

# $\bar{B}^0$ decay into $D^0$ and $f_0(500)$ , $f_0(980)$ , $a_0(980)$ , $\rho$ and $\bar{B}_s^0$ decay into $D^0$ and $\kappa(800)$ , $K^{*0}$

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We make predictions for ratios of branching fractions of  $\bar{B}^0$  decays into  $D^0$  and the scalar mesons  $f_0(500)$ ,  $f_0(980)$ ,  $a_0(980)$ , plus  $\bar{B}_s^0$  decay into  $D^0$  and  $\kappa(800)$ . We also compare the  $\pi^+\pi^-$  production in the scalar channel with that observed in the  $\rho$  channel and make predictions for the  $\bar{B}_s^0$  decay into  $D^0$  and  $K^*(892)$ , comparing the strength of this channel with that of  $\kappa(800)$  production. The work is based on results of the chiral unitary approach where the scalar resonances are generated from the pseudoscalar-pseudoscalar interaction. Up to an arbitrary normalization, the mass distributions and rates for decays into the scalar resonances are predicted with no free parameters. Comparison with experimental data is done when available.

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## I. INTRODUCTION

The weak decay of  $B$  mesons has become an unexpected and most valuable source of information on hadron structure and in particular a powerful instrument to investigate the nature of the scalar mesons, which is a permanent source of debate. The starting point in this line came with the observation in LHCb [1] that in the  $B_s^0$  decay into  $J/\psi$  and  $\pi^+\pi^-$  a pronounced peak for the  $f_0(980)$  was observed, while no signal was seen for the  $f_0(500)$  ( $\sigma$ ). This finding was corroborated by following experiments by the Belle [2], CDF [3], and D0 [4] collaborations. Soon it was also observed that in the  $B^0$  decay into  $J/\psi$  and  $\pi^+\pi^-$  [5,6], a clear signal was seen for  $f_0(500)$  production while no signal, or a very small one, was seen for  $f_0(980)$ .

The low lying scalar mesons have been the subject of study within the unitary extension of chiral perturbation theory, the so-called chiral unitary approach, and a coherent picture emerges where these states are generated from the interaction of pseudoscalar mesons provided by the chiral Lagrangians [7–12]. Some other approaches use different starting points, like assuming a seed of  $q\bar{q}$  [13,14], or a tetraquark component [15,16], but as soon as these original components are allowed to mix with the unavoidable meson-meson components, the large strength of this interaction “eats up” the original seed and the meson-meson cloud becomes the largest component of the states.

The dynamical picture to generate the scalar mesons from the pseudoscalar-pseudoscalar interaction has been tested successfully in a large number of reactions [17] (see a recent update in Ref. [18]). However, the findings of the  $B$  decays have opened a new line of research on this topic, offering new and useful information on the structure of these scalar mesons. Indeed, in Ref. [18] it was shown that the features and ratios obtained from the experiments on  $B$  decays could be well reproduced by the dynamical generation picture of the scalars. It was shown there, that although addressing the full complexity of these and related problems can be rather complicated and require many free parameters [19–25], the evaluation of ratios of decay modes for some of these channels is rather simple and, in particular, allows one to get an insight on the structure of the scalar resonances. We shall also mention that our approach is based on the use of the dominant Cabibbo allowed decay mechanisms at the quark level. The approach does not contain subdominant amplitudes which are also considered, for instance, in studies of  $CP$  violation [26,27], but this is not our purpose here.

A related but different path is followed in Ref. [28], looking at the scalars from the point of view of  $q\bar{q}$  or tetraquarks, but no consideration of the final state interaction of these mesons is done there, while this is at the heart of the generation of the scalar mesons in the chiral unitary approach.

The work of Ref. [18] on  $B_s^0$  and  $B^0$  decays in  $J/\psi$  and  $\pi^+\pi^-$  has followed suit along the same lines and in Ref. [29] the rates for  $B_s^0$  and  $B^0$  decays in  $J/\psi$  and a vector meson were investigated and successfully reproduced, along with predictions for the decays into  $J/\psi$  and

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$\kappa(800)$ . Similarly, in Ref. [30] predictions were done for the ratios of branching fractions of  $\bar{B}^0$  and  $\bar{B}_s^0$  decays into  $J/\psi$  and the scalar mesons  $f_0(1370)$ ,  $f_0(1710)$ , or tensor mesons  $f_2(1270)$ ,  $f_2'(1525)$ ,  $K_2^*(1430)$ . Related work, but on weak  $D$  decays into  $K^0$  and the  $f_0(500)$ ,  $f_0(980)$ , and  $a_0(980)$ , has been done in Ref. [31]. One of the interesting things about these weak decays is that isospin is not conserved and then one can obtain states of different isospin, like the  $f_0(980)$  and  $a_0(980)$ , from the same reaction. The prediction for the rates of these two channels from the same reaction is a new test offered by these weak decays.

In the present paper we undertake a related problem. We study the decay of  $\bar{B}^0$  into  $D^0$  and  $f_0(500)$ ,  $f_0(980)$ , and  $a_0(980)$ . At the same time we study the decay of  $\bar{B}_s^0$  into  $D^0$  and  $\kappa(800)$ . We also relate the rates of production of vector mesons and compare  $\rho$  with  $f_0(500)$  production and  $K^{*0}$  with  $\kappa(800)$  production. Experimentally there is information on  $\rho$  and  $f_0(500)$  production in Ref. [32] for the  $\bar{B}^0$  decay into  $D^0$  and  $\pi^+\pi^-$ . There is also information on the ratio of the rates for  $B^0 \rightarrow \bar{D}^0 K^+ K^-$  and  $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$  [33]. We investigate all these rates and compare them with the experimental information.

## II. FORMALISM

Following Refs. [18] and [28] we show in Fig. 1 the dominant diagrams for  $\bar{B}^0$  [Fig. 1(a)] and  $\bar{B}_s^0$  [Fig. 1(b)] decays at the quark level. The mechanism has the  $b \rightarrow c$  transition, needed for the decay, and the  $u \rightarrow d$  vertex that requires the Cabibbo favored  $V_{ud}$  Cabibbo-Kobayashi-Maskawa (CKM) matrix element ( $V_{ud} = \cos \theta_c$ ). Note that these two processes have the same two weak vertices. Under the assumption that the  $\bar{d}$  in Fig. 1(a) and the  $\bar{s}$  in Fig. 1(b) act as spectators in these processes, these amplitudes are identical.

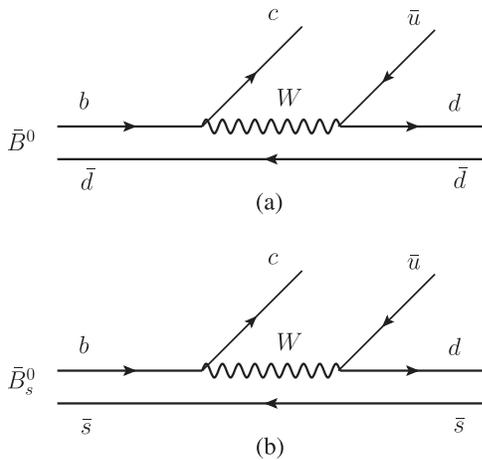


FIG. 1. Diagrammatic representations of  $\bar{B}^0 \rightarrow D^0 \bar{d}$  decay (a) and  $\bar{B}_s^0 \rightarrow D^0 \bar{s}$  decay (b).

### A. $\bar{B}^0$ and $\bar{B}_s^0$ decay into $D^0$ and a vector

Figure 1(a) contains  $d\bar{d}$  from where the  $\rho$  and  $\omega$  mesons can be formed. Figure 1(b) contains  $d\bar{s}$  from where the  $K^{*0}$  emerges. At the quark level, we have

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}); \quad |\omega\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}); \quad (1)$$

$$|K^{*0}\rangle = d\bar{s}. \quad (2)$$

Hence, by taking as reference the amplitude for  $\bar{B}^0 \rightarrow D^0 K^*$  as  $V'_p p_D$ , we can write the rest of the amplitudes as

$$t_{\bar{B}^0 \rightarrow D^0 \rho^0} = -\frac{1}{\sqrt{2}} V'_p p_D, \quad (3)$$

$$t_{\bar{B}^0 \rightarrow D^0 \omega} = \frac{1}{\sqrt{2}} V'_p p_D, \quad (4)$$

$$t_{\bar{B}^0 \rightarrow D^0 \phi} = 0, \quad (5)$$

$$t_{\bar{B}_s^0 \rightarrow D^0 K^{*0}} = V'_p p_D, \quad (6)$$

where  $V'_p$  is a common factor to all  $\bar{B}^0(\bar{B}_s^0) \rightarrow D^0 V_i$  decays, with  $V_i$  being a vector meson, and  $p_D$  the momentum of the  $D^0$  meson in the rest frame of the  $\bar{B}^0$  (or  $\bar{B}_s^0$ ),

$$p_D = \frac{\lambda^{1/2}(M_{\bar{B}_i}^2, M_D^2, M_{V_i}^2)}{2M_{\bar{B}_i}} \quad (7)$$

where  $\lambda$  is the Källén function with  $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ .

The factor  $p_D$  is included to account for a necessary  $P$ -wave vertex to allow the transition from  $0^- \rightarrow 0^- 1^-$ . Although parity is not conserved, angular momentum is, and this requires the angular momentum  $L = 1$ . Note that the angular momentum needed here is different than the one in the  $\bar{B}^0 \rightarrow J/\psi V_i$ , where  $L = 0$  [29]. Hence, a mapping from the situation there to the present case is not possible.

The decay width is given by

$$\Gamma_{\bar{B}_i^0 \rightarrow D^0 V_i} = \frac{1}{8\pi M_{\bar{B}_i}^2} |t_{\bar{B}_i^0 \rightarrow D^0 V_i}|^2 p_D. \quad (8)$$

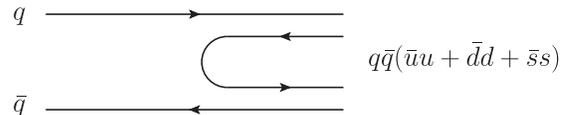


FIG. 2. Schematic representation of the hadronization of a  $q\bar{q}$  pair.

**B.  $\bar{B}^0$  and  $\bar{B}_s^0$  decay into  $D^0$  and a pair of pseudoscalar mesons**

In order to produce a pair of mesons, the final quark-antiquark pair  $d\bar{d}$  or  $d\bar{s}$  in Fig. 1 has to hadronize into two mesons. The flavor content, which is all we need in our study, is easily accounted for in the following way [18,34]: we must add a  $\bar{q}q$  pair with the quantum numbers of the vacuum,  $\bar{u}u + \bar{d}d + \bar{s}s$ , as shown in Fig. 2.

The content of the meson-meson components in the hadronized  $q\bar{q}$  pair is easily done in the following way [18,34]:

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u} \quad \bar{d} \quad \bar{s}), \quad (9)$$

where  $M$  is the  $q\bar{q}$  matrix; then we have the property

$$\begin{aligned} M \cdot M &= \begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u} \quad \bar{d} \quad \bar{s}) \begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u} \quad \bar{d} \quad \bar{s}) \\ &= \begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u} \quad \bar{d} \quad \bar{s}) (\bar{u}u + \bar{d}d + \bar{s}s) \\ &= M(\bar{u}u + \bar{d}d + \bar{s}s). \end{aligned} \quad (10)$$

The next step consists of writing the matrix  $M$  in terms of mesons and we have, using the standard  $\eta$ - $\eta'$  mixing [35,36],

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}. \quad (11)$$

Note that this matrix is different than the standard one used in chiral theory [37] and used in Ref. [7], from where we evaluate the meson-meson amplitudes. The difference between the two matrices is  $\frac{1}{\sqrt{3}}\text{diag}(\eta_1, \eta_1, \eta_1)$  where  $\eta_1$  is the singlet of  $SU(3)$ , which is neglected in the matrix used in chiral theory. The reason is that since the meson-meson interactions are of the type  $(\Phi\partial_\mu\Phi - \partial_\mu\Phi\Phi)^2$ , the singlet contributions are inoperative there.

Hence, we can write

$$d\bar{d}(\bar{u}u + \bar{d}d + \bar{s}s) \rightarrow (\Phi \cdot \Phi)_{22} = \pi^- \pi^+ + \frac{1}{2}\pi^0 \pi^0 + \frac{1}{3}\eta\eta - \sqrt{\frac{2}{3}}\pi^0 \eta + K^0 \bar{K}^0, \quad (12)$$

$$s\bar{d}(\bar{u}u + \bar{d}d + \bar{s}s) \rightarrow (\Phi \cdot \Phi)_{23} = \pi^- K^+ - \frac{1}{\sqrt{2}}\pi^0 K^0, \quad (13)$$

where we have neglected the terms including  $\eta'$  that has too large mass to be relevant in our study.

Eqs. (12) and (13) give us the weight for pairs of two pseudoscalar mesons. The next step consists of letting these mesons interact, which they inevitably will do. This is done in Ref. [18] following the mechanism of Fig. 3.

The  $f_0(500)$  and  $f_0(980)$  will be observed in the  $\bar{B}^0$  decay into  $D^0$  and  $\pi^-\pi^+$  final pairs, the  $a_0(980)$  in  $\pi^0\eta$  pairs, and the  $\kappa(800)$  in the  $\bar{B}_s^0$  decay into  $D^0$  and  $\pi^-K^+$  pairs. Then we have for the corresponding production amplitudes

$$t(\bar{B}^0 \rightarrow D^0 \pi^- \pi^+) = V_P \left( 1 + G_{\pi^- \pi^+} t_{\pi^- \pi^+ \rightarrow \pi^- \pi^+} + \frac{1}{2} G_{\pi^0 \pi^0} t_{\pi^0 \pi^0 \rightarrow \pi^- \pi^+} + \frac{1}{3} G_{\eta\eta} t_{\eta\eta \rightarrow \pi^- \pi^+} + G_{K^0 \bar{K}^0} t_{K^0 \bar{K}^0 \rightarrow \pi^- \pi^+} \right), \quad (14)$$

where  $V_P$  is a common factor of all these processes,  $G_i$  is the loop function of two meson propagators, and we have included the factor  $\frac{1}{2}$  in the intermediate loops involving a pair of identical mesons. The elements of the scattering matrix  $t_{i \rightarrow j}$  are calculated in Refs. [18,31] following the chiral unitary approach in Refs. [7,38]. Note that the use of a common  $V_P$  factor in Eq. (14) is related to the intrinsic  $SU(3)$  symmetric structure of the hadronization  $\bar{u}u + \bar{d}d + \bar{s}s$ , which implicitly assumes that we add an  $SU(3)\bar{q}q$  singlet.

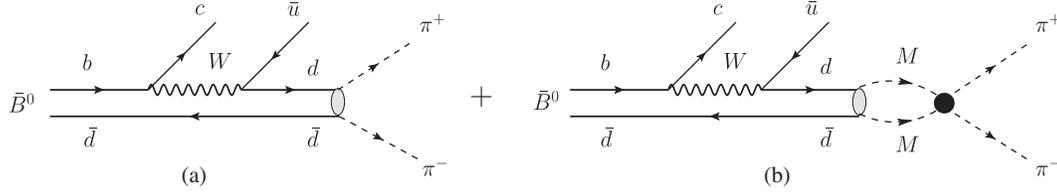


FIG. 3. Diagrammatic representation of the final state interaction of the two mesons produced in a primary step. (a) Direct meson production, (b) meson-meson production through rescattering.

Similarly, we can also produce  $K^+K^-$  pairs and we have

$$t(\bar{B}^0 \rightarrow D^0 K^+ K^-) = V_P \left( G_{\pi^- \pi^+} t_{\pi^- \pi^+ \rightarrow K^+ K^-} + \frac{11}{22} G_{\pi^0 \pi^0} t_{\pi^0 \pi^0 \rightarrow K^+ K^-} + \frac{11}{32} G_{\eta\eta} t_{\eta\eta \rightarrow K^+ K^-} - \sqrt{\frac{2}{3}} G_{\pi^0 \eta} t_{\pi^0 \eta \rightarrow K^+ K^-} + G_{K^0 \bar{K}^0} t_{K^0 \bar{K}^0 \rightarrow K^+ K^-} \right). \quad (15)$$

In the same way we can write<sup>1</sup>

$$t(\bar{B}^0 \rightarrow D^0 \pi^0 \eta) = V_P \left( -\sqrt{\frac{2}{3}} - \sqrt{\frac{2}{3}} G_{\pi^0 \eta} t_{\pi^0 \eta \rightarrow \pi^0 \eta} + G_{K^0 \bar{K}^0} t_{K^0 \bar{K}^0 \rightarrow \pi^0 \eta} \right), \quad (16)$$

and taking into account that the amplitude for  $\bar{B}_s^0 \rightarrow c\bar{u} + d\bar{s}$  in Fig. 1(b) is the same as for  $\bar{B}^0 \rightarrow c\bar{u} + d\bar{d}$  of Fig. 1(a), and using Eq. (13) to account for hadronization, we obtain

$$t(\bar{B}_s^0 \rightarrow D^0 \pi^- K^+) = V_P \left( 1 + G_{\pi^- K^+} t_{\pi^- K^+ \rightarrow \pi^- K^+} - \frac{1}{\sqrt{2}} G_{\pi^0 K^0} t_{\pi^0 K^0 \rightarrow \pi^- K^+} \right), \quad (17)$$

where the amplitudes  $t_{\pi^- K^+ \rightarrow \pi^- K^+}$  and  $t_{\pi^0 K^0 \rightarrow \pi^- K^+}$  are taken from Ref. [38].

In the process of meson-meson scattering in the  $S$ -wave, as we shall study here in order to get the scalar resonances, we have the transition  $0^- \rightarrow 0^- 0^+$  for  $\bar{B}^0 \rightarrow D^0 f_0$ , and now we need  $L = 0$ . Once again the roles of the angular momentum are reversed with respect to the meson pair production in the  $\bar{B}^0 \rightarrow J/\psi \pi^+ \pi^-$  decay [29]. Hence, we can write the differential invariant mass width as

$$\frac{d\Gamma}{dM_{\text{inv}}} = \frac{1}{(2\pi)^3} \frac{p_D \tilde{p}_\pi}{4M_{\bar{B}^0}^2} |t(\bar{B}^0 \rightarrow D^0 \pi^- \pi^+)|^2, \quad (18)$$

where  $\tilde{p}_\pi$  is the pion momentum for the  $\pi^+$  or  $\pi^-$  in the rest frame of the  $\pi^- \pi^+$  system

$$\tilde{p}_\pi = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_\pi^2, m_\pi^2)}{2M_{\text{inv}}}, \quad (19)$$

where  $M_{\text{inv}}$  is the invariant mass of the  $\pi^+ \pi^-$  system, and also write similar formulas for the other decays.

<sup>1</sup>It is worth noting that  $\pi^+ \pi^-$ ,  $\pi^0 \pi^0$ , and  $\eta\eta$  are in isospin  $I = 0$ , while  $\pi^0 \eta$  is in  $I = 1$ .

### III. NUMERICAL RESULTS

In the first place we look for the rates of  $\bar{B}^0$  and  $\bar{B}_s^0$  decay into  $D^0$  and a vector. By looking at Eqs. (3), (4), and (6), we have

$$\frac{\Gamma_{\bar{B}^0 \rightarrow D^0 \rho^0}}{\Gamma_{\bar{B}^0 \rightarrow D^0 \omega}} = \left[ \frac{p_D(\rho^0)}{p_D(\omega)} \right]^3 = 1, \quad (20)$$

$$\frac{\Gamma_{\bar{B}^0 \rightarrow D^0 \rho^0}}{\Gamma_{\bar{B}_s^0 \rightarrow D^0 K^{*0}}} = \left( \frac{M_{\bar{B}_s^0}}{M_{\bar{B}^0}} \right)^2 \frac{1}{2} \left[ \frac{p_D(\rho^0)}{p_D(K^{*0})} \right]^3 \simeq \frac{1}{2}, \quad (21)$$

$$\Gamma_{\bar{B}^0 \rightarrow D^0 \phi} = 0. \quad (22)$$

Experimentally there are no data in the PDG [39] for the branching ratio  $Br(\bar{B}^0 \rightarrow D^0 \phi)$  and we find the branching ratios for  $B^0 \rightarrow \bar{D}^0 \rho^0$  [32],  $B^0 \rightarrow \bar{D}^0 \omega$  [40,41], and  $B_s^0 \rightarrow \bar{D}^0 \bar{K}^{*0}$  [32,42,43] as the following (note the change  $\bar{B}^0 \rightarrow B^0$  and  $D^0 \rightarrow \bar{D}^0$ ,  $\bar{B}_s^0 \rightarrow B_s^0$ ,  $K^{*0} \rightarrow \bar{K}^{*0}$ ):

$$Br(B^0 \rightarrow \bar{D}^0 \rho^0) = (3.2 \pm 0.5) \times 10^{-4}, \quad (23)$$

$$Br(B^0 \rightarrow \bar{D}^0 \omega) = (2.53 \pm 0.16) \times 10^{-4}, \quad (24)$$

$$Br(B_s^0 \rightarrow \bar{D}^0 \bar{K}^{*0}) = (3.5 \pm 0.6) \times 10^{-4}. \quad (25)$$

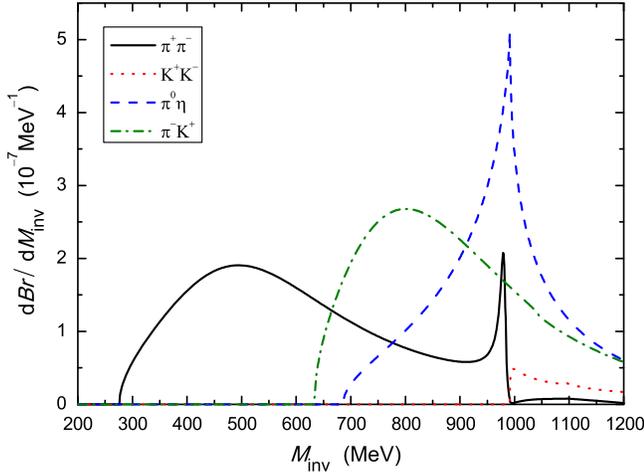


FIG. 4 (color online). Invariant mass distributions for the  $\pi^+\pi^-$ ,  $K^+K^-$ ,  $\pi^0\eta$ , and  $\pi^-K$  in  $\bar{B}^0 \rightarrow D^0\pi^+\pi^-$ ,  $D^0K^+K^-$ ,  $D^0\pi^0\eta$ , and  $\bar{B}_s^0 \rightarrow D^0\pi^-K^+$  decays. The normalization is such that the integral over the  $f_0(500)$  signal gives the experimental branching ratio of Eq. (27).

The ratio  $\frac{\Gamma_{\bar{B}^0 \rightarrow D^0 \rho^0}}{\Gamma_{\bar{B}^0 \rightarrow D^0 \omega}}$  is fulfilled, while the ratio  $\frac{\Gamma_{\bar{B}^0 \rightarrow D^0 \rho^0}}{\Gamma_{\bar{B}_s^0 \rightarrow D^0 K^{*0}}}$  is barely in agreement with data. The branching ratio of Eq. (25) requires combining ratios obtained in different experiments. A direct measure from a single experiment is available in Ref. [44]:

$$\frac{\Gamma_{\bar{B}_s^0 \rightarrow D^0 K^{*0}}}{\Gamma_{\bar{B}^0 \rightarrow D^0 \rho^0}} = 1.48 \pm 0.34 \pm 0.15 \pm 0.12, \quad (26)$$

which is compatible with the factor of 2 that we get from Eq. (21). However, the result of Eq. (25), based on more recent measurements from Refs. [42] and [43], improve on the result of Eq. (26) [45], which means that our prediction for this ratio is a bit bigger than experiment.

We turn now to the production of the scalar resonances. By using Eqs. (14)–(17), we obtain the mass distributions for  $\pi^+\pi^-$ ,  $K^+K^-$ , and  $\pi^0\eta$  in  $\bar{B}^0$  decays and  $\pi^-K$  in  $\bar{B}_s^0$  decay. The numerical results are shown in Fig. 4.

The normalization for all the processes is the same. The scale is obtained demanding that the integrated  $f_0(500)$  distribution has the normalization of the experimental branching ratio of Eq. (27). From Fig. 4, in the  $\pi^+\pi^-$  invariant mass distribution for  $\bar{B}^0 \rightarrow D^0\pi^+\pi^-$  decay, we observe an appreciable strength for  $f_0(500)$  excitation and a less strong, but clearly visible excitation for the  $f_0(980)$ . In the  $\pi^0\eta$  invariant mass distribution, the  $a_0(980)$  is also excited with a strength bigger than that of the  $f_0(980)$ . Finally, in the  $\pi^-K^+$  invariant mass distribution, the  $\kappa(800)$  is also excited with a strength comparable to that of the  $f_0(500)$ . We also plot the mass distribution for  $K^+K^-$  production. It begins at threshold and gets strength from the two underlying  $f_0(980)$  and  $a_0(980)$  resonances; hence we

can see an accumulated strength close to threshold that makes the distribution clearly different from phase space.

There is some experimental information to test some of the predictions of our results. Indeed in Ref. [32] (see Table II of that paper) one can find the rates of production for  $f_0(500)$  [it is called  $f_0(600)$  there] and  $f_0(980)$ . Concretely,

$$\begin{aligned} Br[\bar{B}^0 \rightarrow D^0 f_0(500)] \cdot Br[f_0(500) \rightarrow \pi^+\pi^-] \\ = (0.68 \pm 0.08) \times 10^{-4}, \end{aligned} \quad (27)$$

$$\begin{aligned} Br[\bar{B}^0 \rightarrow D^0 f_0(980)] \cdot Br[f_0(980) \rightarrow \pi^+\pi^-] \\ = (0.08 \pm 0.04) \times 10^{-4}, \end{aligned} \quad (28)$$

where the errors are only statistical. This gives

$$\begin{aligned} \frac{Br[\bar{B}^0 \rightarrow D^0 f_0(980)] \cdot Br[f_0(980) \rightarrow \pi^+\pi^-]}{Br[\bar{B}^0 \rightarrow D^0 f_0(500)] \cdot Br[f_0(500) \rightarrow \pi^+\pi^-]} \Big|_{\text{Exp}} \\ = 0.12 \pm 0.06. \end{aligned} \quad (29)$$

From Fig. 4 it is easy to estimate our theoretical results for this ratio by integrating over the peaks of the  $f_0(500)$  and  $f_0(980)$ . To separate the  $f_0(500)$  and  $f_0(980)$  contributions, a smooth extrapolation of the curve of Fig. 4 is made from 900 to 1000 MeV, as done in Ref. [31]. We find

$$\frac{Br[\bar{B}^0 \rightarrow D^0 f_0(980)] \cdot Br[f_0(980) \rightarrow \pi^+\pi^-]}{Br[\bar{B}^0 \rightarrow D^0 f_0(500)] \cdot Br[f_0(500) \rightarrow \pi^+\pi^-]} \Big|_{\text{Theo}} = 0.08, \quad (30)$$

with an estimated error of about 10%. As we can see, the agreement of the theoretical results with experiment is good within errors.

We have selected  $\bar{B}^0$  decay into  $D^0$  and  $\pi^+\pi^-$  or  $\pi^0\eta$  and  $\bar{B}_s^0$  into  $D^0$  and  $\pi^-K^+$ , which are Cabibbo favored. In this case one does not find competitive mechanisms corresponding to different topologies of the diagrams [46]. Similarly as done in Ref. [18], one could also consider  $\bar{B}_s^0$  into  $D^0$  and  $\pi^+\pi^-$ . In this case we can have this reaction using the mechanism of Fig. 1(b), replacing the final  $d$  quark with an  $s$  quark. Upon hadronization the  $s\bar{s}$  pair will give  $K\bar{K}$ , which upon rescattering can produce  $\pi^+\pi^-$ . The  $udW$  transition is replaced by the  $usW$  transition and hence the  $\cos\theta_c$  into  $\sin\theta_c$ . The evaluation of this diagram is straightforward, but there is a competing diagram of the type of external emission [see Fig. 5 of Ref. [31]] where the  $W$  directly converts into  $s\bar{u}(K^-)$  and the final quark is a  $c$  quark. Upon hadronization of the  $c\bar{s}$  pair we can get  $D^0$  and  $K^+$ . In both mechanisms we have  $K\bar{K}D^0$  in the final state, which through rescattering will give  $D^0\pi^+\pi^-$ , and the two mechanisms interfere. We thus cannot be as predictive as in the other cases where there is only one dominant mechanism and unknown dynamical factors cancel in ratios.

However, we can already say that these two mechanisms are both Cabibbo suppressed, so the ratio of  $f_0(980)$  production in this case would be suppressed with respect to the  $B^0$  case by  $(\sin\theta_c/\cos\theta_c)^2$  with respect to the  $B^0$  case. This is in contrast to the  $B^0$  and  $B_s^0$  decays into  $J/\psi$  and  $f_0(980)$ , where the second decay was favored with respect to the first one [1–6,18]. On the other hand, we see also here that the  $\pi^+\pi^-$  in the  $\bar{B}_s^0$  decay into  $D^0$  and  $\pi^+\pi^-$  proceeds via rescattering of the primary produced  $K\bar{K}$  pair. This is similar to the case of  $B_s^0$  decay into  $J/\psi$  and  $\pi^+\pi^-$  in Ref. [18], and thus we can also predict that in the  $\bar{B}_s^0 \rightarrow D^0\pi^+\pi^-$  the  $f_0(980)$ , although Cabibbo suppressed, could be seen and there would be practically no trace of the  $f_0(500)$  excitation.

It is most instructive to show the  $\pi^+\pi^-$  production combining the  $S$ -wave and  $P$ -wave production. In order to do that, we evaluate  $V_P$  of Eq. (14) and  $V'_P$  of Eq. (3), normalized to obtain the branching fractions given in Eqs. (27) and (23), rather than widths. We shall call the parameters  $\tilde{V}_P$  and  $\tilde{V}'_P$ , suited to this normalization.

We obtain  $\tilde{V}_P = (8.8 \pm 0.5) \times 10^{-2} \text{ MeV}^{-1/2}$  and  $\tilde{V}'_P = (6.8 \pm 0.5) \times 10^{-3} \text{ MeV}^{-1/2}$ .

To obtain the  $\pi^+\pi^-$  mass distribution for the  $\rho$ , we need to convert the total rate for vector production into a mass distribution. This we do by following the steps of Ref. [29], and then we write

$$\begin{aligned} & \frac{d\Gamma_{\bar{B}^0 \rightarrow D^0 \rho^0 \rightarrow D^0 \pi^+ \pi^-}}{dM_{\text{inv}}} \\ &= -\frac{2m_\rho}{\pi} \text{Im} \left[ \frac{1}{M_{\text{inv}}^2 - m_\rho^2 + im_\rho \Gamma_\rho(M_{\text{inv}})} \right] \tilde{\Gamma}_{\bar{B}^0 \rightarrow D^0 \rho^0}, \end{aligned} \quad (31)$$

where

$$\Gamma_\rho(M_{\text{inv}}) = \Gamma_\rho \left( \frac{p_\pi^{\text{off}}}{p_\pi^{\text{on}}} \right) 3 \frac{m_\rho^2}{M_{\text{inv}}^2}, \quad (32)$$

$$p^{\text{off}} = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_\pi^2, m_\pi^2)}{2M_{\text{inv}}} \theta(M_{\text{inv}} - 2m_\pi), \quad (33)$$

$$p^{\text{on}} = \frac{\lambda^{1/2}(m_\rho^2, m_\pi^2, m_\pi^2)}{2m_\rho}, \quad (34)$$

$$\tilde{\Gamma}_{\bar{B}^0 \rightarrow D^0 \rho^0}(M_{\text{inv}}) = \Gamma_{\bar{B}^0 \rightarrow D^0 \rho^0} \left( \frac{p_D^{\text{off}}}{p_D^{\text{on}}} \right)^3, \quad (35)$$

with  $p_D^{\text{off}}$  the  $D^0$  momentum for  $\pi^+\pi^-$  invariant mass  $M_{\text{inv}}$  and  $p_D^{\text{on}}$  for  $M_{\text{inv}} = m_\rho$ . In order to get the  $\pi^-K^+$  mass distribution for  $\bar{B}_s^0 \rightarrow D^0\pi^-K^+$ , we apply the same procedure, changing the mass and width of the vector, and  $\pi K$  instead of  $\pi\pi$  in the formula of the width.

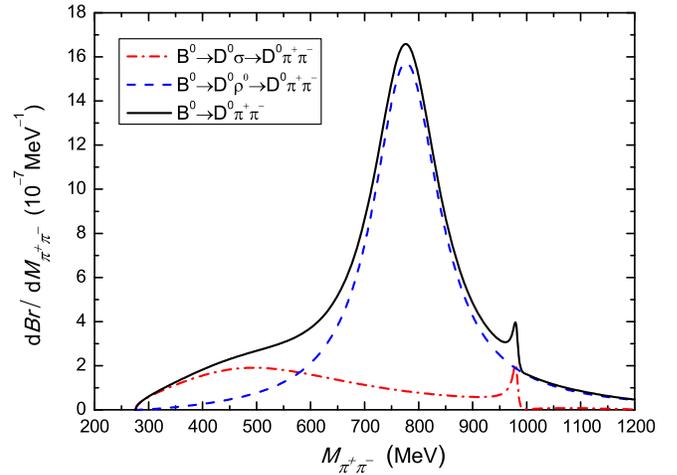


FIG. 5 (color online). Invariant mass distribution for  $\pi^+\pi^-$  in  $\bar{B}^0 \rightarrow D^0\pi^+\pi^-$  decay. The normalization is the same as in Fig. 4.

The formulas are easily generalized for the other decays.

Now we show the results for the  $\pi^+\pi^-$  production in  $\bar{B}^0 \rightarrow D^0\pi^+\pi^-$  in Fig. 5. We see a large contribution from the  $f_0(500)$  and a larger contribution from the  $\rho^0 \rightarrow \pi^+\pi^-$  production. We can see that the  $f_0(500)$  is clearly visible in the distribution of  $\pi^+\pi^-$  invariant mass in the region of 400–600 MeV.

The results of Fig. 5 cannot be directly compared with the experimental ones of Fig. 5 of Ref. [32] because in the experiment a cut for events with  $\pi\pi$  helicity angles with  $\cos(\theta_h) > 0$  has been implemented. We cannot evaluate the helicity angles because our procedure to get the  $\rho$  signal does not explicitly use the pions. Nevertheless, and with this caveat, the shape of the  $\pi\pi$  mass distribution obtained here is remarkably similar to the one of that figure.

The  $V_P$  and  $V'_P$  obtained by fitting the branching ratios of  $f_0(500)$  and  $\rho$  production can be used to obtain the strength of  $K^{*0}$  production versus  $\kappa(800)$  production in the  $\bar{B}_s^0 \rightarrow D^0\pi^-K^+$  decay. For this we use Eqs. (3)–(6) and recall that the rate for  $K^{*0} \rightarrow \pi^-K^+$  is  $\frac{2}{3}$  of the total  $K^{*0}$  production. The results for  $K^{*0} \rightarrow \pi^-K^+$  and  $\kappa(800) \rightarrow \pi^-K^+$  production are shown in Fig. 6, where we see a clear peak for  $K^{*0}$  production, with strength bigger than that for  $\rho^0$  in Fig. 5, due in part to the factor-of-2 bigger strength in Eq. (21) and the smaller  $K^{*0}$  width. The  $\kappa(800)$  is clearly visible in the lower part of the spectrum where the  $K^{*0}$  has no strength.

Finally, although with more uncertainty, we can also estimate the ratio

$$\frac{\Gamma(B^0 \rightarrow \bar{D}^0 K^+ K^-)}{\Gamma(B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-)} = 0.056 \pm 0.011 \pm 0.007 \quad (36)$$

of Ref. [33]. This requires an extrapolation of our results to higher invariant masses where our results would not be accurate, but, assuming that most of the strength for both

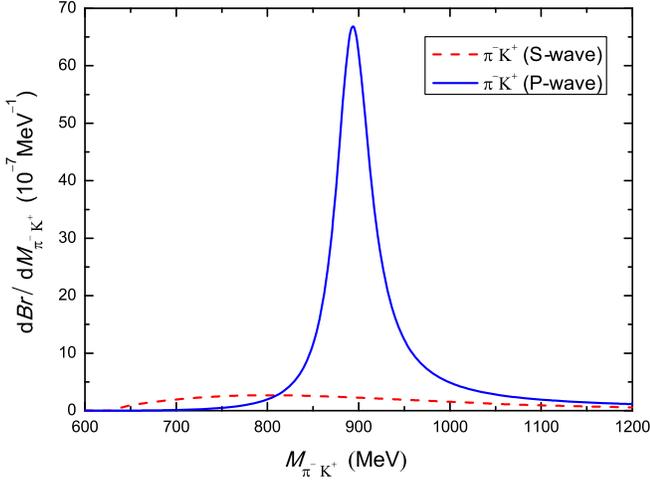


FIG. 6 (color online). Invariant mass distribution for  $\pi^- K^+$  in  $\bar{B}_s^0 \rightarrow D^0 \pi^- K^+$  decay. The normalization is the same as in Fig. 4.

reactions comes from the region close to the  $K^+ K^-$  threshold and from the  $\rho^0$  peak, respectively, we obtain a ratio of the order of 0.03–0.06, which agrees qualitatively with the ratio of Eq. (36).

#### IV. CONCLUSIONS

In this paper we have addressed the study of the  $\bar{B}^0$  decay into  $D^0$  and  $\rho$  or  $f_0(500)$ ,  $f_0(980)$ ,  $a_0(980)$ , and  $\bar{B}_s^0$  decay into  $D^0$  and  $K^*(892)$  or  $\kappa(800)$ . The model used is simple to interpret and allows us to get relative strengths of the different reactions. The Cabibbo favored dominant mechanism at the quark level is identified and then the rates for production of vector mesons are trivially obtained assuming a  $q\bar{q}$  nature for the light vector mesons. The relative rates obtained are in good agreement with experimental data. This in itself is already a good finding, supporting the  $q\bar{q}$  structure for the light vector mesons, which has been advocated from the large  $N_c$  behavior of the amplitudes [47] and from the compositeness sum rule [48,49]. As to the production of the scalar mesons we could predict the invariant mass distributions, up to a common global factor, for the  $\bar{B}^0$  decay into  $D^0 f_0(500)[f_0(500) \rightarrow \pi^+ \pi^-]$ ,  $D^0 f_0(980)[f_0(980) \rightarrow \pi^+ \pi^-]$ ,  $D^0 a_0(980)[a_0(980) \rightarrow \pi^0 \eta]$ , and  $\bar{B}_s^0$  decay into  $D^0 \kappa(800)[\kappa(800) \rightarrow \pi^- K^+]$ . Hence the relative weights of the distributions are predicted with no free parameters under the assumption that these resonances are generated dynamically from the meson-meson interactions, and constitute interesting predictions for future experiments, which are most likely to be performed at LHCb or other facilities.

We would like to abound in this latter comment. The work done here follows a different pattern than the one done in many works in related  $B$  decays on mesons [19–24,26,27]. These papers address explicitly the dynamics of the weak decays, and subsequent strong interaction

involved in the quark matrix elements, which are usually evaluated under the factorization approximation. What makes our work different from other related works, such as Ref. [25] and similar ones, is that we explicitly allow the formation of all meson-meson coupled channels in the weak processes and then allow these meson pairs to interact. The resonances investigated are automatically produced since in our approach it is precisely the interaction that creates these resonances (dynamical generation). In Ref. [25] and related works, some channels, as  $K\bar{K}$  in the study of  $\pi\pi$  production, are automatically incorporated by means of form factors at the price of introducing unknown multiplicative factors to be fitted to the data. These form factors contain the dynamics of the interaction of the mesons. Then, different factors appear when using the  $K\pi$  or  $\pi\pi$  scalar form factors, but in our approach we could relate some processes, like the  $\bar{B}^0 \rightarrow D^0 \pi\pi$  and  $\bar{B}_s^0 \rightarrow D^0 K\pi$ , using a unique unknown factor,  $V_P$ . These different approaches are complementary. As mentioned in the Introduction, our approach, relying on one dominant mechanism, allows us to obtain many ratios with no free parameters, but it cannot be used to study processes like  $CP$  violation which require at least two weak amplitudes, for which approaches like those of Refs. [26,27] are demanded. Our approach is particularly suited to study scalar meson production in cases where we are confident that these states are dynamically generated, and the success of our predictions gives further strength to this hypothesis.

On the other hand, with the information obtained for  $f_0(500)$  and  $\rho$  production and using the experimental rates for these processes, we could make predictions for the strength of  $K^{*0}$  production in  $\bar{B}_s^0$  decay into  $D^0$  and  $K^{*0}$  and compare it with the  $\kappa(800)$  contribution. These are again interesting predictions for future experiments, relative to the production of the  $\rho^0$  in the  $\bar{B}^0$  decay into  $D^0$  and  $\rho$ .

The large amount of information predicted in decays which are Cabibbo favored, and the relevance that this information has on the structure of the scalar mesons, should be a clear motivation for the implementation of these experiments in the near future.

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*Note added.*—Recently, two experimental papers were submitted to the arXiv [50,51]. In Ref. [50] the

$B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$  decay was analyzed and the  $\bar{D}^0 f_0(500)$  and  $\bar{D}^0 f_0(980)$  modes were observed. It is easy to see that the ratio of these two branching ratios agree with our results within errors, and so do the ratios of each of them to  $\rho$  production. In Ref. [51] the  $B_s^0 \rightarrow \bar{D}^0 f_0(980)$  signal, which we discussed is Cabibbo suppressed, was found to be very small, and finally an upper limit was provided. On the other hand, the  $B_s^0 \rightarrow \bar{D}^0 f_0(500)$  mode, which we predict should not be seen, was not observed.

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