



# A combined fit on the annihilation corrections in $B_{u,d,s} \rightarrow PP$ decays within QCDF



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## ARTICLE INFO

### Article history:

Received 11 September 2014

Received in revised form 4 November 2014

Accepted 15 November 2014

Available online 20 November 2014

Editor: B. Grinstein

## ABSTRACT

Motivated by the possible large annihilation contributions implied by recent CDF and LHCb measurements on nonleptonic annihilation  $B$ -meson decays, and the refined experimental measurements on hadronic  $B$ -meson decays, we study the strength of annihilation contributions within QCD factorization (QCDF) in this paper. With the available measurements of two-body  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays, a comprehensive fit on the phenomenological parameters  $X_A^{i,f}$  (or  $\rho_A^{i,f}$  and  $\phi_A^{i,f}$ ) which are used to parameterize the endpoint singularity in annihilation amplitudes is performed with the statistical  $\chi^2$  approach. It is found that (1) flavor symmetry breaking effects are hardly to be distinguished between  $X_{A,s}^i$  and  $X_{A,d}^i$  due to the large experimental errors and theoretical uncertainties, where  $X_{A,s}^i$  and  $X_{A,d}^i$  are related to the nonfactorization annihilation contributions in  $B_s$  and  $B_{u,d}$  decay, respectively. So  $X_{A,s}^i \simeq X_{A,d}^i$  is a good approximation by now. (2) In principle, parameter  $X_A^f$  which is related to the factorization annihilation contributions and independent of the initial state can be regarded as the same variable for  $B_{u,d,s}$  decays. (3) Numerically, two solutions are found, one is  $(\rho_A^i, \phi_A^i [^\circ]) = (2.98_{-0.86}^{+1.12}, -105_{-24}^{+34})$  and  $(\rho_A^f, \phi_A^f [^\circ]) = (1.18_{-0.23}^{+0.20}, -40_{-8}^{+11})$ , the other is  $(\rho_A^i, \phi_A^i [^\circ]) = (2.97_{-0.90}^{+1.19}, -105_{-24}^{+32})$  and  $(\rho_A^f, \phi_A^f [^\circ]) = (2.80_{-0.21}^{+0.25}, 165_{-3}^{+4})$ . Obviously, nonfactorization annihilation parameter  $X_A^i$  is generally unequal to factorization annihilation parameter  $X_A^f$ , which differs from the traditional treatment. With the fitted parameters, all results for observables of  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays are in good agreement with experimental data.

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With the running of the Large Hadron Collider (LHC), many intriguing  $B$ -meson decays are well measured and some interesting phenomena are found by LHCb collaboration in the past years. For example, measurements of branching fractions for the pure annihilation  $B_d \rightarrow K^+K^-$  and  $B_s \rightarrow \pi^+\pi^-$  decays [1]. Their averaged results given by Heavy Flavor Averaging Group (HFAG) are [2]

$$\mathcal{B}(B_d \rightarrow K^+K^-) = (0.12 \pm 0.05) \times 10^{-6}, \quad (1)$$

$$\mathcal{B}(B_s \rightarrow \pi^+\pi^-) = (0.73 \pm 0.14) \times 10^{-6}, \quad (2)$$

which attract much attention recently [3–6].

Theoretically, the branching ratios of pure annihilation nonleptonic  $B$  meson decays are formally  $\Lambda_{\text{QCD}}/m_b$  power suppressed and expected at  $10^{-7}$  level, which roughly agrees with the measurements. In the framework of QCD factorization (QCDF) [7],

the annihilation amplitudes, together with the chirally enhanced power corrections and possible large strong phase involved in them, play an important role in evaluating the observables of  $B$  meson decays. However, due to the endpoint singularities, the amplitudes of annihilation topologies are hardly to be exactly calculated. To estimate the endpoint contributions, phenomenological parameter  $X_A$  is introduced [8] as

$$\int_0^1 \frac{dx}{x} \rightarrow X_A = (1 + \rho_A e^{i\phi_A}) \ln \frac{m_B}{\Lambda_h}, \quad (3)$$

where  $\Lambda_h = 0.5$  GeV. The QCDF approach itself cannot give some information/or constraint on parameters  $\rho_A$  and  $\phi_A$ . To simplify the calculation, one usually takes the same parameters  $\rho_A$  and  $\phi_A$  for factorizable and nonfactorizable annihilation topologies. And as a conservative choice, the values of  $\rho_A \sim 1$  and  $\phi_A \sim -55^\circ$  (named scenario S4) [8–10] are usually adopted in previous

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studies on  $B_{u,d,s} \rightarrow PP$  decays, which leads to the prediction<sup>1</sup>  $\mathcal{B}(B_d \rightarrow K^+K^-) = (0.10^{+0.03+0.03}_{-0.02-0.03}) \times 10^{-6}$  [9] and  $\mathcal{B}(B_s \rightarrow \pi^+\pi^-) = (0.26^{+0.00+0.10}_{-0.00-0.09}) \times 10^{-6}$  [10]. Clearly, the QCDF's prediction on  $\mathcal{B}(B_d \rightarrow K^+K^-)$  agrees well with the current measurements considering the experimental and theoretical errors, while the QCDF's prediction on  $\mathcal{B}(B_s \rightarrow \pi^+\pi^-)$  is much smaller than the experimental data Eq. (2) by about  $3\sigma$ , which implies unexpectedly possible large annihilation corrections and possible large flavor symmetry breaking effects between the annihilation amplitudes of  $B_{u,d}$  and  $B_s$  decays [4,5]. Motivated by such mismatch, some works have been done for possible solutions and implications.

Within the QCDF framework, using the asymptotic light-cone distribution amplitudes, the building blocks of annihilation amplitudes are simplified as [7,8]

$$A_1^i \simeq A_2^i \simeq 2\pi\alpha_s \left[ 9 \left( X_A^i - 4 + \frac{\pi^2}{3} \right) + r_\chi^{M_1} r_\chi^{M_2} (X_A^i)^2 \right], \quad (4)$$

$$A_3^i \simeq 6\pi\alpha_s (r_\chi^{M_1} - r_\chi^{M_2}) \left[ (X_A^i)^2 - 2X_A^i + \frac{\pi^2}{3} \right], \quad (5)$$

$$A_3^f \simeq 6\pi\alpha_s (r_\chi^{M_1} + r_\chi^{M_2}) [2(X_A^f)^2 - X_A^f], \quad (6)$$

where the superscripts “ $i$ ” and “ $f$ ” refer to gluon emission from the initial- and final-states, respectively; the subscripts “1”, “2” and “3” correspond to three possible Dirac structures, with “1” for  $(V-A) \otimes (V-A)$ , “2” for  $(V-A) \otimes (V+A)$ , and “3” for  $(S-P) \otimes (S+P)$ , respectively;  $A_3^i$  is negligible for light final pseudoscalars due to  $r_\chi^{M_1} \simeq r_\chi^{M_2}$ . The explicit expressions of effective annihilation coefficients could be found in Refs. [7,8].

For the annihilation parameters  $X_A$  in Eqs. (4)–(6), although there are no imperative and a priori reasons for it to be the same in the building blocks  $A_k^{i,f}$  ( $k = 1, 2, 3$ ), the simplification  $X_A^i = X_A^f = X_A$  is commonly used in many previous works of nonleptonic  $B$  decays [7–11], independent of mesons involved and topologies. However, the carefully renewed study in Refs. [5,6] shows that it is hardly to accommodate all available observables of charmless  $B \rightarrow PP$  decays simultaneously with the universal  $\rho_A$  and  $\phi_A$ . Recently, a refreshing suggestion was proposed in Refs. [4,5] to cope with the parameters  $X_A$ . The main points of “new treatment” could be briefly summarized as follow:

- (i) As the superscripts of  $A_k^{i,f}$  correspond to different topologies, parameters of  $X_A^i$  and  $X_A^f$  should be treated individually.
- (ii) For the factorizable annihilation topologies, the information of initial state has been included in the decay constant of  $B$  meson and taken outside from the building blocks of  $A_k^f$ . Only the wave functions of final states are involved in the convolution integral of subamplitudes. Additionally, the same asymptotic light cone distribution amplitude is commonly applied to the final pseudoscalar and vector mesons. So, the parameter  $X_A^f$  should be universal for factorizable annihilation amplitudes of both  $B_s$  and  $B_{u,d}$  nonleptonic decays.
- (iii) For the nonfactorizable annihilation topologies, the initial  $B$  meson entangles with the final states via gluon exchange. The wave functions of all participating hadrons, including the initial  $B$  meson, are involved in the convolution integral of subamplitudes. Hence, the parameter  $X_A^i$  might be different from the parameter  $X_A^f$  generally. Moreover, due to the mass relationship  $m_u \simeq m_d \neq m_s$  resulting in the  $SU(3)$  flavor symmetry breaking, it is usually assumed that the momentum fraction of

the valence  $s$  quark in  $B_s$  meson should be larger than that of the spectator  $u, d$  quark in  $B_{u,d}$  meson. The flavor symmetry breaking effects might be embodied in parameter  $X_A^i$ , i.e. two parameters,  $X_{A,d}^i$  and  $X_{A,s}^i$ , should be introduced for nonfactorizable annihilation topologies of  $B_{u,d}$  and  $B_s$  meson decay, respectively, while the isospin symmetry holds approximately. Generally, it is not required that  $X_{A,d}^i$  must be equal or unequal to  $X_{A,s}^i$ , i.e.,  $X_{A,d}^i$  and  $X_{A,s}^i$  are independent variables.

With this assumption, authors of Refs. [4,5] reanalyzed  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays without considering theoretical uncertainties and found that the experimental data on  $\mathcal{B}(B_s \rightarrow \pi^+\pi^-)$  in Eq. (2) could be explained with large  $\rho_{A,s}^i \sim 3$ . Compared with  $\rho_A \sim 1$  in [8,9] for  $\mathcal{B}(B_d \rightarrow K^+K^-)$ , it seems to imply unexpectedly large flavor symmetry breaking effects, then the predictive power of QCDF will be rather limited. Thanks to the large experimental errors,  $\mathcal{B}(B_d \rightarrow K^+K^-)$  can be fitted within a large range of  $(\rho_{A,d}^i, \phi_{A,d}^i)$  including  $\rho_{A,d}^i \sim 3$  [5,13]. Therefore, flavor symmetry might be restored as both aforementioned decays could be accommodated by a common set of  $(\rho_A^i, \phi_A^i)$ . It is interesting and essential to systematically evaluate the exact strength of annihilation contribution and further test the aforementioned points, especially the flavor asymmetry effects.

As it is well known, additional phenomenological parameters  $X_H$  (or  $\rho_H$  and  $\phi_H$ ), like in Eq. (3), were introduced to regulate the endpoint singularity in the hard spectator scattering (HSS) corrections involving the twist-3 light cone distribution amplitudes of light final states [7–11]. The phenomenological importance of HSS corrections to the color-suppressed tree contributions which are enhanced by the large Wilson coefficient  $C_1$  has already been recognized by Refs. [9,12,13] in explicating the current experimental measurements on  $\Delta A_{CP} = A_{CP}(B^+ \rightarrow K^+\pi^0) - A_{CP}(B^0 \rightarrow K^+\pi^-)$  and  $R_{CP}^{\pi\pi} = 2\mathcal{B}(B^0 \rightarrow \pi^0\pi^0)/\mathcal{B}(B^0 \rightarrow \pi^+\pi^-)$ . Because the  $B$  wave functions are also involved in the HSS convolution integral, the flavor symmetry breaking effects might be also embodied in parameter  $X_H$ .

Following the ansatz in Refs. [4,5], we preform a global fit on the annihilation parameters combining available experimental data on  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays with a statistical  $\chi^2$  analysis. Based on our previous analysis [13], the approximation,  $(\rho_{H,d}, \phi_{H,d}) = (\rho_{A,d}^i, \phi_{A,d}^i)$ , is acceptable by current measurements on  $B_{u,d}$  decays (see scenario III in Ref. [13] for details), which lessens effectively the unknown variables. Hence, the approximation  $X_H = X_A^i$  is assumed for  $B_{u,d,s}$  decays in the following analysis. The detailed explanation on the fitting approach could be found in Appendix C of Ref. [13]. The values of input parameters used in our evaluations are summarized in Table 1.

Firstly, to clarify the flavor symmetry breaking effects on parameters  $X_A^{i,f}$ , we perform a fit on the annihilation parameters  $(\rho_{A,d}^i, \phi_{A,d}^i)$  for  $B_{u,d}$  and  $B_s$  decays, respectively. For parameters of  $(\rho_{A,s}^i, \phi_{A,s}^i)$ , the constraints come from observables of the  $\bar{B}_s \rightarrow \pi^-K^+, \pi^+\pi^-, K^+K^-$  decays. The fitted results are shown in Fig. 1. For parameters of  $(\rho_{A,d}^i, \phi_{A,d}^i)$ , they have been fitted with the constraints from  $B_{u,d} \rightarrow \pi K, \pi\pi, KK$  decays, especially, focusing on the so-called “ $\pi K$ ” and “ $\pi\pi$ ” puzzles (see Ref. [13] for details). Their allowed regions (green points) at 68% C.L. are also shown in Fig. 1 for a comparison with  $(\rho_{A,s}^i, \phi_{A,s}^i)$ .

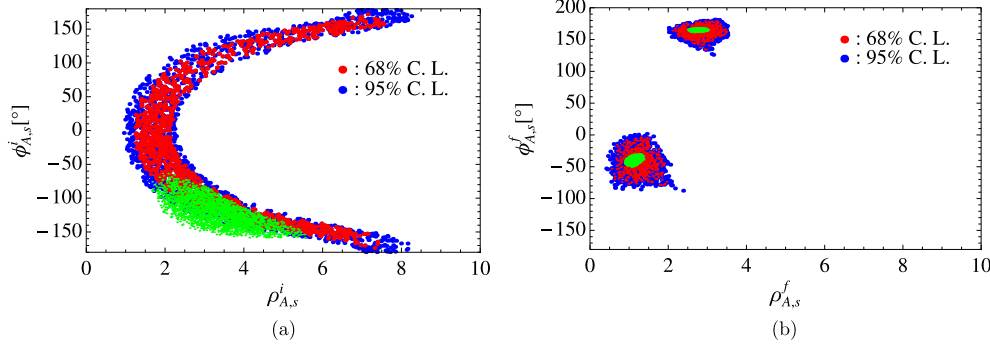
From Fig. 1(a), it is seen clearly that (1) the region of  $(\rho_{A,s}^i, \phi_{A,s}^i)$  cannot be seriously constrained by now, because the current measurements on  $B_s \rightarrow \pi^-K^+, \pi^+\pi^-, K^+K^-$  decays are not accurate enough and the theoretical uncertainties are also still large. Moreover, a relatively large  $\rho_{A,s}^i \sim 3$  with  $\phi_{A,s}^i \sim \pm 100^\circ$  in  $B_s$  system suggested by recent studies [4–6] is allowed. (2) The conven-

<sup>1</sup> The second uncertainty comes from parameters  $\rho_{A,H}$  and  $\phi_{A,H}$ .

**Table 1**

The values of input parameters: CKM matrix elements, pole and running quark masses, decay constants, form factors and Gegenbauer moments.

$\bar{\rho} = 0.1489^{+0.0158}_{-0.0084}$ ,	$\bar{\eta} = 0.342^{+0.013}_{-0.011}$ ,	$A = 0.813^{+0.015}_{-0.027}$ ,	$\lambda = 0.22551^{+0.00068}_{-0.00035}$	[14]
$m_c = 1.67 \pm 0.07$ GeV,	$m_b = 4.78 \pm 0.06$ GeV,	$m_t = 173.21 \pm 0.87$ GeV,		
$\frac{\bar{m}_s(\mu)}{\bar{m}_{u,d}(\mu)} = 27.5 \pm 1.0$ ,	$\bar{m}_s(2 \text{ GeV}) = 95 \pm 5$ MeV,	$\bar{m}_b(\bar{m}_b) = 4.18 \pm 0.03$ GeV	[15]	
$f_{B_d} = (190.6 \pm 4.7)$ MeV,	$f_{B_s} = (227.6 \pm 5.0)$ MeV,	[16]		
$f_\pi = (130.41 \pm 0.20)$ MeV,	$f_K = (156.2 \pm 0.7)$ MeV	[15]		
$F_0^{B \rightarrow \pi}(0) = 0.258 \pm 0.031$ ,	$F_0^{B \rightarrow K}(0) = 0.331 \pm 0.041$ ,	$F_0^{B_s \rightarrow K}(0) = 0.23 \pm 0.06$ ,	[17]	
$a_1^\pi = 0$ ,	$a_2^\pi(2 \text{ GeV}) = 0.17$ ,	$a_1^K(2 \text{ GeV}) = 0.05$ ,	$a_2^K(2 \text{ GeV}) = 0.17$	[18]



**Fig. 1.** The allowed regions of annihilation parameters at 68% C.L. and 95% C.L. for  $B_s$  decays are shown by red and blue points in the planes  $(\rho_{A,s}^{i,f}, \phi_{A,s}^{i,f})$ , respectively. The green pointed regions are the fitted results of  $(\rho_{A,d}^{i,f}, \phi_{A,d}^{i,f})$  at 68% C.L. for  $B_{u,d}$  decays. See text for detail explanation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

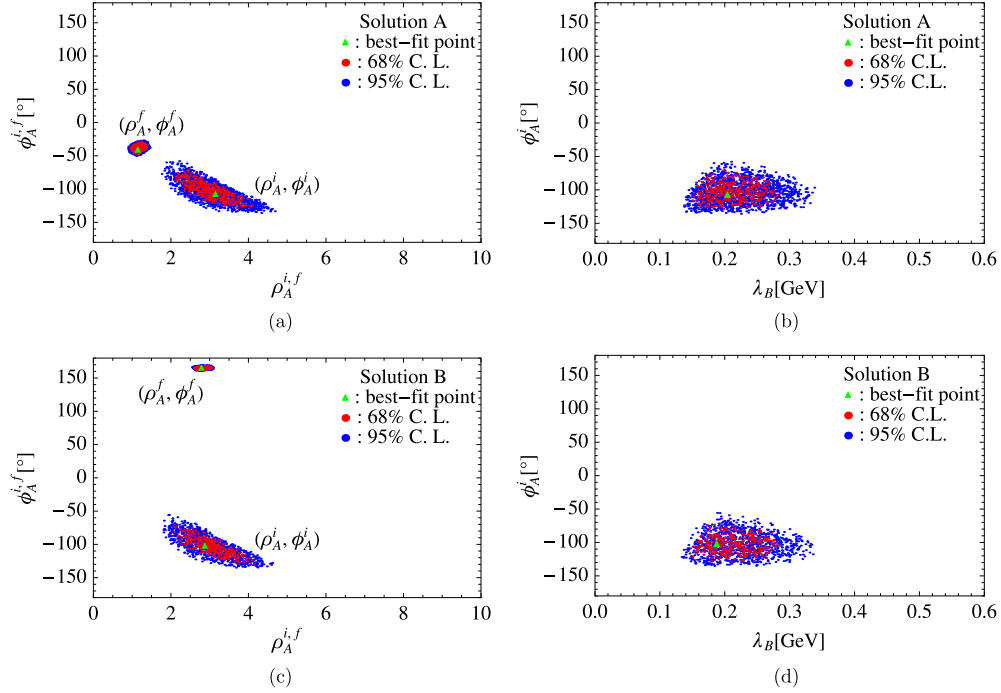
tional choice of  $\rho_{A,d}^i \sim 1$  and  $\phi_{A,d}^i \sim -55^\circ$  [8–10] is ruled out, because the assumption  $X_H = X_A^i$  is used in our study to enhance the magnitude and the strong phase of the color-suppressed tree amplitude  $C$  via spectator interactions and to solve both “ $\pi K$ ” and “ $\pi\pi$ ” puzzles [13]. Besides, a relatively large  $\rho_{A,d}^i \sim 3$  with  $\phi_{A,d}^i \sim 100^\circ$  is allowed by  $\mathcal{B}(B_d \rightarrow K^+ K^-)$  which has large experimental error and theoretical uncertainties until now, and is also consistent with Fig. 7(a) of Ref. [19] for  $B_d \rightarrow K^+ K^-$  decays using the similar statistical fit approach with parameters  $X_A^i = X_A^f$ . (3) The allowed regions of  $(\rho_{A,d}^i, \phi_{A,d}^i)$  can still overlap with the ones of  $(\rho_{A,s}^i, \phi_{A,s}^i)$  in part, around  $(3, -100^\circ)$ , which implies that the treatment  $X_{A,s}^i \neq X_{A,d}^i$  from flavor symmetry breaking effects in Refs. [4,5] is not absolutely sure, at least not necessary with current experimental and theoretical precision.

From Fig. 1(b), it is seen clearly that (1) there are two allowed solutions for parameters of both  $(\rho_{A,d}^f, \phi_{A,d}^f)$  and  $(\rho_{A,s}^f, \phi_{A,s}^f)$ . Besides the commonly used value  $\rho_A^f \sim 1$  [7–11], there is another best-fit value  $\rho_A^f \sim 2.5$ . (2) It is interesting that the allowed regions for  $(\rho_{A,d}^f, \phi_{A,d}^f)$  overlap entirely with those for  $(\rho_{A,s}^f, \phi_{A,s}^f)$ , which confirms the suggestion [4] that  $X_A^f$  (or  $\rho_A^f, \phi_A^f$ ) is universal for  $B_{u,d}$  and  $B_s$  system.

Moreover, comparing Fig. 1(a) with (b), it is seen that (1) generally, the allowed region of  $(\rho_A^f, \phi_A^f)$  is different from that of  $(\rho_A^i, \phi_A^i)$ , and  $X_A^i$  is not always required to be equal to  $X_A^f$ . So, the “new treatment” on parameters  $X_A$  according to either factorizable or nonfactorizable annihilation topologies may be reasonable and appropriate for  $B_{u,d,s}$  decays. (2) The flavor symmetry breaking effects on parameters  $X_A^{i,f}$  could be very small even negligible under the existing circumstances with less available experimental constraints from  $B_s$  decays.

Based on the above analyses and discussions, we present the most simplified (flavor conserving) scenario for the annihilation parameters that both  $(\rho_A^f, \phi_A^f)$  and  $(\rho_A^i, \phi_A^i)$  are universal for both  $B_{u,d} \rightarrow PP$  and  $B_s \rightarrow PP$  decay modes to lessen phenomenological parameters, where  $X^i$  and  $X^f$  are independent variables. To get their exact values, we perform a global fit by combining available experimental data for  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays, which involve 16 decay modes and 42 observables. In our fit, besides of  $(\rho_A^{i,f}, \phi_A^{i,f})$ , the inverse moment  $\lambda_B$ , which is used to parameterize integral of the  $B$  meson distribution amplitude and a hot topic by now (see Ref. [20] for details), is also treated as a free parameter and taken into account. We present the allowed parameter spaces in Fig. 2 and the corresponding numerical results in Table 2.

As Fig. 2 shows, the allowed spaces of  $(\rho_A^{i,f}, \phi_A^{i,f})$  and  $\lambda_B$  are strongly restricted by combined constraints from  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays, especially for  $(\rho_A^f, \phi_A^f)$ . There are two solutions (named solution A and B, respectively). It is easily found that the allowed regions and the best-fit point of  $(\rho_A^i, \phi_A^i)$  are so alike that one can hardly distinguish one from these two solutions. For each solution, there is no common overlap at 68% C.L. between the allowed regions of  $(\rho_A^i, \phi_A^i)$  and  $(\rho_A^f, \phi_A^f)$ , i.e., the nonfactorizable and factorizable annihilation parameters  $X_A^i$  and  $X_A^f$  should be treated as independent parameters, which confirms the suggestion of Refs. [4,5]. Numerically, as listed in Table 2, the fitted result is similar to, but with smaller uncertainties, the results in Ref. [13] where the  $B_s$  decay modes are not considered. In fact, the two sets of parameters values give the same annihilation contributions. From Table 2, it can be seen that a relatively small value of  $\lambda_B \sim 0.2$  GeV which has been found by, for instance, Refs. [8,10,13,20] and a relatively large value of  $\rho_H \sim 3$  with  $\phi_H \sim -105^\circ$  are favored in the phenomenological aspect of  $B$  nonleptonic decays. They will enable the HSS corrections to play an important role in evaluating observables of penguin dominated



**Fig. 2.** The fitted results of annihilation parameters  $\rho_A^{i,f}$ ,  $\phi_A^{i,f}$  and  $B$  wave function parameter  $\lambda_B$  at 68% C.L. and 95% C.L. The best-fit points of solutions A and B correspond to  $\chi_{\min}^2 = 4.70$  and  $\chi_{\min}^2 = 4.77$ , respectively.

**Table 2**

The best-fit values of annihilation parameters and wave function parameter  $\lambda_B$ .

	$\rho_{A,H}^i$	$\phi_{A,H}^i [^\circ]$	$\rho_A^f$	$\phi_A^f [^\circ]$	$\lambda_B$ [GeV]
Solution A	$2.98_{-0.86}^{+1.12}$	$-105_{-24}^{+34}$	$1.18_{-0.23}^{+0.20}$	$-40_{-8}^{+11}$	$0.19_{-0.04}^{+0.09}$
Solution B	$2.97_{-0.90}^{+1.19}$	$-105_{-24}^{+32}$	$2.80_{-0.21}^{+0.25}$	$165_{-3}^{+4}$	$0.19_{-0.04}^{+0.10}$

$B \rightarrow \pi K$  decays, and have significant enhancement, assisted with the large Wilson coefficient  $C_1$ , to the color-suppressed tree amplitude with a large strong phase. As noticed and discussed in Refs. [7,8,21], the vertex corrections, including NLO and NNLO contributions, to the color suppressed tree coefficient  $\alpha_2$  exhibit a serious cancellation of the real part of  $\alpha_2$  (for example, see the first line of Eq. (54) in Ref. [21]), but the HSS mechanism can compensate for the destructive interference and enhance the  $\alpha_2$  with a large magnitude. The value of  $\alpha_2(\pi\pi) \simeq 0.24 - i0.08 \simeq 0.25 e^{-i18^\circ}$  including NNLO vertex and HSS corrections [21] obtained with  $\rho_H = 0$  and  $\lambda_B \sim 0.35$  GeV still cannot accommodate the experimental data on branching ratio  $B_d \rightarrow \pi^0 \pi^0$  decay. So a relatively large HSS corrections arising from  $X_H$  might be a crucial key for the “ $\pi\pi$  puzzle”. The branching rate of  $B_d \rightarrow \pi^0 \pi^0$  decay and the  $CP$  asymmetry of  $B_u \rightarrow \pi^0 K^\pm$  decay, they both are sensitive to the choice of coefficient  $\alpha_2$ , and can provide substantial constraints on parameter  $X_H$ . With the best-fit values of both  $(\rho_H, \phi_H)$  and  $\lambda_B$  in this analysis, one can get  $\alpha_2(\pi\pi) \simeq 0.28 - i0.49 \simeq 0.56 e^{-i60^\circ}$ , which provides a possible solution to the so-called “ $\pi\pi$  and  $\pi K$ ” puzzles simultaneously. Of course, one can have different mechanism for enhancement of the  $\alpha_2$  in QCDF, for example, the final-state rescattering effect<sup>2</sup> advocated [9] and the Principle of Maximum Conformality [22] proposed recently,

<sup>2</sup> Considering the final state interaction effects, the coefficients  $\alpha_2(\pi\pi) \simeq 0.6 e^{-i55^\circ} \simeq 0.34 - i0.49$  and  $\alpha_2(\pi K) \simeq 0.51 e^{-i58^\circ} \simeq 0.27 - i0.43$  [9]. Notice that (1) the above coefficient  $\alpha_2(\pi\pi)$  has similar magnitude module to ours, and the

**Table 3**

The  $CP$ -averaged branching ratios (in the unit of  $10^{-6}$ ) of  $B_s \rightarrow \pi\pi, \pi K, KK$  decays, where the first and second theoretical errors are caused by uncertainties of the CKM and the other parameters (including the quark masses, decay constants and form factors) listed in Table 1, respectively.

Decay mode	Exp. data	This work	Cheng [10]
$\bar{B}_s \rightarrow \pi^- K^+$	$5.4 \pm 0.6$	$5.5_{-0.4-2.5}^{+0.4+3.4}$	$5.3_{-0.8-0.5}^{+0.4+0.4}$
$\bar{B}_s \rightarrow \pi^0 K^0$	–	$1.83_{-0.16-0.20}^{+0.15+0.23}$	$1.7_{-0.8-0.5}^{+2.5+1.2}$
$\bar{B}_s \rightarrow \pi^+ \pi^-$	$0.73 \pm 0.14$	$0.61_{-0.04-0.06}^{+0.02+0.07}$	$0.26_{-0.00-0.09}^{+0.00+0.10}$
$\bar{B}_s \rightarrow \pi^0 \pi^0$	–	$0.31_{-0.02-0.03}^{+0.01+0.03}$	$0.13_{-0.0-0.05}^{+0.0+0.05}$
$\bar{B}_s \rightarrow K^+ K^-$	$24.5 \pm 1.8$	$20.1_{-1.32-5.1}^{+0.78+6.1}$	$25.2_{-7.2-9.1}^{+12.7+12.5}$
$\bar{B}_s \rightarrow K^0 \bar{K}^0$	$< 66$	$21.2_{-1.4-5.7}^{+0.8+6.8}$	$26.1_{-8.1-9.4}^{+13.5+12.9}$

where the allowed regions for parameters  $\rho_{A,H}$  and  $\phi_{A,H}$  might be different.

With the inputs in Table 1 and the best-fit values of parameters listed in Table 2, we present our theoretical results for observables of  $B_s \rightarrow \pi\pi, \pi K, KK$  decays in the third column of Tables 3, 4 and 5. The results [10] with the traditional treatment  $X_A^i = X_A^f$  including flavor symmetry breaking effects are also listed in the last column for comparison. The results for  $B_{u,d} \rightarrow \pi\pi, \pi K, KK$  decays are not listed here, because they are similar to those given in Ref. [13]. From these results, it could be found that (1) all QCDF results of  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays could be accommodated to the experimental data within errors. (2) Our results of branching ratios for  $B_s \rightarrow \pi\pi$  decays are twice as large as those with the traditional treatment [10]. And  $\mathcal{B}(B_s \rightarrow \pi^+ \pi^-) = (0.61_{-0.04-0.06}^{+0.02+0.07}) \times 10^{-6}$  is in good agreement with the data within

large module of  $\alpha_2(\pi\pi)$  is helpful to accommodate the “ $\pi\pi$ ” puzzle. (2) The coefficient  $\alpha_2(\pi K)$  has similar magnitude imaginary to ours, and the large imaginary part of  $\alpha_2(\pi K)$  results in a large strong phase difference to solve the “ $\pi K$ ” puzzle.



**Table 4**

The direct  $CP$  asymmetries (in the unit of  $10^{-2}$ ). The explanation for uncertainties is the same as in Table 3.

Decay mode	Exp. data	This work	Cheng [10]
$\bar{B}_s \rightarrow \pi^- K^+$	$26 \pm 4$	$31^{+1+14}_{-1-8}$	$20.7^{+5.0+3.9}_{-3.0-8.8}$
$\bar{B}_s \rightarrow \pi^0 K^0$	–	$51^{+1+8}_{-2-9}$	$36.3^{+17.4+26.6}_{-18.2-24.3}$
$\bar{B}_s \rightarrow \pi^+ \pi^-$	–	$0^{+0+0}_{-0-0}$	0
$\bar{B}_s \rightarrow \pi^0 \pi^0$	–	$0^{+0+0}_{-0-0}$	0
$\bar{B}_s \rightarrow K^+ K^-$	$-14 \pm 11$	$-11.6^{+0.4+0.4}_{-0.4-0.4}$	$-7.7^{+1.6+4.0}_{-1.2-5.1}$
$\bar{B}_s \rightarrow K^0 \bar{K}^0$	–	$0.54^{+0.02+0.11}_{-0.02-0.13}$	$0.40^{+0.04+0.10}_{-0.04-0.04}$

**Table 5**

The mixing-induced  $CP$  asymmetries (in the unit of  $10^{-2}$ ). The explanation for uncertainties is the same as in Table 3.

Decay mode	Exp. data	This work	Cheng [10]
$\bar{B}_s \rightarrow \pi^0 K^0$	–	$-10.0^{+4.5+7.0}_{-8.4-7.4}$	$8^{+29+23}_{-27-26}$
$\bar{B}_s \rightarrow \pi^+ \pi^-$	–	$16.4^{+0.6+0.0}_{-0.5-0.0}$	$15^{+0+0}_{-0-0}$
$\bar{B}_s \rightarrow \pi^0 \pi^0$	–	$16.4^{+0.6+0.0}_{-0.5-0.0}$	$15^{+0+0}_{-0-0}$
$\bar{B}_s \rightarrow K^+ K^-$	$30 \pm 13$	$18.0^{+0.7+4.3}_{-0.6-5.5}$	$22^{+4+5}_{-5-3}$
$\bar{B}_s \rightarrow K^0 \bar{K}^0$	–	$0.50^{+0.02+0.01}_{-0.02-0.02}$	$0.4^{+0+0.2}_{-0-0.2}$

one experimental error. Meanwhile, our result  $\mathcal{B}(B_s \rightarrow \pi^0 \pi^0) = (0.31^{+0.01+0.03}_{-0.02-0.03}) \times 10^{-6}$  is twice as large as the traditional result  $(0.13^{+0.0+0.05}_{-0.0-0.05}) \times 10^{-6}$ . Moreover, there are also some other differences between the two sets of theoretical results more or less. So, the future accurate measurements on the nonleptonic  $B_s$  meson decays would be helpful to probe the annihilation contributions and to explore the underlying dynamical mechanism.

In summary, we studied the nonfactorizable and factorizable annihilation contributions to  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays with QCDF approach. To clarify the independence of annihilation parameters  $X_A^i$  and  $X_A^f$  and the possible flavor symmetry breaking effects therein, a statistical  $\chi^2$  analysis is performed for nonleptonic  $B_{u,d}$  and  $B_s$  decays. It is found that (1)  $X_A^i$  and  $X_A^f$  are independent parameters, which differs from the traditional treatment with annihilation parameters and verifies the proposal of Ref. [4]. (2) The flavor symmetry breaking effects might be small for nonleptonic  $B_{u,d}$  and  $B_s$  decays by now due to the large experimental errors and theoretical uncertainties. With the simplifications  $X_{A,s}^i = X_{A,d}^i$  and  $X_{A,s}^f = X_{A,d}^f$ , a comprehensive global fit on the annihilation parameters and the  $B$  wave function parameter  $\lambda_B$  is done based on the current available measurements on  $B_{u,d,s} \rightarrow \pi\pi, \pi K, KK$  decays. Two allowed solutions are found. With the best-fit param-

eters summarized in Table 2, the QCDF results for  $B \rightarrow \pi\pi, \pi K, KK$  decays are consistent with the present experimental data within errors. It is expected that the measuremental precision of non-leptonic  $B$  decays could be much improved by LHCb and super-B experiments in the following years, so more information about annihilation contributions could be revealed.

## Acknowledgements

The work is supported by the National Natural Science Foundation of China (Grant Nos. 11105043, 11147008, 11275057, 11475055 and U1232101). Q. Chang is also supported by the Foundation for the Author of National Excellent Doctoral Dissertation of the People's Republic of China (Grant No. 201317) and the Program for Science and Technology Innovation Talents in Universities of Henan Province (Grant No. 14HASTIT036). Thanks Referees for their helpful comments and Xin-Qiang Li for helpful discussions.

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