Communications in Theoretical Physics

PAPER

The $\Xi^*\bar{K}$ and $\Omega\eta$ Interaction Within a Chiral Unitary Approach

To cite this article: Si-Qi Xu *et al* 2016 *Commun. Theor. Phys.* **65** 53

Commun. Theor. Phys. **65** (2016) 53–56 Vol. 65, No. 1, January 1, 2016

The $\Xi^*\bar{K}$ and $\Omega\eta$ Interaction Within a Chiral Unitary Approach*

Si-Qi Xu (徐思琦),[1,2] Ju-Jun Xie (谢聚军),[2,3,†] Xu-Rong Chen (陈旭荣),[2] and Duo-Jie Jia (贾多杰)[1]

[1]College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, China

[2]Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

[3]State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

(Received November 2, 2015; revised manuscript received November 30, 2015)

Abstract *In this work we study the interaction of the coupled channels $\Omega\eta$ and $\Xi^*\bar{K}$ within the chiral unitary approach. The systems under consideration have total isospins 0, strangeness $S = -3$, and spin 3/2. We study the s wave interaction which implies that the possible resonances generated in the system can have spin-parity $J^P = 3/2^-$. The unitary amplitudes in coupled channels develop poles that can be associated with some known baryonic resonances. We find there is a dynamically generated $3/2^-$ Ω state with mass around 1800 MeV, which is in agreement with the predictions of the five-quark model.*

PACS numbers: 13.60.Le, 12.39.Mk, 13.25.Jx
Key words: chiral unitary approach, Ω excited states

1 Introduction

Exploration of exotic hadrons that have more than three valence quarks is an important issue in hadron physics. However, there is no exotic hadron has been established so far, in contrast to the existence of hundreds of ordinary hadrons. Very recently, the new observation of the heavy hidden charm baryonic P_c^+ states[1] has challenged the conventional wisdom that baryons are composed of three quarks from the naive quark model. This observation has attracted a lot of attention from the theoretical side. Various explanations of these states have been proposed, such as molecules, muti-quark states, kinematic effects, or mixtures of components of different nature. Nevertheless, up to now none of them has been accepted unanimously.

In the case of light flavor baryons, only the nucleon and $\Delta(1232)$ excited states have been abundantly studied both on the theoretical and experimental sides, while for the cases of hyperon excited states, the information of them is very scarce. For example, there are only four Ω hyperon states compiled in the "Review of Particle Physics" by the Particle Data Group (PDG),[2] namely the ground state $\Omega(1672)$, and three excitations $\Omega(2250)$, $\Omega(2380)$, and $\Omega(2470)$. Among these four states, only the ground state $\Omega(1672)$ has been clarified to have spin-parity $J^P = 3/2^+$, while the quantum numbers for the other three have not been justified yet.

Although there has not been any further experimental evidence about the Ω excited states sine the 1990s, theorists are always interested in the spectrum of the Ω hyperon, which has been investigated within the traditional three quark models in Refs. [3–6], the large N_c expansion analysis in Refs. [7–11], the algebraic model in Ref. [12], and the Skyrme model in Ref. [13]. Within these models, the predicted mass of the lowest $3/2^-$ Ω excited states is around 2000 MeV, which is always higher than the mass of the lowest $1/2^-$ Ω excited states. In Ref. [14], the $\Xi\bar{K}$ interaction was investigated within an extended chiral $SU(3)$ quark model by solving a resonating group method equation. It was shown that the s wave $I = 0$ $\Xi\bar{K}$ interaction is attractive, and a $1/2^-$ $\Xi\bar{K}$ bound state with 3 MeV binding energy was predicted. Furthermore, in Ref. [15], the Ω excited states in the $\Omega\omega$ system with $J^P = 5/2^-$, $3/2^-$, and $1/2^-$ are dynamically studied in both the chiral SU(3) quark model and the extended chiral SU(3) quark model. The calculated results of that reference show that the $\Omega\omega$ state has an attractive interaction, and in the extended chiral SU(3) quark model such attraction can make for a $\Omega\omega$ quasi-bound state with spin-parity $J^P = 3/2^-$ or $5/2^-$ and the binding energy of about several MeV.

On the other hand, the spectrum of low-lying Ω states with negative parity has been investigated by employing an extended constituent quark model,[16–18] within which the Ω states were cosidered as admixtures of three- and five-quark components. It is shown that the mixing between three- and five-quark components in Ω resonances

*Supported by the National Basic Research Program (973 Program Grant No. 2014CB845406), and the National Natural Science Foundation of China under Grant Nos. 11475227, 11265014, and the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China No. Y5KF151CJ1

†E-mail: xiejujun@impcas.ac.cn

with spin-parity $J^P = 3/2^-$ is very strong, and the mixing decreases the energy of the lowest $3/2^-$ state to be around 1785 ± 25 MeV, which is lower than that of the lowest $1/2^-$ state.[18] Accordingly, five-quark components may be more preferable in the wave function of those Ω excited states.

The $\Xi^* \bar{K}$ and $\Omega\eta$ systems under consideration have total isospin $I = 0$ and spin $J = 3/2$. If we considered only the s wave interaction of Ξ^* and $\bar{K}$ or Ω and η, then the possible resonances generated in the coupled channels of $\Xi^* \bar{K}$ and $\Omega\eta$ can have only $J^P = 3/2^-$. Within the chiral unitary approach, the s wave interaction of the baryon decuplet with the octet of pseudoscalar mesons were studied in Ref. [19]. It was found that in the case of strangeness $S = -3$ and isospin $I = 0$, there is a pole at $(2141, -\mathrm{i}38)$ MeV, which can be identified, by only the mass, with the $\Omega(2250)$ resonance‡ compiled in the PDG. However, as discussed before, until now, the experimental data for the Ω resonances is very poor. No Ω excited states with negative parity have been observed yet. Further studies about the Ω resonances are welcome.

In Ref. [19], it was claimed that the pole position shifts with the value of the subtraction constant. Along this line, in the present work, we re-study the s wave interaction of the coupled channels $\Xi^* \bar{K}$ and $\Omega\eta$ within the chiral unitary approach. By adjusting the value of the subtraction constant, we obtain a pole around $(1800, \mathrm{i}0)$ MeV, which supports the findings in Refs. [16–18]. This is very interesting that the energy of the lowest $3/2^-$ Ω state is lower than the energy of the lowest $1/2^-$ Ω state.

This paper is organized as follows. In next section, we discuss the formalism and the main ingredients of the model. In Sec. 3, we present our main results and, finally, a short summary is given in Sec. 4.

2 Theoretical Framework

We begin with a brief discussion of the formalism of the chiral unitary approach by reviewing the general procedure for calculating the meson-baryon scattering amplitudes since more details can be obtained from Ref. [19]. In the chiral unitary approach, from solving the Bethe–Salpeter equation, the scattering matrix in coupled channels is given by[19]

$$T = [1 - VG]^{-1} V \,, \tag{1}$$

where V is the matrix for the transition potential between the included channels and G, a diagonal matrix, is the loop function for intermediate $\Xi^* \bar{K}$ and $\Omega\eta$ states, which is defined as

$$G = \mathrm{i} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{q^2 - m^2 + \mathrm{i}\epsilon} \frac{2M}{(P-q)^2 - M^2 + \mathrm{i}\epsilon} \,, \tag{2}$$

where m and M are the masses of the $\bar{K}$ or η meson and the Ξ^* or Ω baryon. In the above equation, P is the total incident momentum of the external meson-baryon system.

We study only the s wave interaction, for which, the transition potential for channel i to j reads,[19]

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0) \,, \tag{3}$$

with $f = 93$ MeV the pion decay constant. The k^0 and k'^0 are the energy of the incoming and outgoing meson, respectively. The transition coefficients C_{ij} are symmetric with respect to the indices, and also isospin-dependent. By naming the channels, 1 for $\Xi^* \bar{K}$ and 2 for $\Omega\eta$, the coefficients C_{ij} for the case of isospin $I = 0$ are[19]

$$C_{11} = 0 \,, \; C_{12} = C_{21} = 3 \,, \; C_{22} = 0 \,. \tag{4}$$

From $C_{12} = C_{21} = 3$ and Eq. (3), we find an attractive interaction between $\Xi^* \bar{K}$ and $\Omega\eta$ channels, for which we can expect bound states or resonances in the coupled channels of $\Xi^* \bar{K}$ and $\Omega\eta$.

The loop function G can be regularized either with a cutoff prescription or with dimensional regularization in terms of a subtraction constant. Here we make use of the dimensional regularization scheme. The expression for G is then[19–20]

$$\begin{aligned} G_l = \frac{2M_l}{16\pi^2} \Big\{ & a_l(\mu) + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln\frac{m_l^2}{M_l^2} \\ & + \frac{q_l}{\sqrt{s}} [\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\ & + \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\ & - \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\ & - \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s})] \Big\} \,, \end{aligned} \tag{5}$$

with $\mu = 700$ MeV the scale of the dimensional regularization as used in Ref. [19]. Changes in the scale are reabsorbed in the subtraction constant $a(\mu)$ through $a(\mu') - a(\mu) = \ln(\mu'^2/\mu^2)$ so that the amplitude T is scale independent. In Eq. (5), q_l denotes the three-momentum of meson or baryon in the center of mass frame, which is given by:

$$q_l = \frac{\lambda^{1/2}(s, m_l^2, M_l^2)}{2\sqrt{s}} \,, \tag{6}$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ being the triangular function and m_l and M_l are the masses of the mesons and baryons, respectively. In addition, we take $M_{\Xi^*} = 1533.4$ MeV, $m_{\bar{K}} = 495.6$ MeV, $M_\Omega = 1672.5$ MeV, and $m_\eta = 547.9$ MeV.

The dynamically generated baryon states appear as poles of the scattering amplitudes on the complex energy $\sqrt{s}$ plane. The poles that are found on the second Riemann sheet are identified with resonances. The mass and

‡The other quantum numbers, such as the spin and parity, of this state are unknown.

the width of the state can be found from the position of the pole on the complex energy plane. In the second Riemann sheet, the loop function G in Eq. (5) should be changed, when $\mathrm{Re}(\sqrt{s})$ is above the $\Xi^*\bar{K}$ (2029 MeV) or $\Omega\eta$ (2220 MeV) mass threshold, with

$$G_l^{\rm II} = G_l + 2{\rm i}\frac{q_l}{\sqrt{s}}\frac{M_l}{4\pi}\,, \text{ with } \mathrm{Im}(q_l) > 0\,. \tag{7}$$

We have only two coupled channels, $\Xi^*\bar{K}$ and $\Omega\eta$, and the transition potentials $V_{11} = V_{22} = 0$, then, we search for the pole by looking for zero of the determinant of $|1 - VG|$

$$\det|1 - VG| = 1 - V_{12}^2 G_{11}G_{22} = 0\,, \tag{8}$$

where G_{11} and G_{22} are the G functions for $\Xi^*\bar{K}$ and $\Omega\eta$ channels, respectively. In addition, the scattering amplitudes of $T_{\Xi^*\bar{K}\to\Xi^*\bar{K}}$, and $T_{\Omega\eta\to\Omega\eta}$ are obtained as,

$$T_{\Xi^*\bar{K}\to\Xi^*\bar{K}} = \frac{V_{12}^2 G_{22}}{1 - V_{12}^2 G_{11}G_{22}}\,, \tag{9}$$

$$T_{\Omega\eta\to\Omega\eta} = \frac{V_{12}^2 G_{11}}{1 - V_{12}^2 G_{11}G_{22}}\,. \tag{10}$$

Next, we determine the couplings of the resonance to different channels, $\Xi^*\bar{K}$ and $\Omega\eta$ in the present case. Close to the pole at z_R, the scattering amplitude behaves as

$$T_{ij} = \frac{g_i g_j}{\sqrt{s} - z_R}\,, \tag{11}$$

where g_i is the coupling of the state to the i-channel. We then evaluate the residues of T_{ij} to get the complex valued couplings g_i.

3 Numerical Results

To evaluate the value of the scattering amplitudes T we have to fix the subtraction constants $a_l(\mu)$. Generally, $a(\mu)$ for different channels is different, they should be determined by fitting relevant experimental data. In the present work, we choose the value of $a(\mu)$[§] to get the Ω state at 1785 MeV as corresponding to the estimated mass of $3/2^-$ Ω resonance in Ref. [18].

With $a(\mu) = -3.4$, we obtain a pole of the T matrix at $z_R = (1785.7, -{\rm i}0)$ MeV in the complex plane. The corresponding results of $|T|^2$ as a function of $\sqrt{s}$ for $\Omega\eta \to \Omega\eta$ transition is shown in Fig. 1, where there is a clear peak around 1800 MeV, which can be identified with the $3/2^-$ Ω state that was predicted in Ref. [18]. Since the mass of this state is lower than the mass threshold of $\Xi^*\bar{K}$ and $\Omega\eta$, it is a bound state of $\Xi^*\bar{K}$ and $\Omega\eta$. Besides, because we do not include other lower mass threshold decay channels, the obtained total decay width of the Ω^* state is zero. If we take $a(\mu) = -2.0$, we can also obtain a pole at $z_R = (2142.6, -{\rm i}38.4)$ MeV as in Ref. [19].

[§]We take the same values for $\Xi^*\bar{K}$ and $\Omega\eta$ channels.

In Fig. 2, we show the real and imaginary parts of the loop function G as a function of the total scattering energy. The solid and dashed lines stand the results for the case of the $\Xi^*\bar{K}$, while the red-solid and red-dashed lines represent the case of the $\Omega\eta$. The results are obtained with $a(\mu) = -3.4$.

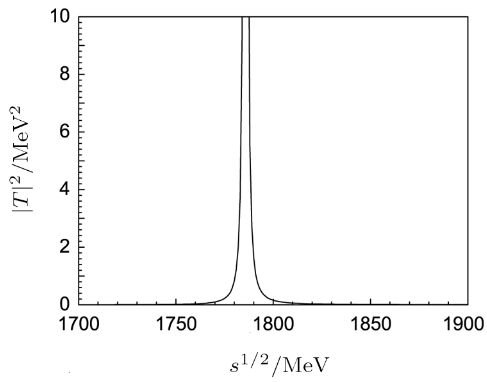

Fig. 1 Modulus squared $|T|^2$ of the $\Omega\eta \to \Omega\eta$ transition.

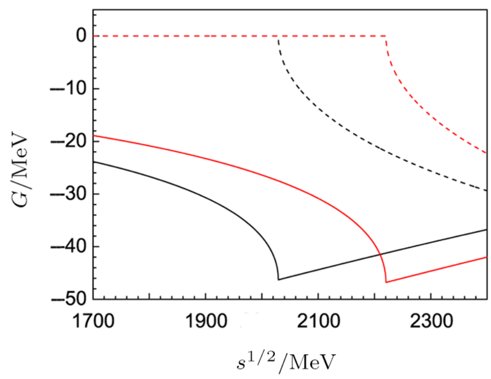

Fig. 2 (Color online) Real (solid lines) and imaginary (dashed lines) parts of the loop function G as a function of the total scattering energy for the cases of $\Xi^*\bar{K}$ (black lines) and $\Omega\eta$ (red lines).

The couplings, of the $3/2^-$ Ω state to $\Xi^*\bar{K}$ (g_{11}) and $\Omega\eta$ (g_{22}) channels are also evaluated around the pole ($z_R = (1785.7, -{\rm i}0)$ MeV) using Eq. (11), which gives,

$$g_{11} = 2.04\,,\ g_{22} = 2.31\,. \tag{12}$$

One can see that the $3/2^-$ $\Omega(1800)$ state has similar couplings to $\Xi^*\bar{K}$ and $\Omega\eta$ channels.

On the other hand, the true free parameter in the model is the $a(\mu)$ subtraction constant in the loop functions, all other parameters are meson masses or meson decay constants which are, in principle, fixed by experiment. In view of this we shall vary the value of $a(\mu)$ in the calculation to study the effect of $a(\mu)$ over the pole position of the $3/2^-$ Ω state. In Fig. 3 one can see the effect of varying $a(\mu)$ over the pole position of the $3/2^-$ Ω state. The pole position moves from 1699 MeV to 2217 MeV with the parameter $a(\mu)$ in the range of $-4.0 \le a(\mu) \le -1.5$. It is not convenient for us to compare our results to the experimental data, because the data are very poor.[2] While comparing to predictions, $M_{\Omega^*} = 1785 \pm 25$ MeV, of Ref. [18], we can get $a(\mu) = -3.4 \pm 0.1$. This value is in line but a bit far from the natural size value of -2 that was used in Ref. [19], where a global fit to the baryonic resonances from baryon decuplet and meson octet interaction was conducted.

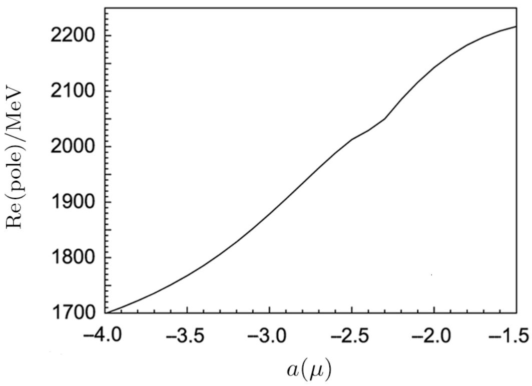

Fig. 3 Results of varying $a(\mu)$ over the $3/2^-$ Ω resonance mass.

The results obtained here partly support the findings in Refs. [16–18] that the meson-baryon components (or five-quark configuration) is more preferable in the $3/2^-$ Ω excited states. This could be tested by future experiments, as pointed in Ref. [18], the BESIII experiment.

4 Summary

In this work, within the chiral unitary approach, we have chosen the $\Xi^* \bar{K}$ and $\Omega\eta$ systems as coupled channels to investigate the dynamical generation of baryon excited states. The systems under consideration have total isospins 0, strangeness $S = -3$, and spin 3/2. We studied the s wave interaction, which implies that the possible resonances generated in the system can have spin-parity $J^P = 3/2^-$. The formalism consists of solving Bethe–Salpeter equations. In the isospin $I = 0$ sector, by adjusting the subtraction constant $a(\mu) = -3.4$, we find a bound Ω excited state with mass around 1800 MeV. This state can be identified with the predicted Ω resonance with mass $M = 1785 \pm 25$ MeV in Ref. [18]. It is shown that the mass of this lowest $3/2^-$ Ω state is lower than the mass of the lowest $1/2^-$ Ω state. Furthermore, there should be more five-quark components in the wave function of the $3/2^-$ Ω state.

Finally, we would like to address that the value of $a(\mu) = -3.4$ is a bit far from the natural size value of -2, which was used in Ref. [19]. However, the experimental data of the Ω^* states is so poor, hence, the value of $a(\mu)$ is still open. It is expected that the future experiments about the Ω^* states can provide more constraints on the value of $a(\mu)$.

Acknowledgments

We would like to thank Xu Cao for the useful discussions and suggestions.

References

[1] R. Aaij, *et al.*, [LHCb Collaboration], Phys. Rev. Lett. **115** (2015) 072001.
[2] K.A. Olive, *et al.*, [Particle Data Group Collaboration], Chin. Phys. C **38** (2014) 090001.
[3] K.T. Chao, N. Isgur, and G. Karl, Phys. Rev. D **23** (1981) 155.
[4] S. Capstick and N. Isgur, Phys. Rev. D **34** (1986) 2809.
[5] L.Y. Glozman and D.O. Riska, Phys. Rept. **268** (1996) 263.
[6] U. Loring, B.C. Metsch, and H.R. Petry, Eur. Phys. J. A **10** (2001) 447.
[7] C.E. Carlson and C.D. Carone, Phys. Lett. B **484** (2000) 260.
[8] C.L. Schat, J.L. Goity, and N.N. Scoccola, Phys. Rev. Lett. **88** (2002) 102002.
[9] J.L. Goity, C. Schat, and N.N. Scoccola, Phys. Lett. B **564** (2003) 83.
[10] N. Matagne and F. Stancu, Phys. Rev. D **71** (2005) 014010.
[11] N. Matagne and F. Stancu, Phys. Rev. D **74** (2006) 034014.
[12] R. Bijker, F. Iachello, and A. Leviatan, Ann. Phys. **284** (2000) 89.
[13] Y. Oh, Phys. Rev. D **75** (2007) 074002.
[14] W.L. Wang, F. Huang, Z.Y. Zhang, and F. Liu, J. Phys. G **35** (2008) 085003.
[15] W.L. Wang, F. Huang, Z.Y. Zhang, Y.W. Yu, and F. Liu, Commun. Theor. Phys. **48** (2007) 695.
[16] S.G. Yuan, C.S. An, K.W. Wei, B.S. Zou, and H.S. Xu, Phys. Rev. C **87** (2013) 025205.
[17] C.S. An, B.C. Metsch, and B.S. Zou, Phys. Rev. C **87** (2013) 065207.
[18] C.S. An and B.S. Zou, Phys. Rev. C **89** (2014) 055209.
[19] S. Sarkar, E. Oset, and M.J. Vicente Vacas, Nucl. Phys. A **750** (2005) 294; [Nucl. Phys. A **780** (2006)] 78.
[20] A. Ramos, E. Oset, and C. Bennhold, Phys. Rev. Lett. **89** (2002) 252001.