by choosing (for set I)

$$\begin{aligned} &\frac{\lambda^Q_{13}}{\lambda^Q_{23}} \sim 0.25, \quad \frac{\lambda^Q_{33}}{\lambda^Q_{23}} \sim 1.4, \quad \lambda^Q_{23} \sim 2.1\times 10^{-6},\\ &\lambda^Q_{13} \sim \lambda^D_{13}, \quad \lambda^Q_{23} \sim \lambda^D_{23}, \quad \lambda^Q_{33} \sim \lambda^D_{33}, \quad \frac{\lambda^H_1}{\lambda^H_2} \sim 1.4,\\ &\frac{\lambda^H_3}{\lambda^H_2} \sim 1, \quad \lambda^H_2 \sim \lambda^Q_{23}, \quad \lambda^E_{13} \sim \lambda^E_{23} \sim \lambda^E_{33} \sim \lambda^Q_{23}, \end{aligned} \quad (6)$$

we have

TABLE I. Numerical illustration for five types of one-loop contributions in our model. For the specific parameter settings, see Appendix C. M^{ν}_{ij} (GeV) stands for the parts in Eqs. (4) and (5) except the LPV trilinear coupling constants.

	$M^{\nu}_{ij}\vert_{\lambda^E_{i3}\lambda^E_{j3}}$	$M^{\nu}_{ij}\vert_{\lambda^D_{i3}\lambda^Q_{j3}}$	$M^{\nu}_{ij}\vert_{\lambda^D_{i3}\lambda^D_{j3}}$	$M^{\nu}_{ij}\vert_{\lambda^Q_{i3}\lambda^Q_{j3}}$	$M^{\nu}_{ij}\vert_{\lambda^H_{i}\lambda^H_{j}}$
Set I	0.0043	0.238	0	0	0.08
Set II	0.0027	0.168	0.004	0.003	0.011

$$\frac{m_{\nu 2}}{m_{\nu 3}} \sim 0.17, \qquad m_{\nu 2} \sim 5.9 \times 10^{-4}\ \text{eV},$$
$$m_{\nu 1} \sim 5.1 \times 10^{-6}\ \text{eV}, \qquad \sin\theta_{13} \sim 0.143, \tag{7}$$
$$\sin\theta_{23} \sim 0.581, \qquad \sin\theta_{12} \sim 0.559.$$

Unlike in the 3G LPV case, where $\lambda'_{i33}\lambda'_{j33}$, $\lambda'_{i23}\lambda'_{j32} + \lambda'_{i32}\lambda'_{j23}$, and $\lambda_{i33}\lambda_{j33}$ type one-loop contributions are dominant, subdominant, and next to subdominant, respectively, here in our model, under the assumptions mentioned before, $\lambda^Q_{i3}\lambda^D_{j3}$, $\lambda^H_i\lambda^H_j$, and $\lambda^E_{i3}\lambda^E_{j3}$ type one-loop contributions are dominant, subdominant, and next to subdominant, respectively. This is because the new fermions τ_1, $t_{1,2}$, $b_{1,2}$ in the internal lines—see Fig. 1—are much heavier than the third-generation fermions τ, t, b.

For the same reason, our requirements of the new LPV couplings we choose are of the order of 10^{-6} and small enough to avoid measurable flavor changing neutral current decays such as $\mu \to e\gamma$ [15]. It worth noting that, by decoupling the vectorlike generation, correct neutrino masses and mixings cannot be obtained via $\lambda\lambda$, $\lambda'\lambda'$ type one-loop contributions.

In addition, $\lambda^H\lambda^H$ type contribution containing up-type (s)quarks in the internal lines is absent in 3G LPV models, because the vectorlike down-type doublet quark Q^b_H mixes with the right-hand singlet top quark.

From Table I, we can also see that, by choosing $\lambda^Q_{i3}\lambda^D_{j3}$, $\lambda^Q_{i3}\lambda^Q_{j3}$, and $\lambda^D_{i3}\lambda^D_{j3}$ type one-loop contributions, the correct neutrino spectrum can also been generated in parameters set II; we do not list the detailed results here.

IV. HIGGS MASS

There are four new up-type Higgs Yukawa couplings in our model, y^U, y^Q, y^{QU}, and $y^{H'}$, corresponding to the Yukawa mass, m^t_{34}, m^t_{43}, m^t_{44}, and m^b_H, separately, and also five new down-type Higgs Yukawa couplings, y^E, y^D, $y^{Q'}$, y^{QD}, and y^H, corresponding to the Yukawa mass, m^τ_{34}, m^b_{34}, m^b_{43}, m^b_{44}, and m^t_H, separately. The related superpotential contributing to the lightest scalar Higgs mass is shown in $\mathcal{W}_0$. According to the assumptions mentioned in the last section, we neglect the down-type Higgs Yukawa contributions and the small up-type contributions between the SM third generations and the extra vectorlike generations. The relevant superpotential can be simplified as

$$\mathcal{W} = \mu^Q Q_4 Q_H + \mu^D D^c_4 D^c_H + y^{H'} Q_H H_u D^c_H + y^{QU} Q_4 H_u U^c_4. \tag{8}$$

So when neglecting the small D term and the two-loop contribution, the new one-loop contribution to the lightest scalar Higgs square mass is [5,16]

$$\triangle m^2_h = \frac{3\times 2}{4\pi^2}(y^t_V)^4 v^2 \sin^4\beta\left[t_V - \frac{1}{6}\left(5 - \frac{1}{x}\right)\left(1 - \frac{1}{x}\right) + 2\frac{X^2_V}{M^2_S}\left(1 - \frac{1}{3x}\right)\right], \tag{9}$$

where $v = 174$ GeV indicates the Higgs VEV and

$$y^{H'} = y^{QU} \equiv y^t_V, \qquad x = M^2_S/M^2_V, \qquad t_V = \log\frac{M^2_S}{M^2_V},$$
$$(A_{H'} - \mu_{H'}\cot\beta)^2 = (A_{QU} - \mu_{QU}\cot\beta)^2 = (A_V - \mu_V\cot\beta)^2 \equiv X^2_V, \tag{10}$$

in which, for simplicity, $\mu^Q = \mu^D \equiv M_V$ stands for the vectorlike mass of the new up-type quarks, $M^2_Q = M^2_D \equiv m^2$ [see Eq. (4)], and $M_S = \sqrt{M^2_V + m^2}$ stands for the average mass of the new up-type squarks.

In MSSM, the Higgs mass from the t, $\tilde{t}$ one-loop contributions is about 110 GeV, for $A^t = \mu = 400$ GeV, $m_{\tilde{t}} = 400$ GeV, and $\tan\beta = 10$. Direct search bounds from CMS for exotic heavy toplike quark set limits of $M_{t'} > 557$ GeV if $B(t' \to Wb) = 1$ [17] and $M_{t'} > 475$ GeV if $B(t' \to Zt) = 1$ [18]. When considering the mass mixing between the vectorlike quarks and the SM third-generation quarks, in other words, considering the realistic branch ratios, the mass limit is adjusted to be $M_{t'} > 415$ GeV [19,20]. So if we set the vectorlike fermion masses in our model to be $M_V \sim 500$ GeV, the soft supersymmetry-breaking parameters to be $m \sim 700$ GeV, $A_V = \mu_V \sim 500$ GeV, and $B_V\mu_V \sim 500^2$ GeV^2, then from Eq. (10), in order to get approximately 125 GeV Higgs mass, for about $M_V = 500$ GeV and $M_S = 850$ GeV, we just need to set $y^t_V \sim 1$ or, say, need to set $m^t_{44} = m^b_H \equiv m^t_V \sim 174$ GeV. These values are just near their infrared quasifix point, as mentioned in the last section.

Evoked by the ATLAS and CMS discovery of the enhancement in the $\gamma\gamma$ channel and little deviation in the ZZ channel [21,22], the effects of the exotic vectorlike quarks to the Higgs production and decay have been extensively studied recently [21]. In general, in a theory with N vectorlike generations extension, the new fermion contributions are suppressed by $N^2 m^{t2}_V/M^2_V$ [21,22]. So only the very large couplings to the Higgs can obviously enhance the Higgs production and decay in the $\gamma\gamma$ channel [21], but as we have mentioned, these couplings have a quasifix point which limits their TeV values to be about 1 [5]. This value is large enough to accommodate $m_h \sim 125$ GeV but too small to influence the Higgs decay; one cannot depend on vectorlike fermions by themselves to modify the Higgs decay branching ratios. As far as the Higgs problem is concerned, extra vectorlike fermions are mainly introduced to adjust the Higgs mass. However, the $\gamma\gamma$ and ZZ channel anomalies, if they persist, can be realized through the light top squark scenario [23], which is beyond the scope of this paper.

V. THE EXTRA VECTORLIKE FERMION DECAYS

To be clear, we list the new extra vectorlike fermions below:

$$\Psi_E = \begin{pmatrix} E_H^c \\ \bar{E}_4^c \end{pmatrix}, \qquad \Psi_Q = \begin{pmatrix} Q_4^{t,b} \\ \bar{Q}_H^{t,b} \end{pmatrix}, \qquad \Psi_U = \begin{pmatrix} U_4^c \\ U_H \end{pmatrix}, \qquad \Psi_D = \begin{pmatrix} D_4^c \\ D_H \end{pmatrix}, \tag{11}$$

in which E_H^c mixes with τ_L; E_4^c mixes with τ_R; Q_4^b, D_H^c mixes with b_L; D_4^c, Q_H^t mixes with b_R, Q_4^t, U_H^c mixes with t_L; and U_4^c, Q_H^b mixes with t_R. These exotic heavy fermions can decay into SM bosons—see Fig. 2—which we will analyze below. Our analyses agree with the results given in Ref. [5]. However, the slight difference comes from their neglect of the contributions proportional to s_W^2 in the vertex of Feynman rules.

Note that theoretically speaking, when kinematically allowed, the exotic fermions predicted in our model have the other two decay modes: (i) through supersymmetric gauge kinetic iterations or the supersymmetric Yukawa interactions, decay into chargino or neutralino and sfermions, such as $\tau_1 \to \tilde{C}^+ \tilde{\nu}_\tau$, $b_1 \to \tilde{N}_i \tilde{b}$, $t_1 \to \tilde{C}^- \tilde{b}$, where $\tilde{N}_i$, $i = 1$–4, is a neutralino and $\tilde{C}^\pm$ is

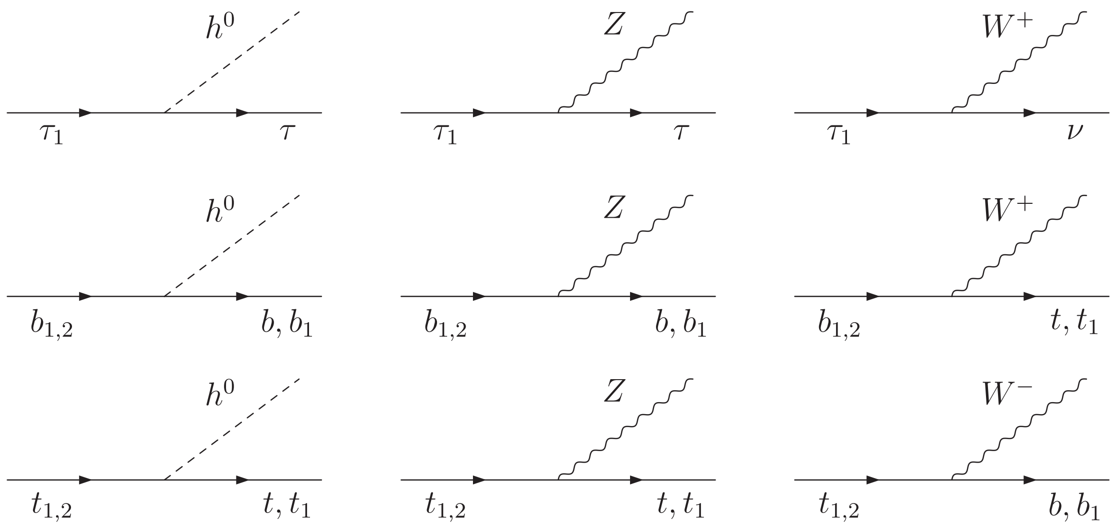

FIG. 2. Tree-level decay of new exotic fermions in our model; all fermions stay in mass eigenstates.

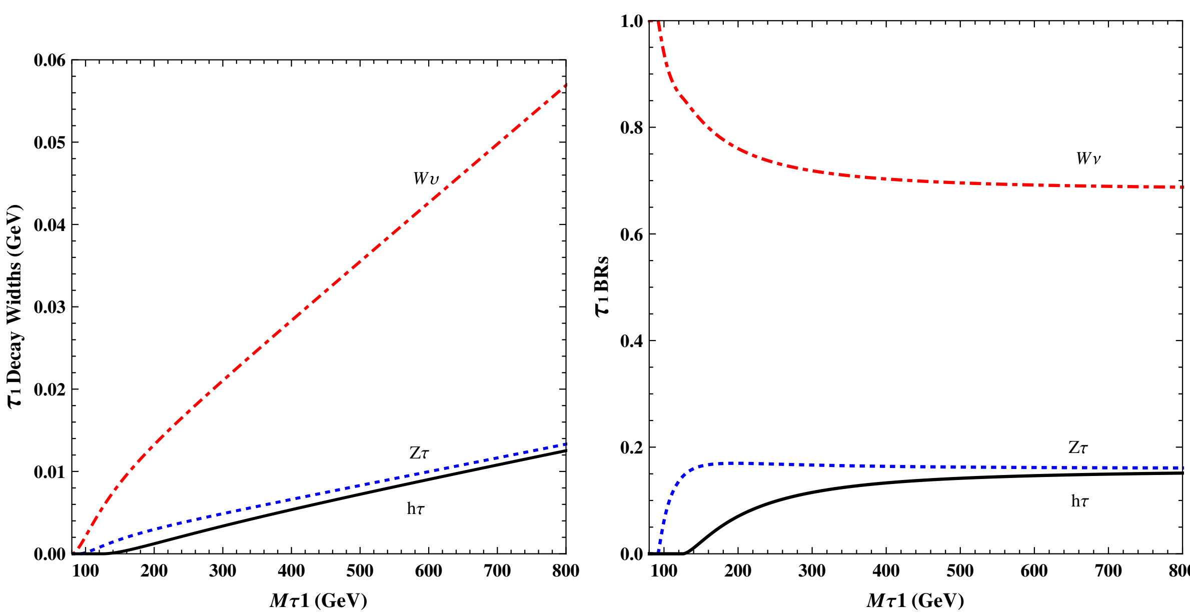

FIG. 3 (color online). The decay widths of the new lepton τ_1 (left panel) and its branching ratios (right panel) with $y^E = 0.04$.

a chargino, and (ii) through LPV interactions—see Eq. (2)—decay into fermions and sfermions, such as $\tau_1 \to e\tilde{\mu}$, $b_1 \to \tilde{\tau}t$, $t_1 \to \tilde{e}b$. Although the kinematical conditions for the latter decay mode are easy to satisfy, we have already seen in Sec. III that the LPV couplings in our model, in order to explain the neutrino spectrum, are of the order of 10^{-6}, so we can neglect this kind of decay channels reasonably. On the other hand, for simplicity here in our work, we assume that the former decay mode is not kinematically allowed. Therefore, the exotic fermions can decay only into SM bosons.

A. τ_1 decays

The weak boson interaction Lagrangian to τ, τ_1 is

$$\mathcal{L} \supset g^W_{\bar\tau_{1L}\nu_\tau}\bar\tau_{1L}\gamma^\mu\nu_{\tau L}W^-_\mu + g^Z_{\bar\tau_{1L}\tau_L}\bar\tau_{1L}\gamma^\mu\tau_L Z_\mu + g^Z_{\bar\tau_{1R}\tau_R}\bar\tau_{1R}\gamma^\mu\tau_R Z_\mu + g^{h^0}_{\bar\tau_{1L}\tau_R}\bar\tau_{1L}\tau_R h^0 + g^{h^0}_{\bar\tau_L\tau_{1R}}\bar\tau_L\tau_{1R}h^0 + \text{H.c.} \tag{12}$$

The couplings and the decay widths of τ_1 are given in Appendix D.

The main characteristic of the lepton sector is that there must be mass mixing between the third and the vectorlike lepton; otherwise, the new heavy charged leptons τ_1 will be stable and give an unacceptable cosmological heavy relic [24]. For specific, when $y^E = 0$, the off-diagonal elements of L^τ, R^τ are equal to zero. That is why we set $y^E \neq 0$ in Sec. II while discussing the neutrino spectrum; more specifically, we set $y^E \leq 0.04$. Under these parameter settings, numerical results of τ_1 decay into W, Z, h^0 are shown in Fig. 3, we can see in the limit of large m_{τ_1}, and the branching ratios are $\text{BR}(\tau_1 \to W\nu_\tau) \sim 0.7$ and $\text{BR}(\tau_1 \to Z\tau) = \text{BR}(\tau_1 \to h\tau) \sim 0.15$.

B. $t_{1,2}$ decays

The weak boson interaction Lagrangian to t, t_1, t_2 is

$$\begin{aligned}\mathcal{L} \supset\, & g^W_{\bar t_{1L}b_L}\bar t_{1L}\gamma^\mu b_L W^-_\mu + g^W_{\bar t_{2L}b_L}\bar t_{2L}\gamma^\mu b_L W^-_\mu + g^W_{\bar t_{1L}b_R}\bar t_{1R}\gamma^\mu b_R W^-_\mu + g^W_{\bar t_{2R}b_R}\bar t_{2R}\gamma^\mu b_R W^-_\mu g^W_{\bar t_{2L}b_{1L}}\bar t_{2L}\gamma^\mu b_{1L}W^-_\mu \\ & + g^W_{\bar t_{2L}b_{R1}}\bar t_{2R}\gamma^\mu b_{1R}W^-_\mu + g^Z_{\bar t_{1L}t_L}\bar t_{1L}\gamma^\mu t_L Z_\mu + g^Z_{\bar t_{2L}t_L}\bar t_{2L}\gamma^\mu t_L Z_\mu + g^Z_{\bar t_{2L}t_{1L}}\bar t_{2L}\gamma^\mu t_{1L}Z_\mu + g^Z_{\bar t_{1R}t_R}\bar t_{1R}\gamma^\mu t_R Z_\mu \\ & + g^Z_{\bar t_{2R}t_R}\bar t_{2R}\gamma^\mu t_R Z_\mu + g^Z_{\bar t_{2R}t_{1R}}\bar t_{2R}\gamma^\mu t_{1R}Z_\mu + g^{h^0}_{\bar t_{1L}t_R}\bar t_{1L}t_R h^0 + g^{h^0}_{\bar t_L t_{1R}}\bar t_L t_{1R}h^0 + g^{h^0}_{\bar t_{2L}t_R}\bar t_{2L}t_R h^0 \\ & + g^{h^0}_{\bar t_L t_{2R}}\bar t_L t_{2R}h^0 + g^{h^0}_{\bar t_{2L}t_{1R}}\bar t_{2L}t_{1R}h^0 + g^{h^0}_{\bar t_{1L}t_{2R}}\bar t_{1L}t_{2R}h^0 + \text{H.c.}\end{aligned} \tag{13}$$

The couplings and the decay widths of $t_{1,2}$ are given in Appendix D.

As mentioned in Sec. II, we take $y^U \sim y^Q \leq 0.08$, $y^{QU} \sim y^{H'} \leq 1$; the numerical results are shown in Figs. 4 and 5. We can see that, in the limit of large $M_{t_{1,2}}$, the branching ratios of t_1 are $\text{BR}(t_1 \to Wb) \sim 0.4$ and $\text{BR}(t_1 \to Zt) = \text{BR}(t_1 \to h^0 t) \sim 0.3$, and the branching ratios of t_2 are $\text{BR}(t_2 \to Wb_1) \sim 0.85$ and $\text{BR}(t_2 \to Zt_1) \sim 0.15$.

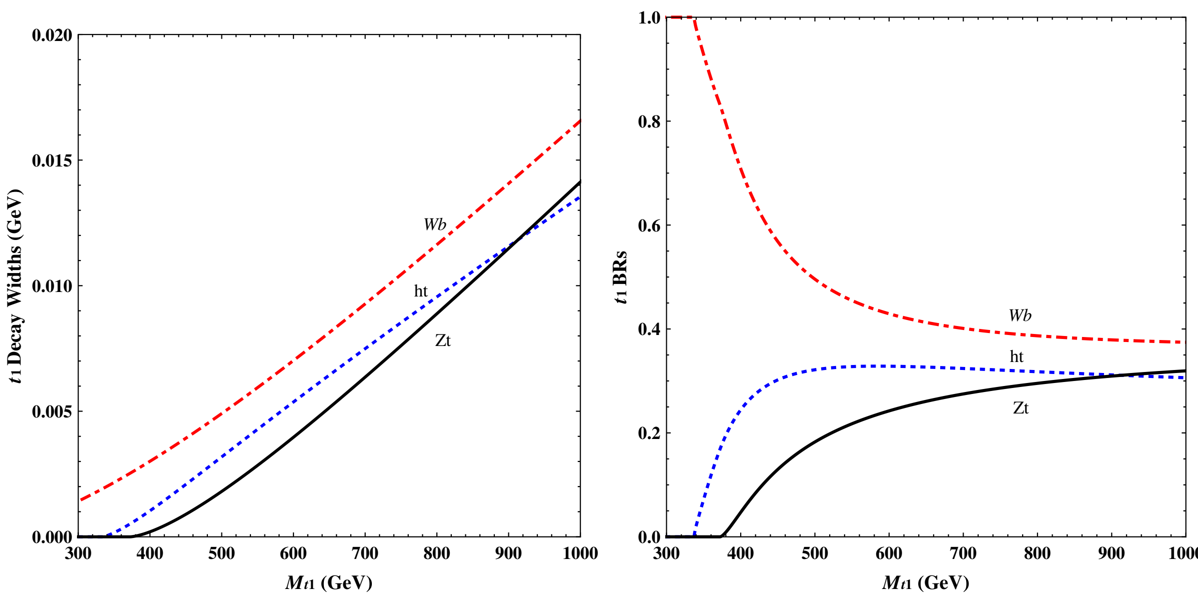

FIG. 4 (color online). The decay widths of the lightest new up-type quark t_1 (left panel) and its branching ratios (right panel) with $y^{QD} = y^H = y^D = 0$, $y^U \sim y^Q = 0.08$, and $y^{QU} \sim y^{H'} = 1$.

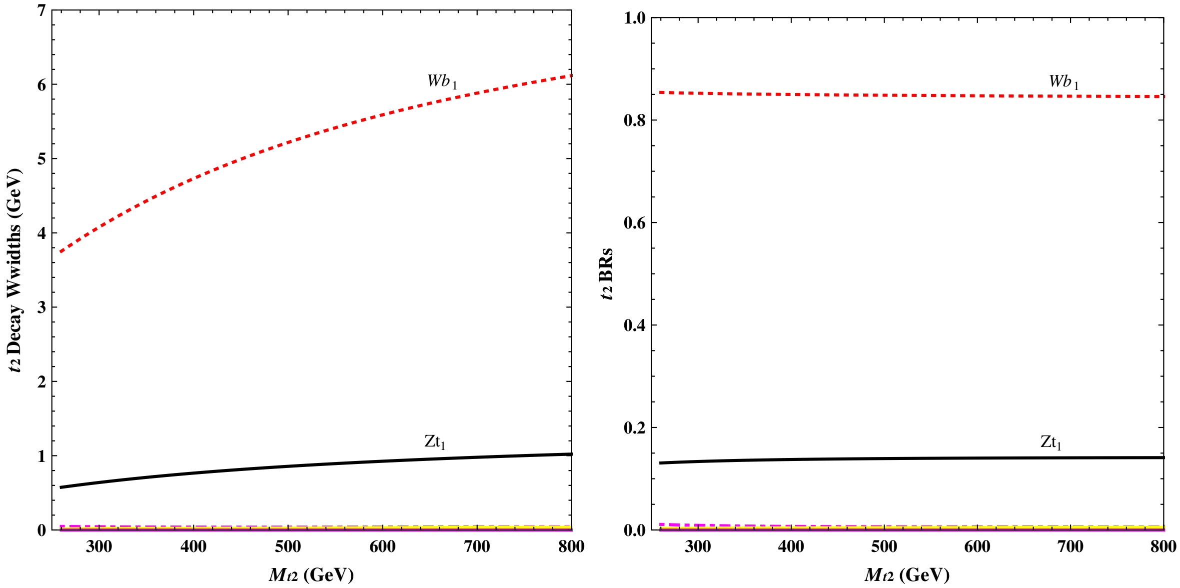

FIG. 5 (color online). The decay widths of the heaviest new up-type quark t_2 (left panel) and its branching ratios (right panel) with $y^{QD} = y^H = y^D = 0$, $y^U \sim y^Q = 0.08$, and $y^{QU} \sim y^{H'} = 0.98$.

C. $b_{1,2}$ decays

The weak boson interaction Lagrangian to b, b_1, b_2 is

$$\begin{aligned}\mathcal{L} \supset\ & g^W_{\bar b_{1L} t_L} \bar b_{1L}\gamma^\mu t_L W^-_\mu + g^W_{\bar b_{2L} t_L} \bar b_{2L}\gamma^\mu t_L W^-_\mu + g^W_{\bar b_{1L} t_R} \bar b_{1R}\gamma^\mu t_R W^-_\mu + g^W_{\bar b_{2R} t_R} \bar b_{2R}\gamma^\mu t_R W^-_\mu + g^W_{\bar b_{2L} t_{1L}} \bar b_{2L}\gamma^\mu t_{1L} W^-_\mu \\ & + g^W_{\bar b_{2R} t_{R1}} \bar b_{2R}\gamma^\mu t_{1R} W^-_\mu + g^Z_{\bar b_{1L} b_L} \bar b_{1L}\gamma^\mu b_L Z_\mu + g^Z_{\bar b_{2L} b_L} \bar b_{2L}\gamma^\mu b_L Z_\mu + g^Z_{\bar b_{2L} b_{1L}} \bar b_{2L}\gamma^\mu b_{1L} Z_\mu + g^Z_{\bar b_{1R} b_R} \bar b_{1R}\gamma^\mu b_R Z_\mu \\ & + g^Z_{\bar b_{2R} b_R} \bar b_{2R}\gamma^\mu b_R Z_\mu + g^Z_{\bar b_{2R} b_{1R}} \bar b_{2R}\gamma^\mu b_{1R} Z_\mu + g^{h^0}_{\bar b_{1L} b_R} \bar b_{1L} b_R h^0 + g^{h^0}_{\bar b_L b_{1R}} \bar b_L b_{1R} h^0 + g^{h^0}_{\bar b_{2L} b_R} \bar b_{2L} b_R h^0 \\ & + g^{h^0}_{\bar b_L b_{2R}} \bar b_L b_{2R} h^0 + g^{h^0}_{\bar b_{2L} b_{1R}} \bar b_{2L} b_{1R} h^0 + g^{h^0}_{\bar b_{1L} b_{2R}} \bar b_{1L} b_{2R} h^0 + \text{H.c.} \end{aligned} \quad (14)$$

The couplings and the decay widths of $b_{1,2}$ are given in Appendix D.

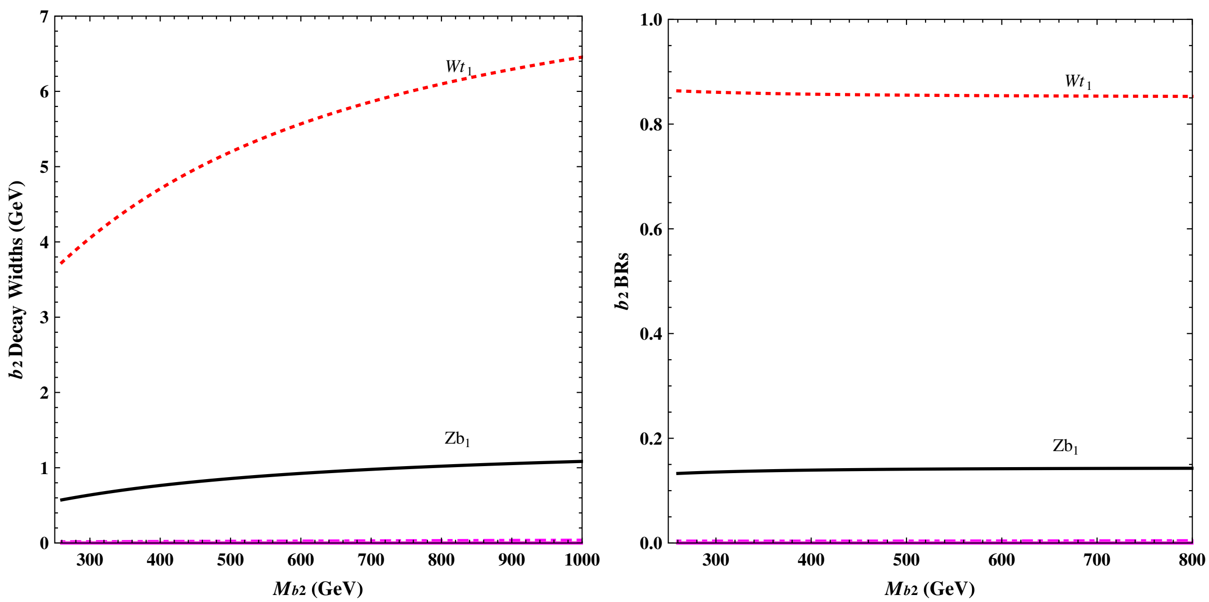

FIG. 6 (color online). The decay widths of the heaviest new down-type quark b_2 (left panel) and its branching ratios (right panel) with $y^{QD} = y^H = y^D = 0$, $y^U \sim y^Q = 0.08$, and $y^{QU} \sim y^{H'} = 0.98$.

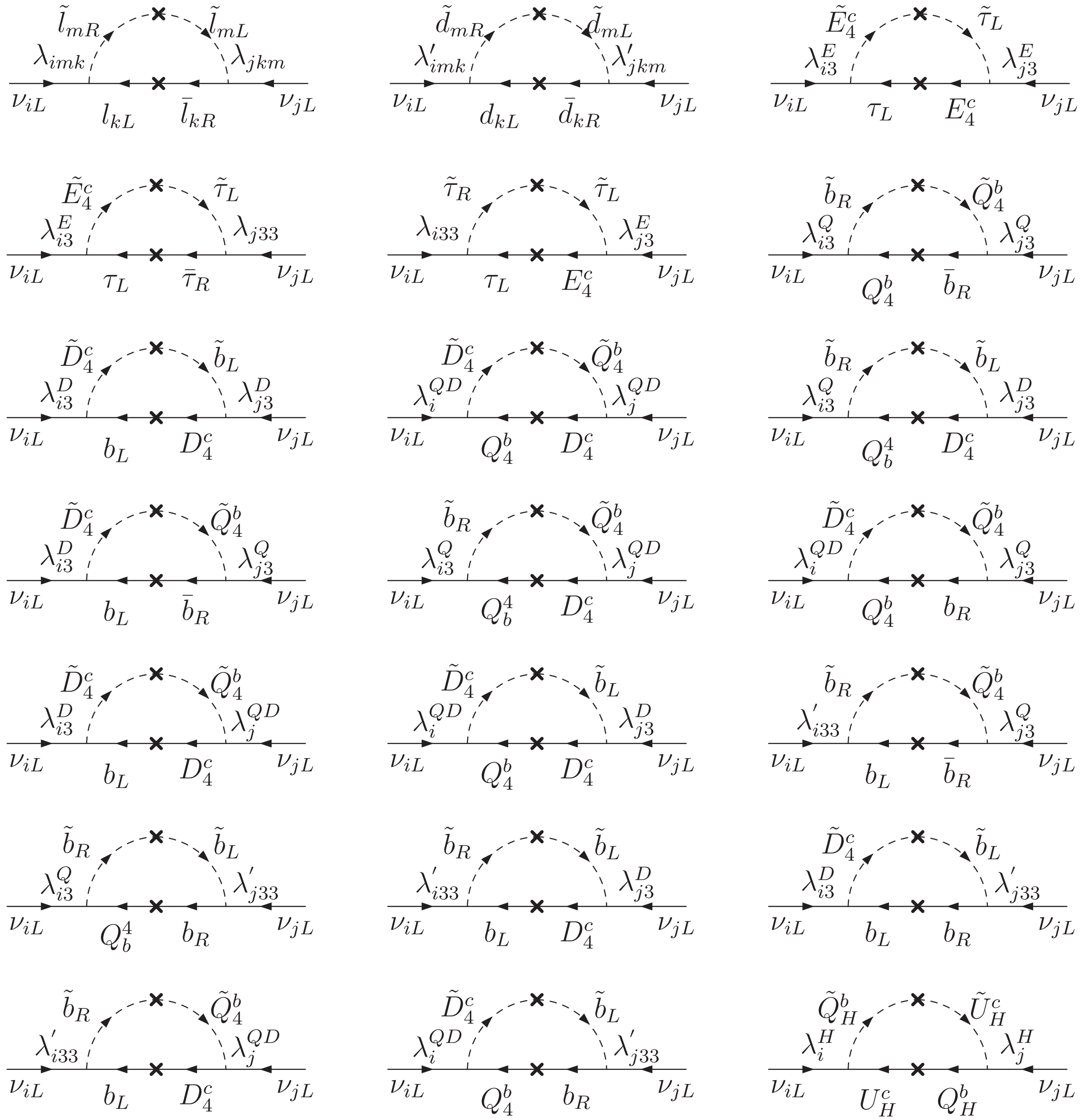

FIG. 7. One-loop contributions to the neutrino masses and mixings in our model.

The numerical results of b_2 decay widths and branching ratios are shown in Fig. 6 under the parameter settings mentioned before; we can see $BR(b_2 \to Wb_1) \sim 0.85$ and $BR(b_2 \to Zb_1) \sim 0.15$. The branching ratios of b_1 are $BR(b_1 \to Wt) = 1$, which are not shown here.

VI. SUMMARY AND DISCUSSION

We have studied several phenomenological aspects of the LPV MSSM model with a vectorlike extra generation: the neutrino spectrum, the Higgs mass, and the LHC phenomenology of the new predicted fermions. The results are

(i) The correct neutrino masses and mixings, especially the relatively large θ_{13}, can be generated from trilinear LPV couplings. The new trilinear R-parity violating couplings make it easy to generate the proper value of θ_{13}. These coupling constants need to be about 10^{-6}.

(ii) The two new up-type Higgs Yukawa couplings, $y^{H'}$ and y^{QU}, between the vectorlike quarks and the SM third-generation quarks, with values about 1 near to their infrared quasifixed point in the TeV scale, can give rise to 125 GeV Higgs mass with no need of a very heavy new superpartner.

(iii) There are five new heavy fermions, τ_1, $t_{1,2}$, $b_{1,2}$, predicted in this model. They can only decay into SM bosons by some kinematic assumptions. The branching ratio depends on the mass mixing between the vectorlike fermions and the SM third-generation fermions. These charged exotic fermions would be quasistable if such mass mixings are very small.

Based on our previous work about bilinear LPV couplings, further research on the renormalization group of them is worthy to be studied in the future. There is also plenty to be further analyzed in the area of new fermion LHC phenomenology based on this model.

ACKNOWLEDGMENTS

We thank Dr. Guang-Zhi Xu for a very helpful discussion. This work was supported in part by the National Natural Science Foundation of China under Grants No. 11075193 and No. 10821504 and by the National Basic Research Program of China under Grant No. 2010CB833000.

APPENDIX A: THE (S)FERMION MASS MIXING MATRIXES

Because the CP violation is not considered in this paper, we have taken all the masses to be real. In this model, the mass matrix M of the third-generation lepton and the vectorlike lepton is given as the following:

$$\mathcal{L} \supset -(\tau, E_H^c)\mathcal{M}^\tau \begin{pmatrix} \tau^c \\ E_4^c \end{pmatrix} \tag{A1}$$

and

$$\mathcal{M}^l = \begin{pmatrix} m_{33}^\tau & m_{34}^\tau \\ 0 & \mu^E \end{pmatrix}, \tag{A2}$$

where $m_{33}^\tau \equiv y_{33}^l \frac{v}{\sqrt{2}} \cos\beta$ and $m_{34}^\tau \equiv y_3^E \frac{v}{\sqrt{2}} \cos\beta$. By taking $|\mu^E| \gg |m_{34}^\tau|$, then the biunitary matrix to diagonalize $\mathcal{M}^\tau$ is

$$L^{\tau *}\mathcal{M}^\tau R^{\tau\dagger} = (m_\tau, \mu^E) \equiv \mathrm{diag}(m_\tau, m_{\tau 1}), \tag{A3}$$

where

$$L^\tau = \begin{pmatrix} 1 & \frac{m_{34}^\tau}{\mu^E} \\ -\frac{m_{34}^\tau}{\mu^E} & 1 \end{pmatrix}, \qquad R^\tau = \begin{pmatrix} 1 & \frac{m_\tau m_{34}^\tau}{(\mu^E)^2} \\ -\frac{m_\tau m_{34}^\tau}{(\mu^E)^2} & 1 \end{pmatrix}. \tag{A4}$$

The mass matrix $\mathcal{M}^b$ of the third-generation down quark and vectorlike down-type quarks is given as the following:

$$\mathcal{L} \supset -(b, Q_4^b, D_H^c)\mathcal{M}^b \begin{pmatrix} b^c \\ D_4^c \\ Q_H^t \end{pmatrix}, \tag{A5}$$

where

$$\mathcal{M}^b = \begin{pmatrix} m_{33}^b & m_{34}^b & 0 \\ m_{43}^b & m_{44}^b & \mu^Q \\ 0 & \mu^D & m_H^b \end{pmatrix}, \tag{A6}$$

where $m_{33}^b \equiv y_{33}^d \frac{v}{\sqrt{2}} \cos\beta$, $m_{34}^b \equiv y_3^D \frac{v}{\sqrt{2}} \cos\beta$, $m_{43}^b \equiv y_3^{Q\prime} \frac{v}{\sqrt{2}} \cos\beta$, and $m_{44}^b \equiv y^{QD} \frac{v}{\sqrt{2}} \cos\beta$. By taking that $|\mu^Q| \sim |\mu^D| \gg |m_{4b}^b|, |m_{44}^b|, |m_{33}^b|, |m_{34}^b|$, then the biunitary matrix diagonalize $\mathcal{M}^t$ is

$$L^{b*}\mathcal{M}^b R^{b\dagger} = (m_b, \mu^Q, \mu^D) \equiv \mathrm{diag}(m_b, m_{b1}, m_{b2}), \tag{A7}$$

where

$$L^b = \begin{pmatrix} 1 & 0 & -\frac{m_{34}^b}{\mu^D} \\ 0 & 1 & \frac{\mu^D m_{44}^t + \mu^Q m_H^b}{(\mu^D)^2 + (\mu^Q)^2 - (m_H^b)^2} \\ \frac{m_{34}^b}{\mu^D} & \frac{(m_H^b)^2 + (m_{34}^b)^2}{\mu^Q m_H^b + \mu^D m_{44}^b} & 1 \end{pmatrix} \tag{A8}$$

and

$$R^b = \begin{pmatrix} 1 & \frac{m_{34}^b}{\mu^D} & 0 \\ 0 & \frac{\mu^Q m_{44}^b + \mu^D m_H^b}{(\mu^D)^2 + (\mu^Q)^2 - (m_H^b)^2} & 1 \\ -\frac{m_{34}^b}{\mu^D} & 1 & \frac{(m_H^b)^2 + (m_{34}^b)^2}{\mu^D m_H^b + \mu^Q m_{44}^b} \end{pmatrix}. \tag{A9}$$

The mass matrix $\mathcal{M}^t$ of the top quark and vectorlike up-type generations is given as the following:

$$\mathcal{L} \supset -(t, Q_4^t, U_H^c)\mathcal{M}^t \begin{pmatrix} t^c \\ U_4^c \\ Q_H^b \end{pmatrix}, \tag{A10}$$

where

$$\mathcal{M}^t = \begin{pmatrix} m_{33}^t & tm_{34} & 0 \\ m_{43}^t & m_{44}^t & \mu^Q \\ 0 & \mu^U & m_H^t \end{pmatrix}, \tag{A11}$$

where $m_{33}^t \equiv y_{33}^u \frac{v}{\sqrt{2}} \sin\beta$, $m_{34}^t \equiv y_3^U \frac{v}{\sqrt{2}} \sin\beta$, $m_{43}^t \equiv y_3^Q \frac{v}{\sqrt{2}} \sin\beta$, $m_{44}^t \equiv y^{QU} \frac{v}{\sqrt{2}} \sin\beta$, and $m_H^t \equiv y \frac{v}{\sqrt{2}} \cos\beta$. By taking that $|\mu^Q| \sim |\mu^U| \gg |m_{43}^t|, |m_{44}^t|, |m_{33}^t|, |m_{34}^t|, |m_H^t|$, then the biunitary matrix diagonalize $\mathcal{M}^t$ is

$$L^{t*}\mathcal{M}^t R^{t\dagger} = (m_t, \mu^Q, \mu^U) \equiv \mathrm{diag}(m_t, m_{t1}, m_{t2}), \tag{A12}$$

where

$$L^t = \begin{pmatrix} 1 & 0 & -\frac{m_{34}^t}{\mu^U} \\ 0 & 1 & \frac{\mu^U m_{44}^t + \mu^Q m_H^t}{(\mu^U)^2 + (\mu^Q)^2 - (m_H^t)^2} \\ \frac{m_{34}^t}{\mu^U} & \frac{(m_H^t)^2 + (m_{34}^t)^2}{\mu^Q m_H^t + \mu^U m_{44}^t} & 1 \end{pmatrix} \tag{A13}$$

and

$$R^t = \begin{pmatrix} 1 & \frac{m_{34}^t}{\mu^U} & 0 \\ 0 & \frac{\mu^Q m_{44}^t + \mu^U m_H^t}{(\mu^U)^2 + (\mu^Q)^2 - (m_H^t)^2} & 1 \\ -\frac{m_{34}^t}{\mu^U} & 1 & \frac{(m_H^t)^2 + (m_{34}^t)^2}{\mu^U m_H^t + \mu^Q m_{44}^t} \end{pmatrix}. \tag{A14}$$

The charged slepton mass-squared matrix $\tilde{\mathcal{M}}_\tau^2$ of $\tilde{\tau}$ and the superpartners of the vectorlike leptons is given as the following:

$$\mathcal{L} \supset (\tilde{L}_3^{-*}, \tilde{E}_3^c, \tilde{E}_4^c, \tilde{E}_H^{c*})\tilde{\mathcal{M}}_l^2 \begin{pmatrix} \tilde{L}_3^- \\ \tilde{E}_3^{c*} \\ \tilde{E}_4^{c*} \\ \tilde{E}_H^c \end{pmatrix}, \tag{A15}$$

where

$$\begin{aligned}
&(\tilde{\mathcal{M}}_\tau^2)_{11} = M^2 + \left(\frac{m_Z^2}{2} - m_W^2\right)\cos 2\beta + m_\tau^2 + |m_{34}^\tau|^2, \quad (\tilde{\mathcal{M}}_\tau^2)_{12} = (m_0 - \mu\tan\beta)m_\tau, \quad (\tilde{\mathcal{M}}_\tau^2)_{13} = (m_0 - \mu\tan\beta)m_{34}^\tau,\\
&(\tilde{\mathcal{M}}_\tau^2)_{14} = \mu^e m_{34}^\tau, \quad (\tilde{\mathcal{M}}_\tau^2)_{21} = (m_0 - \mu\tan\beta)m_\tau, \quad (\tilde{\mathcal{M}}_\tau^2)_{22} = M_E^2 - (m_Z - m_W^2)\cos 2\beta + m_\tau^2, \quad (\tilde{\mathcal{M}}_\tau^2)_{23} = m_\tau m_{34}^\tau,\\
&(\tilde{\mathcal{M}}_\tau^2)_{24} = 0, \quad (\tilde{\mathcal{M}}_\tau^2)_{31} = (m_0 - \mu\tan\beta)m_{34}^\tau, \quad (\tilde{\mathcal{M}}_\tau^2)_{32} = m_\tau m_{34}^\tau, \quad (\tilde{\mathcal{M}}_\tau^2)_{33} = |\mu^E|^2 + M_E^2 - (m_Z^2 - m_W^2)\cos 2\beta + |m_{34}^\tau|^2,\\
&(\tilde{\mathcal{M}}_\tau^2)_{34} = B^E\mu^E, \quad (\tilde{\mathcal{M}}_\tau^2)_{41} = \mu^E m_{34}^\tau, \quad (\tilde{\mathcal{M}}_\tau^2)_{42} = 0, \quad (\tilde{\mathcal{M}}_\tau^2)_{43} = B^E\mu^E, \quad (\tilde{\mathcal{M}}_\tau^2)_{44} = |\mu^E|^2 + M_{EH}^2 + (m_Z^2 - m_W^2)\cos 2\beta.
\end{aligned} \tag{A16}$$

The corresponding unitary scalar matrix is defined as

$$V^\tau \tilde{\mathcal{M}}_\tau^2 V^{\tau\dagger} = \mathrm{diag}(\tilde{M}_\tau^2, \tilde{M}_{\tau 1}^2, \tilde{M}_{\tau 2}^2, \tilde{M}_{\tau 3}^2). \tag{A17}$$

The mass-squared matrix $\tilde{\mathcal{M}}_b^2$ of $\tilde{b}$ and the superpartners of the down-type vectorlike fermions is given as the following:

$$\mathcal{L} \supset (\tilde{b}^*, \tilde{D}_3^c, \tilde{D}_4^c, \tilde{D}_H^{c*}, \tilde{Q}_4^{b*}, \tilde{Q}_H^t)\tilde{\mathcal{M}}_b^2 \begin{pmatrix} \tilde{b} \\ \tilde{D}_3^{c*} \\ \tilde{D}_4^{c*} \\ \tilde{D}_H^c \\ \tilde{Q}_4^b \\ \tilde{Q}_H^{t*} \end{pmatrix}, \tag{A18}$$

where

$$\begin{aligned}
&(\tilde{\mathcal{M}}_b^2)_{11} = M_Q^2 - \frac{m_Z^2 + 2m_W^2}{6}\cos 2\beta + m_b^2 + |m_{34}^b|^2, \quad (\tilde{\mathcal{M}}_{b^2})_{12} = (m_0 - \mu\tan\beta)m_b, \quad (\tilde{\mathcal{M}}_b^2)_{13} = (m_0 - \mu\tan\beta)m_{34}^b,\\
&(\tilde{\mathcal{M}}_b^2)_{14} = \mu^D m_{34}^b, \quad (\tilde{\mathcal{M}}_b^2)_{15} = m_b m_{43}^b + m_{34}^b m_{44}^b, (\tilde{\mathcal{M}}_b^2)_{16} = 0, \quad (\tilde{\mathcal{M}}_b^2)_{21} = (m_0 - \mu\tan\beta)m_b,\\
&(\tilde{\mathcal{M}}_b^2)_{22} = M_D^2 - \frac{m_Z^2 - m_W^2}{3}\cos 2\beta + m_b^2 + |m_{43}^d|^2, \quad (\tilde{\mathcal{M}}_b^2)_{23} = m_b m_{34}^b + m_{43}^b m_{44}^{b*}, \quad (\tilde{\mathcal{M}}_b^2)_{24} = 0,\\
&(\tilde{\mathcal{M}}_b^2)_{25} = (m_0 - \mu\tan\beta)m_{43}^b, \quad (\tilde{\mathcal{M}}_b^2)_{26} = \mu^Q m_{43}^b, \quad (\tilde{\mathcal{M}}_b^2)_{31} = (m_0 - \mu\tan\beta)m_{34}^b,\\
&(\tilde{\mathcal{M}}_b^2)_{32} = m_b m_{34}^b + m_{43}^b m_{44}^b, (\tilde{\mathcal{M}}_b^2)_{33} = |\mu^D|^2 + M_D^2 - \frac{m_Z^2 - m_W^2}{3}\cos 2\beta + |m_{34}^b|^2 + |m_{44}^b|^2, \quad (\tilde{\mathcal{M}}_b^2)_{34} = -B^D\mu^D,\\
&(\tilde{\mathcal{M}}_b^2)_{35} = (m_0 - \mu\tan\beta)m_{44}^b, \quad (\tilde{\mathcal{M}}_b^2)_{36} = \mu^Q m_{44}^b + \mu^D m_H^b, \quad (\tilde{\mathcal{M}}_b^2)_{41} = \mu^D m_{34}^b, \quad (\tilde{\mathcal{M}}_b^2)_{42} = 0, \quad (\tilde{\mathcal{M}}_b^2)_{43} = -B^D\mu^D,\\
&(\tilde{\mathcal{M}}_b^2)_{44} = |\mu^D|^2 + M_{DH}^2 + \frac{m_Z^2 - m_W^2}{3}\cos 2\beta + +|m_H^b|^2, \quad (\tilde{\mathcal{M}}_b^2)_{45} = \mu^D m_{44}^b + \mu^Q m_H^b, \quad (\tilde{\mathcal{M}}_b^2)_{46} = (m_0 - \mu\cot\beta)m_H^b,\\
&(\tilde{\mathcal{M}}_b^2)_{51} = m_b m_{43}^d + m_{34}^b m_{44}^b, \quad (\tilde{\mathcal{M}}_b^2)_{52} = (m_0 - \mu\tan\beta)m_{43}^b, \quad (\tilde{\mathcal{M}}_b^2)_{53} = (m_0 - \mu\tan\beta)m_{44}^b,\\
&(\tilde{\mathcal{M}}_b^2)_{54} = \mu^D m_{44}^b + \mu^Q m_H^b, \quad (\tilde{\mathcal{M}}_b^2)_{55} = |\mu^Q|^2 + M_Q^2 - \frac{m_Z^2 + 2m_W^2}{6}\cos 2\beta + |m_{44}^b|^2 + |m_{43}^b|^2, \quad (\tilde{\mathcal{M}}_b^2)_{56} = B^Q\mu^Q,\\
&(\tilde{\mathcal{M}}_b^2)_{61} = 0, \quad (\tilde{\mathcal{M}}_b^2)_{62} = \mu^Q m_{43}^b, \quad (\tilde{\mathcal{M}}_b^2)_{63} = \mu^Q m_{44}^b + \mu^D m_H^b, \quad (\tilde{\mathcal{M}}_b^2)_{64} = (m_0 - \mu\cot\beta)m_H^b, \quad (\tilde{\mathcal{M}}_b^2)_{65} = B^Q\mu^Q,\\
&(\tilde{\mathcal{M}}_b^2)_{66} = |\mu^Q|^2 + M_{QH}^2 + |m_H^b|^2 + \frac{m_{Z^2} + 2m_W^2}{6}\cos 2\beta.
\end{aligned} \tag{A19}$$

The corresponding unitary scalar matrix is defined as

$$V^b \tilde{\mathcal{M}}_b^2 V^{b\dagger} = \mathrm{diag}(\tilde{M}_b^2, \tilde{M}_{b1}^2, \tilde{M}_{b2}^2, \tilde{M}_{b3}^2, \tilde{M}_{b4}^2, \tilde{M}_{b5}^2). \tag{A20}$$

The mass-squared matrix $\tilde{\mathcal{M}}_t^2$ of $\tilde{t}$ and the superpartners of the up-type vectorlike fermions is given as the following:

$$\mathcal{L} \supset (\tilde{t}^*, \tilde{U}_3^c, \tilde{U}_4^c, \tilde{U}_H^{c*}, \tilde{Q}_4^{t*}, \tilde{Q}_H^b)\tilde{\mathcal{M}}_t^2 \begin{pmatrix} \tilde{t} \\ \tilde{U}_3^{c*} \\ \tilde{U}_4^{c*} \\ \tilde{U}_H^c \\ \tilde{Q}_4^t \\ \tilde{Q}_H^{b*} \end{pmatrix}, \tag{A21}$$

where

$$\begin{aligned}
&(\tilde{\mathcal{M}}_t^2)_{11} = M_Q^2 + \frac{4m_W^2 - m_Z^2}{6}\cos 2\beta + m_t^2 + |m_{34}^t|^2, \quad (\tilde{\mathcal{M}}_t^2)_{12} = (m_0 - \mu\cot\beta)m_t, \quad (\tilde{\mathcal{M}}_t^2)_{13} = (m_0 - \mu\cot\beta)m_{34}^t, \\
&(\tilde{\mathcal{M}}_t^2)_{14} = \mu^U m_{34}^t, \quad (\tilde{\mathcal{M}}_t^2)_{15} = m_t m_{43}^t + m_{34}^t m_{44}^t, \quad (\tilde{\mathcal{M}}_t^2)_{16} = 0, \quad (\tilde{\mathcal{M}}_t^2)_{21} = (m_0 - \mu\cot\beta)m_t, \\
&(\tilde{\mathcal{M}}_t^2)_{22} = M_U^2 + \frac{2}{3}(m_Z^2 - m_W^2)\cos 2\beta + m_t^2 + |m_{43}^t|^2, \quad (\tilde{\mathcal{M}}_t^2)_{23} = m_t m_{34}^t + m_{43}^t m_{44}^t, \quad (\tilde{\mathcal{M}}_t^2)_{24} = 0, \\
&(\tilde{\mathcal{M}}_t^2)_{25} = (m_0 - \mu\cot\beta)m_{43}^t, \quad (\tilde{\mathcal{M}}_t^2)_{26} = \mu^Q m_{43}^t, \quad (\tilde{\mathcal{M}}_t^2)_{31} = (m_0 - \mu\cot\beta)m_{34}^t, \quad (\tilde{\mathcal{M}}_t^2)_{32} = m_t^* m_{34}^u + m_{43}^t m_{44}^t, \\
&(\tilde{\mathcal{M}}_t^2)_{33} = |\mu^U|^2 + M_U^2 + \frac{2}{3}(m_Z^2 - m_W^2)\cos 2\beta + |m_{34}^t|^2 + |m_{44}^t|^2, \quad (\tilde{\mathcal{M}}_t^2)_{34} = -B^U\mu^U, \quad (\tilde{\mathcal{M}}_t^2)_{35} = (m_0 - \mu\cot\beta)m_{44}^t, \\
&(\tilde{\mathcal{M}}_t^2)_{36} = \mu^Q m_{44}^t + \mu^U m_H^t, \quad (\tilde{\mathcal{M}}_t^2)_{41} = \mu^U m_{34}^t, (\tilde{\mathcal{M}}_t^2)_{42} = 0, \quad (\tilde{\mathcal{M}}_t^2)_{43} = -B^U\mu^U, \\
&(\tilde{\mathcal{M}}_t^2)_{44} = |\mu^U|^2 + M_{UH}^2 - \frac{2}{3}(m_Z^2 - m_W^2)\cos 2\beta + |m_H^t|^2, (\tilde{\mathcal{M}}_t^2)_{45} = \mu^{U*} m_{44}^t + \mu^Q m_H^t, \quad (\tilde{\mathcal{M}}_t^2)_{46} = (m_0 - \mu\tan\beta)m_H^t, \\
&(\tilde{\mathcal{M}}_t^2)_{51} = m_t m_{43}^t + m_{34}^t m_{44}^t, \quad (\tilde{\mathcal{M}}_t^2)_{52} = (m_0 - \mu\cot\beta)m_{43}^t, \quad (\tilde{\mathcal{M}}_t^2)_{53} = (m_0 - \mu\cot\beta)m_{44}^t, \\
&(\tilde{\mathcal{M}}_t^2)_{54} = \mu^U m_{44}^t + \mu^Q m_H^t, \quad (\tilde{\mathcal{M}}_t^2)_{55} = |\mu^Q|^2 + M_Q^2 + \frac{4m_W^2 - m_Z^2}{6}\cos 2\beta + |m_{44}^t|^2 + |m_{43}^t|^2, \quad (\tilde{\mathcal{M}}_t^2)_{56} = -B^Q\mu^Q, \\
&(\tilde{\mathcal{M}}_t^2)_{61} = 0, \quad (\tilde{\mathcal{M}}_t^2)_{62} = \mu^Q m_{43}^t, \quad (\tilde{\mathcal{M}}_t^2)_{63} = \mu^Q m_{44}^t + \mu^U m_H^t, \quad (\tilde{\mathcal{M}}_t^2)_{64} = (m_0 - \mu\tan\beta)m_H^t, \\
&(\tilde{\mathcal{M}}_t^2)_{65} = -B^Q\mu^Q, \quad (\tilde{\mathcal{M}}_t^2)_{66} = |\mu^Q|^2 + M_{QH}^2 + |m_H^t|^2 - \frac{4m_W^2 - m_Z^2}{6}\cos 2\beta.
\end{aligned} \tag{A22}$$

The corresponding unitary scalar matrix is defined as

$$V^t \tilde{\mathcal{M}}_t^2 V^{t\dagger} = \mathrm{diag}(\tilde{M}_t^2, \tilde{M}_{t1}^2, \tilde{M}_{t2}^2, \tilde{M}_{t3}^2, \tilde{M}_{t4}^2, \tilde{M}_{t5}^2). \tag{A23}$$

APPENDIX B: NEUTRINO MASSES IN OUR MODEL

All fourteen types of one-loop Feynman diagrams which can contribute to the neutrino mass and mixing in our model are shown in Fig. 7.

The corresponding analytical results are listed below:

$$M_{ij}^\nu|_{\lambda\lambda} \simeq \frac{1}{8\pi^2}\sum_{k,m}\lambda_{i33}\lambda_{j33}R_{m1}^{*\tau}L_{m1}^{*\tau}V_{k1}^{*\tau}V_{k2}^{\tau}m_{\tau_m}b(m_{\tau_m}, M_{\tilde{\tau}_{L(R)k}}), \tag{B1}$$

$$M_{ij}^\nu|_{\lambda\lambda^E} \simeq \frac{1}{8\pi^2}\sum_{k,m}\lambda_{i3}^E\lambda_{j33}[R_{m1}^{*\tau}L_{m1}^{*\tau}V_{k1}^{*\tau}V_{k3}^{\tau}m_{\tau_m}b(m_{\tau_m}, M_{\tilde{\tau}_{L(R)k}}) + R_{m2}^{*\tau}L_{m1}^{*\tau}V_{k1}^{*\tau}V_{k2}^{\tau}m_{\tau_m}b(m_{\tau_m}, M_{\tilde{\tau}_{L(R)k}})], \tag{B2}$$

$$M_{ij}^\nu|_{\lambda^E\lambda^E} \simeq \frac{1}{8\pi^2}\sum_{k,m}\lambda_{i3}^E\lambda_{j3}^E R_{m2}^{*\tau}L_{m1}^{*\tau}V_{k1}^{*\tau}V_{k3}^{\tau}m_{\tau_m}b(m_{\tau_m}, M_{\tilde{\tau}_{L(R)k}}), \tag{B3}$$

$$M_{ij}^\nu|_{\lambda'\lambda'} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda'_{i33}\lambda'_{j33}[R_{m1}^{*b}L_{m1}^{*b}V_{k1}^{*b}V_{k2}^{b}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}) + \lambda'_{i32}\lambda'_{j23}R_{m1}^{*b}L_{m1}^{*b}m_{b_m}\sin\alpha_{s1(2)}\cos\alpha_{s1(2)}b(m_{b_m}, M_{\tilde{s}_{1,2}})], \tag{B4}$$

$$M^{\nu}_{ij}|_{\lambda^Q\lambda^Q} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda^Q_{i3}\lambda^Q_{j3}R^{*b}_{m1}L^{*b}_{m2}V^{*b}_{k2}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}), \tag{B5}$$

$$M^{\nu}_{ij}|_{\lambda^D\lambda^D} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda^D_{i3}\lambda^D_{j3}R^{*b}_{m2}L^{*b}_{m1}V^{*b}_{k1}V^{b}_{k3}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}), \tag{B6}$$

$$M^{\nu}_{ij}|_{\lambda^{QD}\lambda^{QD}} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda^{QD}_{i}\lambda^{QD}_{j}R^{*b}_{m2}L^{*b}_{m2}V^{*b}_{k3}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}), \tag{B7}$$

$$M^{\nu}_{ij}|_{\lambda^Q\lambda^D} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda^Q_{i3}\lambda^D_{j3}[R^{*b}_{m2}L^{*b}_{m2}V^{*b}_{k1}V^{b}_{k2}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}) + R^{*b}_{m1}L^{*b}_{m1}V^{*b}_{k3}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}})], \tag{B8}$$

$$M^{\nu}_{ij}|_{\lambda^Q\lambda^{QD}} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda^Q_{i3}\lambda^{QD}_{j}[R^{*b}_{m1}L^{*b}_{m2}V^{*b}_{k3}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}) + R^{*b}_{m2}L^{*b}_{m2}V^{*b}_{k2}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}})], \tag{B9}$$

$$M^{\nu}_{ij}|_{\lambda^D\lambda^{QD}} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda^D_{i3}\lambda^{QD}_{j}[R^{*b}_{m2}L^{*b}_{m1}V^{*b}_{k3}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}) + R^{*b}_{m2}L^{*b}_{m2}V^{*b}_{k1}V^{b}_{k3}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}})], \tag{B10}$$

$$M^{\nu}_{ij}|_{\lambda'\lambda^Q} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda'_{i33}\lambda^Q_{j3}[R^{*b}_{m1}L^{*b}_{m1}V^{*b}_{k2}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}) + R^{*b}_{m1}L^{*b}_{m2}V^{*b}_{k1}V^{b}_{k2}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}})], \tag{B11}$$

$$M^{\nu}_{ij}|_{\lambda'\lambda^D} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda'_{i33}\lambda^D_{j3}[R^{*b}_{m2}L^{*b}_{m1}V^{*b}_{k1}V^{b}_{k2}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}) + R^{*b}_{m1}L^{*b}_{m1}V^{*b}_{k1}V^{b}_{k3}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}})], \tag{B12}$$

$$M^{\nu}_{ij}|_{\lambda'\lambda^{QD}} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda'_{i33}\lambda^{QD}_{j}[R^{*b}_{m2}L^{*b}_{m1}V^{*b}_{k2}V^{b}_{k5}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}}) + R^{*b}_{m1}L^{*b}_{m2}V^{*b}_{k1}V^{b}_{k3}m_{b_m}b(m_{b_m}, M_{\tilde{b}_{L(R)k}})], \tag{B13}$$

$$M^{\nu}_{ij}|_{\lambda^H\lambda^H} \simeq \frac{3}{8\pi^2}\sum_{k,m}\lambda^H_{i}\lambda^H_{j}R^{*t}_{m3}L^{*t}_{m3}V^{*u}_{k6}V^{u}_{k4}m_{t_m}b(m_{t_m}, M_{\tilde{t}_{L(R)k}}), \tag{B14}$$

in which $L^{\tau,b,t}$, $R^{\tau,b,t}$ are biunitary matrices of mass matrices between (τ, b, t) and the vectorlike fermions (see Appendix A), while m_{τ_m}, m_{b_m}, m_{t_m} indicate the corresponding mass eigenvalues. $V^{\tilde{\tau},\tilde{d},\tilde{t}}$ are the square mass mixing unitary matrices of their superpartners, while $M_{\tilde{\tau}_{L(R)k}}$, $M_{\tilde{b}_{L(R)k}}$, $M_{\tilde{t}_{L(R)k}}$ stand for the corresponding mass eigenvalues. $\sin\alpha_{s1(2)}$, $\cos\alpha_{s1(2)}$ are the unitary matrix elements of $\tilde{s}$. $b(m_1, m_2)$ is the loop integral factor: $b(m_1, m_2) \equiv \frac{1}{m_1^2-m_2^2}(m_1^2 \ln m_1^2 - m_2^2 \ln m_2^2 - m_2^2 + m_1^2)$. The value range of the indices in Eqs. (B1)–(B3) is $m = 1, 2$, $k = 1$–4, while in Eqs. (B4)–(B14), it is $m = 1, 2, 3$, $k = 1$–6.

APPENDIX C: NEUTRINO SPECTRUM-CALCULATING METHOD AND PARAMETER SETTINGS

The methods to generate neutrino masses and mixing angles with one-loop trilinear $\not{L}$ couplings actually involves the following three matrices:

$$m_1\begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}, \qquad m_2\begin{pmatrix} d^2 & de & df \\ de & e^2 & ef \\ df & ef & f^2 \end{pmatrix},$$
$$m_3\begin{pmatrix} g^2 & gh & gl \\ gh & h^2 & hl \\ gl & hl & l^2 \end{pmatrix}, \tag{C1}$$

where we name each of the matrices above $\mathcal{M}_{1,2,3}$ separately. We assume $m_1 > m_{2,3}$, $m_2 \sim m_3$ and there is no strong hierarchy between $a, b, c, d, e, f, g, h, l$.

$\mathcal{M}_1$ has only one eigenvalue after diagonalized by an unitary rotation

$$X^T\mathcal{M}_1X = \mathrm{diag}(0, 0, M_1), \tag{C2}$$

where

$$M_1 = m_1(a^2 + b^2 + c^2) \tag{C3}$$

and

$$X = \begin{pmatrix} c_2 & s_2 c_3 & s_2 s_3 \\ -s_2 & c_2 c_3 & s_2 s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}, \quad \text{(C4)}$$

$$s_2 = \frac{a}{\sqrt{a^2 + b^2}}, \qquad c_3 = \frac{c}{\sqrt{a^2 + b^2 + c^2}}. \quad \text{(C5)}$$

If we rotate the sum over $\mathcal{M}_{1,2,3}$ by matrix X, it becomes

$$X^T(\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3)X \approx m_1(a^2 + b^2 + c^2)\begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & 1 \end{pmatrix}, \quad \text{(C6)}$$

where ϵ_{ij} are some small values related with m_2/m_1, m_3/m_1 and the other elements of $\mathcal{M}_{1,2,3}$. We can then define another unitary matrix X' to diagonalize the matrix in Eq. (B6) in an approximate way:

$$X'^T X^T(\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3)XX' \approx m_1(a^2 + b^2 + c^2)\mathrm{diag}(\delta'_3, \delta'_2, 1), \quad \text{(C7)}$$

where

$$X' = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{(C8)}$$

and

$$\tan 2\theta_1 = \frac{2\epsilon_{12}}{\epsilon_{22} - \epsilon_{11}}. \quad \text{(C9)}$$

Then from Eq. (B7), we get all three mass eigenvalues

$$M_1 \sim m_1(a^2 + b^2 + c^2), \qquad M_2 \sim M_1\delta'_2, \qquad M_3 \sim M_1\delta'_3, \quad \text{(C10)}$$

and from Eqs. (B4) and (B8), we get all three mixing angles

$$s_{13} = s_2 s_3 = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad s_{23} = c_2 s_3/c_{13} = \frac{b}{\sqrt{b^2 + c^2}}, \quad s_{12} = (s_1 c_2 + c_1 s_2 c_3)/c_{13}. \quad \text{(C11)}$$

The parameter settings we used in Table I are given as the following:

Set I:

$$m^\tau_{34} = 10, \quad M_E = M_{EH} = 600\ \text{GeV},$$
$$B^E\mu^E = 400^2\ \text{GeV}^2; \quad m^b_H = 170\ \text{GeV},$$
$$m^b_{34} = m^b_{43} = m^b_{44} = 0,$$
$$M_Q = M_D = M_{DH} = M_{QH} = 700\ \text{GeV},$$
$$B^D\mu^D = B^Q\mu^Q = 500^2\ \text{GeV}^2;$$
$$m^t_{34} = m^t_{43} = m^t_H = 13\ \text{GeV}, \quad m^t_{44} = 174\ \text{GeV},$$
$$M_U = M_{UH} = 700\ \text{GeV}, \quad B^U\mu^U = 500^2\ \text{GeV}^2,$$
$$\tan\beta = 10, \quad A = \mu = 500\ \text{GeV}.$$

Set II:

$$m^\tau_{34} = 10, \quad M_E = M_{EH} = 1000\ \text{GeV},$$
$$B^E\mu^E = 600^2\ \text{GeV}^2; \quad m^b_H = 170\ \text{GeV},$$
$$m^b_{34} = m^b_{43} = m^b_{44} = 10\ \text{GeV},$$
$$M_Q = M_D = M_{DH} = M_{QH} = 1000\ \text{GeV},$$
$$B^D\mu^D = B^Q\mu^Q = 600^2\ \text{GeV}^2;$$
$$m^t_{34} = m^t_{43} = m^t_H = 13\ \text{GeV}, \quad m^t_{44} = 174\ \text{GeV},$$
$$M_U = M_{UH} = 1000\ \text{GeV}, \quad B^U\mu^U = 600^2\ \text{GeV}^2,$$
$$\tan\beta = 10, \quad A = \mu = 600\ \text{GeV}.$$

APPENDIX D: EXOTIC QUARK AND LEPTON COUPLINGS TO W, Z, h^0 AND DECAY WIDTHS

The couplings for the W, Z, h^0 boson with leptons in Eq. (13) are

$$\begin{aligned} g^W_{\tilde{\tau}_{1L}\nu_{\tau L}} &= \frac{g}{\sqrt{2}}L^\tau_{21}, \\ g^Z_{\tilde{\tau}_{1L}\tau_L} &= \frac{g s^2_W}{c_W}L^\tau_{22}L^\tau_{12} - \frac{g}{4c_W}[(2 - 4s^2_W)L^\tau_{21}L^\tau_{11}], \\ g^Z_{\tilde{\tau}_{1R}\tau_R} &= \frac{g s^2_W}{c_W}R^\tau_{22}R^\tau_{12} + \frac{g}{4c_W}(4s^2_W R^\tau_{21}R^\tau_{11}), \\ g^{h^0}_{\tilde{\tau}_{1L}\tau_R} &= -\frac{s_\alpha}{\sqrt{2}}(y^\tau_{33}L^\tau_{21}R^\tau_{11} + y^E_3 L^\tau_{21}R^\tau_{12}), \\ g^{h^0}_{\tilde{\tau}_L\tau_{1R}} &= -\frac{s_\alpha}{\sqrt{2}}(y^\tau_{33}L^\tau_{11}R^\tau_{21} + y^E_3 L^\tau_{11}R^\tau_{22}), \end{aligned} \quad \text{(D1)}$$

where $c_\alpha = s_\beta$, $s_\alpha = -c_\beta$ is the elements of the rotation matrix related with the real parts of (H^0_u, H^0_d).

Then the decay widths of τ_1 are

$$\begin{aligned}
\Gamma(\tau_1 \to W\nu_\tau) &= \frac{m_{\tau_1}}{32\pi}(1 + x_W^4 - 2x_W^2)^{1/2}(1 - 2x_W^2 + x_W^{-2})(g^W_{\tilde{\tau}_1\nu_\tau})^2,\\
\Gamma(\tau_1 \to Z\tau) &= \frac{m_{\tau_1}}{32\pi}(1 + x_Z^4 + x_\tau^4 - 2x_Z^2 - 2x_\tau^2 - 2x_Z^2x_\tau^2)^{1/2}\Big\{(1 + x_\tau^2 - 2x_Z^2 + (1 - x_\tau^2)^2x_Z^{-2})[(g^Z_{\tilde{\tau}_{1L}\tau_L})^2 + (g^Z_{\tilde{\tau}_{1R}\tau_R})^2]\\
&\quad + 12x_\tau g^Z_{\tilde{\tau}_{1L}\tau_L}g^Z_{\tilde{\tau}_{1R}\tau_R}\Big\},\\
\Gamma(\tau_1 \to h^0\tau) &= \frac{m_{\tau_1}}{32\pi}(1 + x_{h^0}^4 + x_\tau^4 - 2x_{h^0}^2 - 2x_\tau^2 - 2x_{h^0}^2x_\tau^2)^{1/2}\Big\{(1 + x_\tau^2 - x_{h^{02}})\Big[(g^{h^0}_{\tilde{\tau}_{1L}\tau_R})^2 + (g^{h^0}_{\tilde{\tau}_L\tau_{1R}})^2\Big] + 4x_\tau g^{h^0}_{\tilde{\tau}_{1L}\tau_R}g^{h^0}_{\tilde{\tau}_L\tau_{1R}}\Big\},
\end{aligned} \tag{D2}$$

where $x_i = m_i/m_{\tau_1}$ for $i = W, Z, \tau, h^0$.

The couplings for the W, Z, h^0 boson with t, t_1, t_2 in Eq. (14) are

$$\begin{aligned}
g^W_{\tilde{t}_{1L}b_L} &= \frac{g}{\sqrt{2}}(L^t_{21}L^b_{11} + L^t_{22}L^b_{12}), \qquad g^W_{\tilde{t}_{1R}b_R} = \frac{g}{\sqrt{2}}R^t_{23}R^b_{13},\\
g^W_{\tilde{t}_{2L}b_L} &= \frac{g}{\sqrt{2}}(L^t_{31}L^b_{11} + L^t_{32}L^b_{12}), \qquad g^W_{\tilde{t}_{2R}b_R} = \frac{g}{\sqrt{2}}R^t_{33}R^b_{13},\\
g^W_{\tilde{t}_{2L}b_{1L}} &= \frac{g}{\sqrt{2}}(L^t_{31}L^b_{21} + L^t_{32}L^b_{22}), \qquad g^W_{\tilde{t}_{2R}b_{1R}} = \frac{g}{\sqrt{2}}R^t_{33}R^b_{23},\\
g^Z_{\tilde{t}_{1L}t_L} &= \frac{-2gs_W^2}{3c_W}L^t_{23}L^t_{13} + \frac{g}{4c_W}\Big[\Big(2 - \frac{8}{3}s_W^2\Big)(L^t_{21}L^t_{11} + L^t_{22}L^t_{12})\Big],\\
g^Z_{\tilde{t}_{1R}t_R} &= -\frac{g}{4c_W}\Big[\frac{8}{3}s_W^2(R^t_{21}R^t_{11} + R^t_{22}R^t_{12}) + \Big(2 - \frac{4}{3}s_W^2\Big)R^t_{23}R^t_{13}\Big],\\
g^Z_{\tilde{t}_{2L}t_L} &= \frac{-2gs_W^2}{3c_W}L^t_{33}L^t_{13} + \frac{g}{4c_W}\Big[\Big(2 - \frac{8}{3}s_W^2\Big)(L^t_{31}L^t_{11} + L^t_{32}L^t_{12})\Big],\\
g^Z_{\tilde{t}_{2R}t_R} &= -\frac{g}{4c_W}\Big[\frac{8}{3}s_W^2(R^t_{31}R^t_{11} + R^t_{32}R^t_{12}) + \Big(2 - \frac{4}{3}s_W^2\Big)R^t_{33}R^t_{13}\Big],\\
g^Z_{\tilde{t}_{2L}t_{1L}} &= \frac{-2gs_W^2}{3c_W}L^t_{33}L^t_{23} + \frac{g}{4c_W}\Big[\Big(2 - \frac{8}{3}s_W^2\Big)(L^t_{31}L^t_{21} + L^t_{32}L^t_{22})\Big],\\
g^Z_{\tilde{t}_{2R}t_{1R}} &= -\frac{g}{4c_W}\Big[\frac{8}{3}s_W^2(R^t_{31}R^t_{21} + R^t_{32}R^t_{22}) + \Big(2 - \frac{4}{3}s_W^2\Big)R^t_{33}R^t_{23}\Big],\\
g^{h^0}_{\tilde{t}_{1L}t_R} &= \frac{c_\alpha}{\sqrt{2}}(y^u_{33}L^t_{21}R^t_{11} + y^Q_3L^t_{22}R^t_{11} + y^U_3L^t_{21}R^t_{12} + y^{QU}_3L^t_{22}R^t_{12}) - \frac{s_\alpha}{\sqrt{2}}y^HL^t_{23}R^t_{13},\\
g^{h^0}_{\tilde{t}_Lt_{1R}} &= \frac{c_\alpha}{\sqrt{2}}(y^u_{33}L^t_{11}R^t_{21} + y^Q_3L^t_{12}R^t_{21} + y^U_3L^t_{11}R^t_{22} + y^{QU}_3L^t_{12}R^t_{22}) - \frac{s_\alpha}{\sqrt{2}}y^HL^t_{13}R^t_{23},\\
g^{h^0}_{\tilde{t}_{2L}t_R} &= \frac{c_\alpha}{\sqrt{2}}(y^u_{33}L^t_{31}R^t_{11} + y^Q_3L^t_{32}R^t_{11} + y^U_3L^t_{31}R^t_{12} + y^{QU}_3L^t_{32}R^t_{12}) - \frac{s_\alpha}{\sqrt{2}}y^HL^t_{33}R^t_{13},\\
g^{h^0}_{\tilde{t}_Lt_{2R}} &= \frac{c_\alpha}{\sqrt{2}}(y^u_{33}L^t_{11}R^t_{31} + y^Q_3L^t_{12}R^t_{31} + y^U_3L^t_{11}R^t_{32} + y^{QU}_3L^t_{12}R^t_{32}) - \frac{s_\alpha}{\sqrt{2}}y^HL^t_{13}R^t_{33},\\
g^{h^0}_{\tilde{t}_{2L}t_{1R}} &= \frac{c_\alpha}{\sqrt{2}}(y^u_{33}L^t_{31}R^t_{21} + y^Q_3L^t_{32}R^t_{21} + y^U_3L^t_{31}R^t_{22} + y^{QU}_3L^t_{32}R^t_{22}) - \frac{s_\alpha}{\sqrt{2}}y^HL^t_{33}R^t_{23},\\
g^{h^0}_{\tilde{t}_{1L}t_{2R}} &= \frac{c_\alpha}{\sqrt{2}}(y^u_{33}L^t_{21}R^t_{31} + y^Q_3L^t_{22}R^t_{31} + y^U_3L^t_{21}R^t_{32} + y^{QU}_3L^t_{22}R^t_{32}) - \frac{s_\alpha}{\sqrt{2}}y^HL^t_{23}R^t_{33}.
\end{aligned} \tag{D3}$$

The decay widths of the lightest new up-type quark t_1 are

$$\begin{aligned}
\Gamma(\mathrm{t}_1 \to \mathrm{Wb}) &= \frac{m_{t_1}}{32\pi}(1 + x_W^4 + x_b^4 - 2x_W^2 - 2x_b^2 - 2x_W^2 x_b^2)^{1/2}\Big\{(1 + x_b^2 - 2x_W^2 + (1 - x_b^2)^2 x_W^{-2})[(g^W_{\bar t_{1L} b_L})^2 + (g^W_{\bar t_{1R} b_R})^2] \\
&\quad + 12 x_b g^W_{\bar t_{1L} b_L} g^W_{\bar t_{1R} b_R}\Big\}, \\
\Gamma(\mathrm{t}_1 \to \mathrm{Zt}) &= \frac{m_{t_1}}{32\pi}(1 + x_Z^4 + x_t^4 - 2x_Z^2 - 2x_t^2 - 2x_Z^2 x_t^2)^{1/2}\Big\{(1 + x_t^2 - 2x_Z^2 + (1 - x_t^2)^2 x_Z^{-2})[(g^{\prime Z}_{\bar t_{1L} t_L})^2 + (g^Z_{\bar t_{1R} t_R})^2] \\
&\quad + 12 x_t g^Z_{\bar t_{1L} t_L} g^Z_{\bar t_{1R} t_R}\Big\}, \\
\Gamma(\mathrm{t}_1 \to \mathrm{h}^0\mathrm{t}) &= \frac{m_{t_1}}{32\pi}(1 + x_{h^0}^4 + x_t^4 - 2x_{h^0}^2 - 2x_t^2 - 2x_{h^0}^2 x_t^2)^{1/2}\Big\{(1 + x_t^2 - x_{h^0}^2)\Big[(g^{h^0}_{\bar t_{1L} t_R})^2 + (g^{h^0}_{\bar t_L t_{1R}})^2\Big] + 4 x_t g^{h^0}_{\bar t_{1L} t_R} g^{h^0}_{\bar t_L t_{1R}}\Big\},
\end{aligned} \tag{D4}$$

where $x_i = m_i/m_{t_1}$ for $i = W, Z, t, h^0$. The heaviest new up-type quark t_2 has six decay channels. The decay widths have similar forms and can be deduced straightforwardly.

The couplings for the W, Z, h^0 boson with b, b_1, b_2 in Eq. (A1) are

$$\begin{aligned}
g^W_{\bar b_{1L} t_L} &= \frac{g}{\sqrt{2}}(L^b_{21} L^t_{11} + L^b_{22} L^t_{12}), \qquad g^W_{\bar b_{1R} t_R} = \frac{g}{\sqrt{2}} R^b_{23} R^t_{13}, \\
g^W_{\bar b_{2L} t_L} &= \frac{g}{\sqrt{2}}(L^b_{31} L^t_{11} + L^b_{32} L^t_{12}), \qquad g^W_{\bar b_{2R} t_R} = \frac{g}{\sqrt{2}} R^b_{33} R^t_{13}, \\
g^W_{\bar b_{2L} t_{1L}} &= \frac{g}{\sqrt{2}}(L^b_{31} L^t_{21} + L^b_{32} L^t_{22}), \qquad g^W_{\bar b_{2R} t_{1R}} = \frac{g}{\sqrt{2}} R^b_{33} R^t_{23}, \\
g^Z_{\bar b_{1L} b_L} &= \frac{g s_W^2}{3 c_W} L^b_{23} L^b_{13} - \frac{g}{4 c_W}\Big[\Big(2 - \frac{4}{3} s_W^2\Big)(L^b_{21} L^b_{11} + L^b_{22} L^b_{12})\Big], \\
g^Z_{\bar b_{1R} b_R} &= \frac{g}{4 c_W}\Big[\frac{4}{3} s_W^2 (R^b_{21} R^b_{11} + R^b_{22} R^b_{12}) + \Big(2 - \frac{8}{3} s_W^2\Big) R^b_{23} R^b_{13}\Big], \\
g^Z_{\bar b_{2L} b_L} &= \frac{g s_W^2}{3 c_W} L^b_{33} L^b_{13} - \frac{g}{4 c_W}\Big[\Big(2 - \frac{4}{3} s_W^2\Big)(L^b_{31} L^b_{11} + L^b_{32} L^b_{12})\Big], \\
g^Z_{\bar b_{2R} b_R} &= \frac{g}{4 c_W}\Big[\frac{4}{3} s_W^2 (R^b_{31} R^b_{11} + R^b_{32} R^b_{12}) + \Big(2 - \frac{8}{3} s_W^2\Big) R^b_{33} R^b_{13}\Big], \\
g^Z_{\bar b_{2L} b_{1L}} &= \frac{g s_W^2}{3 c_W} L^b_{33} L^b_{23} - \frac{g}{4 c_W}\Big[\Big(2 - \frac{4}{3} s_W^2\Big)(L^b_{31} L^b_{21} + L^b_{32} L^b_{22})\Big], \\
g^Z_{\bar b_{2R} b_{1R}} &= \frac{g}{4 c_W}\Big[\frac{4}{3} s_W^2 (R^b_{31} R^b_{21} + R^b_{32} R^b_{22}) + \Big(2 - \frac{8}{3} s_W^2\Big) R^b_{33} R^b_{23}\Big], \\
g^{h^0}_{\bar b_{1L} b_R} &= -\frac{s_\alpha}{\sqrt{2}}(y^d_{33} L^b_{21} R^b_{11} + y^{Q'}_3 L^b_{22} R^b_{11} + y^D_3 L^b_{21} R^b_{12} + y^{QD}_3 L^b_{22} R^b_{12}) + \frac{c_\alpha}{\sqrt{2}} y^{H'} L^b_{23} R^b_{13}, \\
g^{h^0}_{\bar b_L b_{1R}} &= -\frac{s_\alpha}{\sqrt{2}}(y^d_{33} L^b_{11} R^b_{21} + y^{Q'}_3 L^b_{12} R^b_{21} + y^D_3 L^b_{11} R^b_{22} + y^{QD}_3 L^b_{12} R^b_{22}) + \frac{c_\alpha}{\sqrt{2}} y^{H'} L^b_{13} R^b_{23}, \\
g^{h^0}_{\bar b_{2L} b_R} &= -\frac{s_\alpha}{\sqrt{2}}(y^d_{33} L^b_{31} R^b_{11} + y^{Q'}_3 L^b_{32} R^b_{11} + y^D_3 L^b_{31} R^b_{12} + y^{QD}_3 L^b_{32} R^b_{12}) + \frac{c_\alpha}{\sqrt{2}} y^{H'} L^b_{33} R^b_{13}, \\
g^{h^0}_{\bar b_L b_{2R}} &= -\frac{s_\alpha}{\sqrt{2}}(y^d_{33} L^b_{11} R^b_{31} + y^{Q'}_3 L^b_{12} R^b_{31} + y^D_3 L^b_{11} R^b_{32} + y^{QD}_3 L^b_{12} R^b_{32}) + \frac{c_\alpha}{\sqrt{2}} y^{H'} L^b_{13} R^b_{33}, \\
g^{h^0}_{\bar b_{2L} b_{1R}} &= -\frac{s_\alpha}{\sqrt{2}}(y^d_{33} L^b_{31} R^b_{21} + y^{Q'}_3 L^b_{32} R^b_{21} + y^D_3 L^b_{31} R^b_{22} + y^{QD}_3 L^b_{32} R^b_{22}) + \frac{c_\alpha}{\sqrt{2}} y^{H'} L^b_{33} R^b_{23}, \\
g^{h^0}_{\bar b_{1L} b_{2R}} &= -\frac{s_\alpha}{\sqrt{2}}(y^d_{33} L^b_{21} R^b_{31} + y^{Q'}_3 L^b_{22} R^b_{31} + y^D_3 L^b_{21} R^b_{32} + y^{QD}_3 L^b_{22} R^b_{32}) + \frac{c_\alpha}{\sqrt{2}} y^{H'} L^b_{23} R^b_{33}.
\end{aligned} \tag{D5}$$

The decay widths of the lightest new down-type quark b_1 are

$$\Gamma(\mathrm{b}_1 \to \mathrm{Wt}) = \frac{m_{b_1}}{32\pi}(1 + x_W^4 + x_t^4 - 2x_W^2 - 2x_t^2 - 2x_W^2 x_t^2)^{1/2}\Big\{(1 + x_t^2 - 2x_W^2 + (1 - x_t^2)^2 x_W^{-2})[(g^W_{\bar{b}_{1L}t_L})^2 + (g^W_{\bar{b}_{1R}t_R})^2] + 12x_t g^W_{\bar{b}_{1L}t_L} g^W_{\bar{b}_{1R}t_R}\Big\},$$

$$\Gamma(\mathrm{b}_1 \to \mathrm{Zb}) = \frac{m_{b_1}}{32\pi}(1 + x_Z^4 + x_b^4 - 2x_Z^2 - 2x_b^2 - 2x_Z^2 x_b^2)^{1/2}\Big\{(1 + x_b^2 - 2x_Z^2 + (1 - x_b^2)^2 x_Z^{-2})[(g^Z_{\bar{t}_{1L}t_L})^2 + (g^Z_{\bar{b}_{1R}b_R})^2] + 12x_b g^Z_{\bar{b}_{1L}b_L} g^Z_{\bar{b}_{1R}b_R}\Big\},$$

$$\Gamma(\mathrm{b}_1 \to \mathrm{h}^0\mathrm{b}) = \frac{m_{b_1}}{32\pi}(1 + x_{h^0}^4 + x_b^4 - 2x_{h^0}^2 - 2x_b^2 - 2x_{h^0}^2 x_b^2)^{1/2}\Big\{(1 + x_b^2 - x_{h^0}^2)\Big[(g^{h^0}_{\bar{b}_{1L}b_R})^2 + (g^{h^0}_{\bar{b}_L b_{1R}})^2\Big] + 4x_b g^{h^0}_{\bar{b}_{1L}b_R} g^{h^0}_{\bar{b}_L b_{1R}}\Big\}, \tag{D6}$$

where $x_i = m_i/m_{b_1}$ for index $i = W, Z, b, h^0$. The heaviest new down-type quark b_2 has six decay channels; the decay widths have similar forms and can be deduced straightforwardly.

[1] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012); S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).
[2] K. S. Babu, J. C. Pati, and H. Stremnitzer, Phys. Lett. B **256**, 206 (1991); T. Moroi and Y. Okada, Mod. Phys. Lett. A **07**, 187 (1992); Phys. Lett. B **295**, 73 (1992); K. S. Babu and J. C. Pati, Phys. Lett. B **384**, 140 (1996); M. Bastero-Gil and B. Brahmachari, Nucl. Phys. **B575**, 35 (2000); Q. Shafi and Z. Tavartkiladze, Nucl. Phys. **B580**, 83 (2000); K. S. Babu, I. Gogoladze, and C. Kolda, arXiv:hep-ph/0410085; V. Barger, J. Jiang, P. Langacker, and T.-J. Li, Int. J. Mod. Phys. A **22**, 6203 (2007); K. S. Babu, I. Gogoladze, M. U. Rehman, and Q. Shafi, Phys. Rev. D **78**, 055017 (2008); P. W. Graham, A. Ismail, S. Rajendran, and P. Saraswat, Phys. Rev. D **81**, 055016 (2010); Z.-f. Kang and T.-j. Li, J. High Energy Phys. 10 (2012) 150; G. Krnjaic and D. Stolarski, arXiv:1212.4860.
[3] T. Ibrahim and P. Nath, Phys. Rev. D **78**, 075013 (2008).
[4] C. Liu, Phys. Rev. D **80**, 035004 (2009).
[5] S. P. Martin, Phys. Rev. D **81**, 035004 (2010).
[6] S. W. Ham, S.-a. Shim, and S. K. Oh, arXiv:1001.1129; C. Liu and S. Yang, Phys. Rev. D **81**, 093009 (2010); J. M. Arnold, B. Fornal, and M. Trott, J. High Energy Phys. 08 (2010) 059; T. Moroi, R. Sato, and T. T. Yanagida, Phys. Lett. B **709**, 218 (2012); M. Endo, K. Hamaguchi, S. Iwamoto, and N. Yokozaki, Phys. Rev. D **85**, 095012 (2012); K. Nakayama and N. Yokozaki, J. High Energy Phys. 11 (2012) 158; S. P. Martin and J. D. Wells, Phys. Rev. D **86**, 035017 (2012); Y. Okada and L. Panizzi, arXiv:1207.5607.
[7] R. Sundrum, J. High Energy Phys. 01 (2011) 062; T. Gherghetta and A. Pomarol, J. High Energy Phys. 12 (2011) 069; C. Bouchart, A. Knochel, and G. Moreau, Phys. Rev. C **83**, 048801 (2011); M. McGarrie, arXiv:1109.6245.
[8] F. P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).
[9] S.-B. Kim *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).
[10] C. Aulakh and R. Mohapatra, Phys. Lett. **119B**, 136 (1982); F. Zwirner, Phys. Lett. **132B**, 103 (1983); L. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984); I. H. Lee, Phys. Lett. **138B**, 121 (1984); G. Ross and J. Valle, Phys. Lett. **151B**, 375 (1985); J. Ellis, G. Gelmini, C. Jarlskog, G. G. Ross, and J. W. Valle, Phys. Lett. **150B**, 142 (1985); S. Dawson, Nucl. Phys. **B261**, 297 (1985); R. Barbieri and A. Masiero, Nucl. Phys. **B267**, 679 (1986); S. Dimopoulos and L. Hall, Phys. Lett. B **207**, 210 (1988); V. D. Barger, G. F. Giudice, and T. Han, Phys. Rev. D **40**, 2987 (1989); L. E. Ibáñez and G. G. Ross, Nucl. Phys. **B368**, 3 (1992); C. C. J. Louis and G. Moreau, J. Phys. G **37**, 105015 (2010); M. Chemtob and P. N. Pandita, Phys. Rev. D **76**, 095019 (2007); A. Abada, G. Bhattacharyya, and G. Moreau, Phys. Lett. B **642**, 503 (2006); A. Abada and G. Moreau, J. High Energy Phys. 08 (2006) 044; for reviews, see M. Chemtob, Prog. Part. Nucl. Phys. **54**, 71 (2005); R. Barbier *et al.*, Phys. Rep. **420**, 1 (2005).
[11] A. Y. Smirnov and F. Vissani, Nucl. Phys. **B460**, 37 (1996); C. Liu, Mod. Phys. Lett. A **12**, 329 (1997); M. Drees, S. Pakvasa, X. Tata, and T. der Veldhuis, Phys. Rev. D **57**, R5335 (1998); G. Bhattacharyya, H. V. K. Kleingrothaus, and H. Päs, Phys. Lett. B **463**, 77 (1999); A. S. Joshipura and S. K. Vempati, Phys. Rev. D **60**, 111303 (1999); C.-H. Chang and T.-F. Feng, Eur. Phys. J. C **12**, 137 (2000); M. Bisset, O. C. W. Kong, C. Macesanu, and L. H. Orr, Phys. Rev. D **62**, 035001 (2000); E. J. Chun and S. K. Kang, Phys. Rev. D **61**, 075012 (2000); A. S. Joshipura, R. D. Vaidya, and S. K. Vempati, Phys. Rev. D **65**, 053018 (2002); A. Abada, S. Davidson, and M. Losada, Phys. Rev. D **65**, 075010 (2002); A. Abada, G. Bhattacharyya, and M. Losada, Phys. Rev. D **66**, 071701 (2002); F. Borzumati and J. S. Lee, Phys. Rev. D **66**, 115012 (2002); M. A. Diaz, M. Hirsch, W. Porod, J. C. Romao, and J. W. F. Valle, Phys. Rev. D **68**, 013009 (2003); **71**, 059904(E) (2005); S. Bar-Shalom and S. Roy, Phys. Rev. D **69**, 075004 (2004); B. C. Allanach, A. Dedes, and H. K. Dreiner, Phys. Rev. D **69**, 115002 (2004); **72**, 079902(E) (2005); G. Bhattacharyya,

P. B. Pal, H. Päs, and T. J. Weiler, Phys. Rev. D **74**, 053006 (2006); M. Gozdz, W. A. Kaminski, F. Simkovic, and A. Faessler, Phys. Rev. D **74**, 055007 (2006); S. Choubey and M. Mitra, J. High Energy Phys. 05 (2010) 021; E. Halyo, arXiv:1101.6044; C. Frugiuele and T. Gregoire, Phys. Rev. D **85**, 015016 (2012); E. Peinado and A. Vicente, Phys. Rev. D **86**, 093024 (2012).

[12] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).

[13] J. W. F. Valle, arXiv:hep-ph/9712277.

[14] K. S. Babu and R. N. Mohapatra, Phys. Rev. D **42**, 3778 (1990); R. Barbieri, M. M. Guzzo, A. Masiero, and D. Tommasini, Phys. Lett. B **252**, 251 (1990).

[15] A. de Gouvea, S. Lola, and K. Tobe, Phys. Rev. D **63**, 035004 (2001).

[16] K. S. Babua, I. Gogoladze, M. Rehman, and Q. Shafi, Phys. Rev. D **78**, 055017 (2008).

[17] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 103 (2012).

[18] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Rev. Lett. **107**, 271802 (2011).

[19] K. Rao and D. Whiteson, Phys. Rev. D **86**, 015008 (2012).

[20] S. P. Martin and J. D. Wells, Phys. Rev. D **86**, 035017 (2012).

[21] N. Arkani-Hamed, K. Blum, R. T. D'Agnolo, and J. J. Fan, J. High Energy Phys. 01 (2013) 149; M. A. Ajaib, I. Gogoladze, and Q. Shafi, Phys. Rev. D **86**, 095028 (2012); N. Bonne and G. Moreau, Phys. Lett. B **717**, 409 (2012); K. Kumar, R. V. Morales, and F. Yu, Phys. Rev. D **86**, 113002 (2012); A. Joglekar, P. Schwallerb, and C. E. M. Wagnera, arXiv:1303.2969; W. Z. Feng and P. Nath, arXiv:1303.0289; J. Kearney, A. Pierce, and N. Weiner, Phys. Rev. D **86**, 113005 (2012).

[22] K. Ishiwata and M. B. Wise, Phys. Rev. D **84**, 055025 (2011).

[23] M. R. Buckley and D. Hooper, Phys. Rev. D **86**, 075008 (2012); A. Joglekar, P. Schwaller, and C. E. M. Wagner, J. High Energy Phys. 12 (2012) 064; N. D. Christensen, T. Han, and S. Su, Phys. Rev. D **85**, 115018 (2012); R. Benbrik, M. G. Bock, S. Heinemeyer, O. Stal, G. Weiglein, and L. Zeune, Eur. Phys. J. C **72**, 2171 (2012).

[24] M. Fairbairn, A. C. Kraan, D. A. Milstead, T. Sjöstrand, P. Skands, and T. Sloan, Phys. Rep. **438**, 1 (2007).