Suppressing explosive synchronization by contrarians

2016 EPL 113 28005

(http://iopscience.iop.org/0295-5075/113/2/28005)

A Letters Journal Exploring the Frontiers of Physics

January 2016

EPL, **113** (2016) 28005 www.epljournal.org
doi: 10.1209/0295-5075/113/28005

Suppressing explosive synchronization by contrarians

Xiyun Zhang[1], Shuguang Guan[1,2], Yong Zou[1,2], Xiaosong Chen[2] and Zonghua Liu[1,2(a)]

[1] *Department of Physics, East China Normal University - Shanghai, 200062, China*
[2] *State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences Beijing 100190, China*

received 10 November 2015; accepted in final form 29 January 2016
published online 16 February 2016

PACS 89.75.-k – Complex systems
PACS 05.45.Xt – Synchronization; coupled oscillators

Abstract – Explosive synchronization (ES) has recently received increasing attention and studies have mainly focused on the conditions of its onset so far. However, its inverse problem, *i.e.* the suppression of ES, has not been systematically studied so far. As ES is usually considered to be harmful in certain circumstances such as the cascading failure of power grids and epileptic seizure, etc., its suppression is definitely important and deserves to be studied. We here study this inverse problem by presenting an efficient approach to suppress ES from a first-order to second-order transition, without changing the intrinsic network structure. We find that ES can be suppressed by only changing a small fraction of oscillators into contrarians with negative couplings and the critical fraction for the transition from the first order to the second order increases with both the network size and the average degree. A brief theory is presented to explain the underlying mechanism. This finding underlines the importance of our method to improve the understanding of neural interactions underlying cognitive processes.

Synchronization is one of the key problems in nonlinear science and has been studied for a long time. It was previously focused on the case of two coupled chaotic oscillators and then moved to the case of coupled lattices [1]. Since the concept of complex networks was introduced in 1999, the main topic of synchronization has been shifted to the influence of the structure of complex networks [2–4]. One common feature for all these cases is that the routes to synchronization are continuous, *i.e.*, the size of synchronized clusters grows gradually, presenting a second-order transition. However, there are exceptions of the first-order transition such as the cascading failure in power grids and financial crisis, etc. [5,6], whose underlying mechanism has remained unclear for a long time. A remarkable progress on this challenging problem is the finding of explosive synchronization (ES) in 2011 in the networked Kuramoto oscillators [7], where the transition from a non-synchronized state to a synchronized state is abrupt/discontinuous, namely, the global synchronization appears explosively. In that paper, two conditions were thought to be necessary for the onset of ES: i) a scale-free (SF) network topology and ii) the existence of a positive correlation between the natural frequency of an oscillator and its degree. After that, great attention has been paid to this problem and is mainly focused on the conditions of its onset [6,8–17].

The phenomenon of the first-order transition was in fact revealed before 2011 [18–20] but received attention only after the work of ref. [7] in 2011, because of the new feature of an extremely fast cascading process named by ES. Then, ES was immediately confirmed by an experiment in a circuit of star graph of coupled Rössler oscillators [8]. After that, it was found that ES can be also observed in a generic network (either SF or non-SF) in a modified Kuramoto model, provided that a positive correlation between the natural frequencies of oscillators and their coupling strengths is preserved [13–16] or there is a frequency disorder [21]. To illustrate the underlying mechanism of these two different ways of ES in refs. [7] and [13], ref. [15] shows that they can be unified into a common root of suppressing the formation of giant clusters, called *suppressive rule of ES*, *i.e.* the small synchronization clusters are prevented from merging gradually into larger clusters and thus cannot induce a second-order transition. According to this suppressive rule, it is also possible to have other ways to ES, provided that they can prevent the gradual growing and merging of small clusters. Fortunately, such a scenario is reported recently in multilayer networks where

(a) E-mail: zhliu@phy.ecnu.edu.cn

the suppressive rule is implemented by adaptively adjusting the behaviors of individual oscillators by using their local order parameter [22].

In sum, all these works focus on finding the conditions for the onset of ES. Thus, an open question is how to suppress ES, *i.e.* the inverse problem of the onset of ES, which will make the phase transition be degraded from the first order to the second order. This problem does deserve to be studied as many first-order transitions are harmful to human beings such as the cascading failure of power grids [23] and epileptic seizures [24], etc. To make the first-order transition change into the second-order transition, an intuitive idea is to make the suppressive rule break. Reference [15] showed a way to break the suppressive rule by randomly exchanging the frequencies of two nodes i and j. In reality, however, the natural frequencies of oscillators are usually not changeable. Thus, an interesting question would be whether it is possible to suppress ES but keeping their original characteristic features unchanged such as the natural frequencies and the network topology, etc.

We here study this problem. We find that ES can be effectively suppressed by introducing a small fraction of contrarians with negative coupling strengths, in contrast to conformists with positive coupling strengths [25–31]. There is a critical fraction for the transition from the first order to the second order and it increases with both the network size and the average degree. To understand the role of contrarians, we consider two typical ways to introduce them into the network. One is to let the contrarians be distributed randomly and homogeneously among the conformists, while the other is to let the contrarians be heterogeneously distributed in the network, *e.g.*, only distributed to a local area of the network. We find that both ways can effectively suppress ES, indicating that the contrarians take a positive role in suppressing ES.

We consider a network of N coupled Kuramoto oscillators. Each oscillator is characterized by its phase $\theta_i(t), i = 1, \cdots, N$ and obeys an equation of motion defined as

$$\dot{\theta}_i = \omega_i + \frac{\lambda|\omega_i|}{k_i}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i), \quad i = 1, \ldots, N, \quad (1)$$

where λ is the overall coupling strength, ω_i and k_i are the natural frequency and the degree of oscillator i, respectively, and A_{ij} are the elements of the adjacency matrix A, so that $A_{ij} = 1$ when nodes i and j are connected and $A_{ij} = 0$ otherwise. Equation (1) can be considered as a frequency-weighted network and reflects the feature of several natural and social systems [5,32]. It has been recently shown that eq. (1) shows ES at a critical point λ_c [13,15–17] for a general complex network with a symmetric distributed ω_i. Our aim here is to present an efficient and practical approach to suppress the ES in eq. (1).

For this purpose, our idea is to introduce a small fraction of contrarians to suppress the growth of larger clusters in eq. (1). In detail, we assume that the oscillators of eq. (1) can be either conformists or contrarians, which can be distinguished from the role of λ in the coupling. In the literatures, there are two approaches to define a conformist or a contrarian. In the first method, a contrarian oscillator will receive interactions from its neighbors by a negative coupling strength while a conformist oscillator will receive interactions from its neighbors by a positive coupling strength [25–27]. Thus, eq. (1) can be rewritten as

$$\dot{\theta}_i = \omega_i + \frac{\lambda_i|\omega_i|}{k_i}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i), \quad i = 1, \ldots, N, \quad (2)$$

where the conformists have positive λ_i while the contrarians have negative λ_i. For simplicity, we let all the coupling strength λ_i have the same amplitude $\lambda > 0$ with $\lambda_i = \lambda$ for all the conformists and $\lambda_i = -\lambda$ for all the contrarians.

In the second method, a contrarian oscillator will give negative coupling to each of its neighbors while a conformist oscillator will give a positive coupling to each of its neighbors [28–30]. Thus, eq. (1) can be rewritten as

$$\dot{\theta}_i = \omega_i + \frac{|\omega_i|}{k_i}\sum_{j=1}^{N} \lambda_j A_{ij}\sin(\theta_j - \theta_i), \quad i = 1, \ldots, N, \quad (3)$$

where the conformists contribute positive λ_j while the contrarians contribute negative λ_j. Doing the same as in the first way of defining conformists and contrarians, we let all the coupling strength λ_j have the same amplitude $\lambda > 0$ with $\lambda_j = \lambda$ for all the conformists and $\lambda_j = -\lambda$ for all the contrarians. We will consider both these two definitions in this work.

We let the fraction of contrarians be f, which represents the ratio between the number of the contrarians and the total number of oscillators. Thus, we have fN contrarians and $(1 - f)N$ conformists in the network. Then, a key point is how to distribute these contrarians among the conformists. For convenience, we here consider two typical ways. In the first one, we let the contrarians be randomly mixed with the conformists in the network, *i.e.* we randomly choose fN nodes to be contrarians and let the remaining $(1-f)N$ nodes to be conformists. The advantage of this way is that each node will have approximately the same fraction of neighboring conformists or contrarians. In the second one, we let the contrarians be distributed only at a local part of the network, resulting that the contrarians are highly heterogeneously distributed in the network. The aim of choosing these two typical ways is to see how the distribution of contrarians influences the controlling effect, which may provide a new insight into understanding how the inhibitory neurons play a role in the function of neuron networks.

To measure the coherence of the collective motion, we introduce an order parameter R [7,29],

$$Re^{i\Psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}, \quad (4)$$

where Ψ denotes the average phase, and R $(0 \leq R \leq 1)$ is a measure of phase coherence. R will reach unity when the system is fully synchronized and it will be 0 for an incoherent state. As the conformists will attract each other to form synchronized clusters and the contrarians will repel each other to be out of phase, the competition between them will form different collective behaviors. For the case of $f = 0$, it has been revealed that the value of R will show a jumping behavior at the critical point $\lambda = \lambda_c$ [13,15–17], *i.e.* the onset of ES. Then, an interesting question will be: What will happen for the case $f > 0$?

In numerical simulations, we take the random Erdős-Rényi (ER) network with size $N = 500$ and average degree $\langle k \rangle = 6$ as an example, unless with specific illustration. We let the frequency ω_i in eq. (1) be the symmetric Lorentzian distribution of $g(\omega) = \frac{1}{\pi}[\frac{\gamma}{\omega^2+\gamma^2}]$ with γ being the half-width at half-maximum [33]. We let $\gamma = 0.5$ in this paper. Then, we choose a small fraction f of network's nodes to be the contrarians and the remaining fraction $1 - f$ of nodes to be the conformists by the two ways defined by eq. (2) and eq. (3), respectively.

To measure the relationship between the order parameter R and the coupling strength λ, we increase (decrease) the coupling strength λ adiabatically with an increment (decrement) $\delta\lambda = 0.01$ from $\lambda = 0$ $(\lambda = 5)$ and compute the stationary value of R for each λ during the forward (backward) transition from the incoherent to the phase synchronized state. We first consider the case of randomly distributed contrarians and the definition of contrarian in eq. (2). We call it *the random-case1*. Figure 1(a) shows the results of how R changes with λ, where the "squares" and "circles" represent the forward and backward cases of $f = 0$, respectively, the "up triangles" and "down triangles" represent the forward and backward cases of $f = 0.04$, respectively, and the "left triangles" and "right triangles" represent the forward and backward cases of $f = 0.08$, respectively. It is easy to see that there are hysteretic loops for the cases of $f = 0$ and $f = 0.04$ but no loop for the case of $f = 0.08$, indicating that the first-order transition of synchronization has been changed into a second-order transition by a small fraction of contrarians of $f = 0.08$. Similarly, we show the corresponding case of randomly distributed contrarians and the definition of contrarian in eq. (3) in fig. 1(b) and call it *the random-case2*. Then we turn to the case in which the contrarians are highly heterogeneously distributed in the network, *i.e.* in a local part of the network. Figure 1(c) shows the case of heterogeneously distributed contrarians and the definition of contrarian in eq. (2), called *the hetero-case1*. And fig. 1(d) shows the case of heterogeneously distributed contrarians and the definition of contrarian in eq. (3), called *the hetero-case2*. Comparing the four panels of fig. 1, interestingly, we find that all of them show the second-order phase transition when $f = 0.08$, indicating the success of controlling ES by a small fraction of contrarians. This result may be explained as follows. As the nodes in the ER network are randomly connected, the links of the contrarians will go to different nodes, including both the conformists and contrarians. The difference between the case of randomly distributed contrarians and the case of highly heterogeneously distributed contrarians is that the latter has more links among the contrarians than the former. As the links among the contrarians take only a small fraction of the total links of contrarians, most of the total links of contrarians will go to the conformists, resulting in the fact that the two cases have the same function of preventing the onset of ES. We also notice from fig. 1 that the transition point λ_c will increase with the increase of f. In this way, the induced second-transition point λ_{c2} (*i.e.* corresponding to f_c) will be much larger than the original first-transition point λ_{c1}. This result is important as it not only suppresses the first-order transition but it also makes the transition point be postponed a lot, which is needed by the real systems with potential cascading risk.

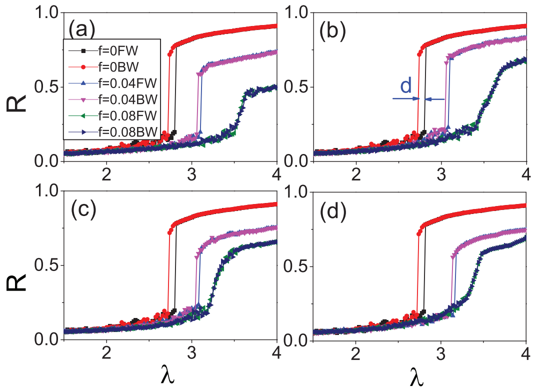

Fig. 1: (Color online) Forward and backward synchronization transitions for the Erdős-Rényi network with $N = 500$ and $\langle k \rangle = 6$, where the "squares" and "circles" represent the forward and backward cases of $f = 0$, respectively, the "up triangles" and "down triangles" represent the forward and backward cases of $f = 0.04$, respectively, and the "left triangles" and "right triangles" represent the forward and backward cases of $f = 0.08$, respectively. Panels (a)–(d) correspond to the random-case1, the random-case2, the hetero-case1, and the hetero-case2, respectively.

Let d be the width of the hysteretic loop, see fig. 1(b). To figure out the role of contrarians, we measure the dependence of d on f. Figure 2(a) shows the results for taking an average on 30 realizations, where the four curves pointed by the arrow with $N = 500$ represent the four corresponding cases in fig. 1(a)–(d), respectively. From them we see that there exists a critical value f_c in all the four cases, where d changes from zero to non-zero. All the values of f_c are smaller than 0.1, indicating that the introduced contrarians are very efficient to prevent the onset of ES.

To see the size effect, we show two more cases of $N = 1000$ and $N = 1500$ in fig. 2(a) where the other parameters are taken the same as in the cases of $N = 500$. It is easy to see that all the three groups of curves have similar behavior, *i.e.* all of them decrease with the increase

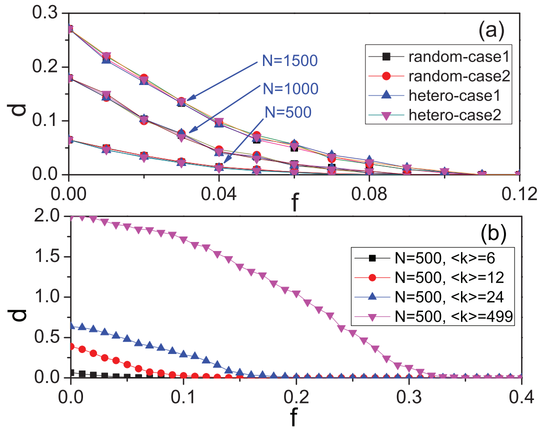

Fig. 2: (Color online) Dependence of d on f for the Erdős-Rényi network, where the results are obtained by taking an average on 30 realizations. (a) The influence of size N on f_c with fixed $\langle k \rangle = 6$ where the "squares", "circles", "up triangles" and "down triangles" represent the corresponding cases in fig. 1(a)–(d), respectively, and the three groups of curves denote the cases of $N = 500, 1000$ and 1500, respectively. (b) The influence of the average degree $\langle k \rangle$ on f_c with fixed $N = 500$ and the random-case2 where the "squares", "circles", "up triangles" and "down triangles" represent the cases of $\langle k \rangle = 6, 12, 24$ and 499, respectively.

of f and their critical values f_c are around 0.1. This common feature implies that ES can be controlled by a small fraction of contrarians, no matter the contrarian's definitions and distributions. On the other hand, from fig. 2(a) we also notice that f_c increases with N, which will be explained in the next section.

To see the influence of the average degree, we take the random-case2 as an example. We let $N = 500$ and gradually increase $\langle k \rangle$ from 6 to 12, then to 24 and finally to the fully connected case of 499. The results are shown in fig. 2(b). It is easy to see that f_c increases with $\langle k \rangle$.

To check the robustness of the controlling of ES, we have changed the network topology into a scale-free (SF) network with a power law degree distribution and find similar results as in fig. 2, indicating the robustness of our approach to the network topologies. On the other hand, it would be interesting to show how the approach works in a realistic neuron network. For this purpose, we take the neuronal network of *C. elegans* as an example. We take the data of the network of *C. elegans* from ref. [34] where each node represents a neuron and two nodes are connected if there is a synaptic connection between the corresponding nodes. The obtained network has 170 nodes and the average degree $\langle k \rangle = 32.75$. Doing the same numerical simulations as in ER and SF networks, we obtain a similar result as in fig. 2, indicating that our approach can be successfully used to the realistic neuron networks.

It is well known that there are both excitatory and inhibitory neurons in cortical neural networks. The excitatory neurons comprise the majority (80–90%) of the neuronal population and show to be largely homogeneous. The inhibitory neurons comprise only 10% of the neuron population, but show to be extremely heterogeneous and play a role in controlling and coordinating the activity of large populations of local neurons [35–37]. It is also pointed out that the non-identity of neurons is necessary for the diversity of memory [38]. In these neuron networks, the positive (negative) coupling inherently represents exciting (inhibitory) coupling. Our finding thus implies that the existence of inhibitory neurons in the brain network is necessary not only for sustaining its normal function but also for preventing its abnormal behaviors, *i.e.* ES. That is, the revealed dependence of phase transition on f has a twofold meaning in neuron networks. The first one is that the existence of contrarians can prevent the occurrence of ES such as the epileptic seizures. The second one is that different f may result in different degrees of phase transition, which is of significance in explaining how the neuron network with excitable and inhibitory neurons implements a diversity of cognitive processes.

We now present a brief theoretical analysis. For the system of eq. (1) without contrarians, ref. [15] shows that there is a suppressive rule,

$$Y_{ij} \equiv \frac{|\omega_i - \omega_j|}{|\omega_i| + |\omega_j|} \leq \lambda R, \tag{5}$$

which controls the formation of synchronized clusters. According to this rule, two oscillators i and j will be synchronized when their scaled frequency difference Y_{ij} satisfies eq. (5), *i.e.* it is smaller than λR, and unsynchronized otherwise. That is, those pairs of neighboring nodes i and j violating eq. (5) will take the role of preventing the growing up of the formed small synchronized clusters. When the coupling λ is large enough, all the small synchronization clusters will suddenly merge together and thus result in the first-order transition, *i.e.* ES.

For the systems of equations (2) and (3) with contrarians, we now derive their corresponding suppressive rule. We let r_i be the instantaneous local order parameter for the oscillator i, defined as $r_i e^{i\phi} = \frac{1}{k_i}\sum_{j=1}^{k_i}\langle e^{i\theta_j}\rangle$, where $\langle \cdots \rangle$ denotes a time average. By definition, $0 \leq r_i \leq 1$, and ϕ denotes the phase averaged over the ensemble of neighbors. The relationship between the global order parameter R and the local order parameter r_i can be represented as [39,40]

$$R = \frac{\sum_{i=1}^{N} k_i r_i}{\sum_{i=1}^{N} k_i}. \tag{6}$$

In the case of $f = 0$, there is no contrarians in the network. We let R_0 and r_{i0} be its global and local order parameters, respectively, and thus have $Y_{ij}(f=0) = \frac{|\omega_i - \omega_j|}{|\omega_i| + |\omega_j|} \leq \lambda R_0$.

We now focus on the case of $f > 0$. For the contrarians of eq. (2), eq. (4) can be rewritten as

$$Re^{i\Psi} = \frac{1}{N}\left(\sum_{j \in conformists} e^{i\theta_j} + \sum_{j \in contrarians} e^{i\theta_j}\right). \tag{7}$$

As the contrarians of eq. (2) have a negative coupling λ, their phases are generally different from those conformists with a phase difference π [31]. In this sense, we have

$$R \approx (1-2f)R_0. \tag{8}$$

For the contrarians of eq. (3), we have

$$r_i e^{i\phi} = \frac{1}{k_i}\left(\sum_{j\in conformists}\langle e^{i\theta_j}\rangle + \sum_{j\in contrarians}\langle e^{i\theta_j}\rangle\right) \tag{9}$$

which gives $r_i \approx (1-2f)r_{i0}$. Substituting it into eq. (6), we also have $R \approx (1-2f)R_0$. That is, eq. (8) works for both the systems of equations (2) and (3). As the contrarians are randomly distributed in the network, we may assume $r_i \approx (1-2f)R_0$ in the mean-field framework.

Note that $r_i \sin(\Psi-\theta_i) = \frac{1}{k_i}\sum_{j=1}^{k_i}\sin(\theta_j-\theta_i)$. Plugging it into eq. (2), one obtains

$$\begin{aligned}\dot{\theta}_i &= \omega_i + \lambda_i|\omega_i| r_i \sin(\Psi-\theta_i)\\ &\approx \omega_i + \lambda_i|\omega_i|(1-2f)R_0\sin(\Psi-\theta_i).\end{aligned} \tag{10}$$

The evolution of the phase difference $\Delta\theta_{ij} \equiv \theta_i - \theta_j$ is then given by

$$\begin{aligned}\Delta\dot{\theta}_{ij} &= \omega_i + (1-2f)R_0\lambda_i|\omega_i|\sin(\Psi-\theta_i)\\ &\quad -\omega_j - (1-2f)R_0\lambda_j|\omega_j|\sin(\Psi-\theta_j).\end{aligned} \tag{11}$$

When two conformists i and j are phase-locked, one has $\Delta\dot{\theta}_{ij}=0$, and thus

$$\frac{\omega_i-\omega_j}{(1-2f)R_0\lambda} = [|\omega_j|\sin(\Psi-\theta_j) - |\omega_i|\sin(\Psi-\theta_i)]. \tag{12}$$

The maximum value of the right-hand side of eq. (12) is $|\omega_i| + |\omega_j|$, which gives a necessary condition of phase-locking between two conformists,

$$\frac{|\omega_i-\omega_j|}{|\omega_i|+|\omega_j|} \le (1-2f)\lambda R_0. \tag{13}$$

This is the suppressive rule of the case with both conformists and contrarians, corresponding to eq. (5). Comparing eq. (13) with eq. (5) we see that the factor $(1-2f)$ will make it more difficult for the conformists to form small synchronized clusters in the case of $f>0$ than in that of $f=0$, indicating that the core of the synchronized giant cluster will not be generated from the conformists.

In the case of eq. (3), both the conformists and contrarians have the same expression and can be rewritten as

$$\begin{aligned}\dot{\theta}_i &= \omega_i + \frac{\lambda|\omega_i|}{k_i}\left[\sum_{j=1}^{N} A_{ij}\sin(\theta_j-\theta_i)\right.\\ &\qquad \left. -2\sum_{j\in contrarians} A_{ij}\sin(\theta_j-\theta_i)\right]\\ &= \omega_i + \lambda|\omega_i| r_i\sin(\Psi-\theta_i) - \sigma_i\\ &\approx \omega_i + (1-2f)R_0\lambda|\omega_i|\sin(\Psi-\theta_i) - \sigma_i,\end{aligned} \tag{14}$$

where $\sigma_i \equiv \frac{2\lambda|\omega_i|}{k_i}\sum_{j\in contrarians} A_{ij}\sin(\theta_j-\theta_i)$ represents the fluctuation from the fraction f of contrarians. The evolution of the phase difference $\Delta\theta_{ij} \equiv \theta_i-\theta_j$ is then given by

$$\begin{aligned}\Delta\dot{\theta}_{ij} &= \omega_i - \sigma_i - \omega_j + \sigma_j\\ &\quad + (1-2f)R_0\lambda[|\omega_i|\sin(\Psi-\theta_i) - |\omega_j|\sin(\Psi-\theta_j)].\end{aligned} \tag{15}$$

The necessary condition of phase-locking will be

$$\frac{|\omega_i-\omega_j+\sigma_j-\sigma_i|}{|\omega_i|+|\omega_j|} \le (1-2f)\lambda R_0. \tag{16}$$

When $\sigma_i=\sigma_j$, it will become eq. (13). In general, as both σ_i and σ_j are not very large, the fluctuation $\sigma_j-\sigma_i$ will be a small quantity. Thus, eq. (16) is approximately equivalent to eq. (13). In this sense, what we obtained from eq. (13) will also work here.

Based on the modified suppressive rule of eq. (13), we now explain the numerical results of figs. 1 and 2. Letting $\lambda' = (1-2f)\lambda$ represent the effective coupling strength, eq. (13) becomes $\frac{|\omega_i-\omega_j|}{|\omega_i|+|\omega_j|} \le \lambda' R_0$, which is exactly the suppressive rule of ref. [15]. When $f=0$, we have $\lambda'=\lambda$, indicating that λ' can be considered as the value of λ in the case of $f=0$. Thus, when λ' is located in the region of the hysteretic loop, we may expect a hysteretic loop in the case of contrarians for $\lambda = \lambda'/(1-2f) > \lambda'$. This prediction has been confirmed by fig. 1 where the value of λ needed for the case of $f=0.04$ in the hysteretic loop is larger than that for the case of $f=0$.

Notice that eq. (13) comes from the mean-field theory, which is precise only when $N\to\infty$. For a finite size N, the order parameter R generally has a fluctuation of $O(N^{-1/2})$ [41,42], indicating that a larger N will have a smaller fluctuation. In the region of the hysteretic loop, there are two attractors with one having a larger R and the other having a smaller R. Both of them have their own basins of attraction [17]. Fluctuation will induce R to jump between the two attractors, resulting in the hysteretic loop. In this sense, a larger fluctuation will make the jumping occur easily and thus the loop width d will decrease with the increase of fluctuation, which explains the increase of f_c with N in fig. 2(a). We also notice from fig. 2(b) that f_c increases with $\langle k\rangle$, which can be similarly explained. By numerical simulations we find that the fluctuation of the order parameter decreases with the increase of $\langle k\rangle$, thus resulting in a larger f_c for larger $\langle k\rangle$, as shown in fig. 2(b).

In conclusions, we have presented an approach to control ES in both artificial and realistic networks by adding a small fraction of contrarians into the network. We find that the approach is robust to both the different definitions of contrarians and the different distributions of contrarians. By a theoretical analysis we show that the contrarians can reduce the number of small synchronized clusters. The critical f_c increases with both the network size N and the average degree $\langle k \rangle$. This finding may provide new insights into the diversity of cognitive processes.

* * *

ZL thanks H. Zhou for critical comments. This work was partially supported by the NNSF of China under Grant Nos. 11135001, 11375066, 11305062 and 81471651, 973 Program under Grant No. 2013CB834100, and the Open Project Program of the State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (No. Y4KF151CJ1).

REFERENCES

[1] Pikovsky A., Rosenblum M. and Kurths J., *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge) 2001.
[2] Albert R. and Barabási A.-L., *Rev. Mod. Phys.*, **74** (2002) 47.
[3] Dorogovtsev S. N., Goltsev A. V. and Mendes J. F. F., *Rev. Mod. Phys.*, **80** (2008) 1275.
[4] Arenas A., Diaz-Guilera A., Kurths J., Moreno Y. and Zhou C., *Phys. Rep.*, **469** (2008) 93.
[5] Dorfler F. and Bullo F., *Proceedings of the American Control Conference, Baltimore* (IEEE) 2010, p. 930.
[6] Ji P., Peron T. K. DM., Menck P. J., Rodrigues F. A. and Kurths J., *Phys. Rev. Lett.*, **110** (2013) 218701.
[7] Gómez-Gardeñes J., Gómez S., Arenas A. and Moreno Y., *Phys. Rev. Lett.*, **106** (2011) 128701.
[8] Leyva I., Sevilla-Escoboza R., Buldú J. M., Sendiña-Nadal I., Gómez-Gardeñes J., Arenas A., Moreno Y., Gómez S., Jaimes-Reategui R. and Boccaletti S., *Phys. Rev. Lett.*, **108** (2012) 168702.
[9] Peron T. K. D. M. and Rodrigues F. A., *Phys. Rev. E*, **86** (2012) 056108; 016102; Coutinho B. C., Goltsev A. V., Dorogovtsev S. N. and Mendes J. F. F., *Phys. Rev. E*, **87** (2013) 032106; Liu W., Wu Y., Xiao J. and Zhan M., *EPL*, **101** (2013) 38002.
[10] Leyva I., Sendiña-Nadal I., Almendral J. A., Navas A., Olmi S. and Boccaletti S., *Phys. Rev. E*, **88** (2013) 042808.
[11] Li P., Zhang K., Xu X., Zhang J. and Small M., *Phys. Rev. E*, **87** (2013) 042803; Zhu L., Tian L. and Shi D., *Phys. Rev. E*, **88** (2013) 042921.
[12] Su G., Ruan Z. and Liu Z., *EPL*, **103** (2013) 48004.
[13] Zhang X., Hu X., Kurths J. and Liu Z., *Phys. Rev. E*, **88** (2013) 010802(R).
[14] Leyva I., Navas A., Sendiña-Nadal I., Almendral J. A., Buldú J. M., Zanin M., Papo D. and Boccaletti S., *Sci. Rep.*, **3** (2013) 1281.
[15] Zhang X., Zou Y., Boccaletti S. and Liu Z., *Sci. Rep.*, **4** (2014) 5200.
[16] Hu X., Boccaletti S., Huang W., Zhang X., Liu Z., Guan S. and Lai C., *Sci. Rep.*, **4** (2014) 7262.
[17] Zou Y., Pereira T., Small M., Liu Z. and Kurths J., *Phys. Rev. Lett.*, **112** (2014) 114102.
[18] Heath T. and Wiesenfeld K., *Ann. Phys.*, **9** (2000) 689.
[19] Pazó D., *Phys. Rev. E*, **72** (2005) 046211.
[20] Lee W. S., Ott E. and Antonsen T. M., *Phys. Rev. Lett.*, **103** (2009) 044101.
[21] Skardal P. S. and Arenas A., *Phys. Rev. E*, **89** (2014) 062811.
[22] Zhang X., Boccaletti S., Guan S. and Liu Z., *Phys. Rev. Lett.*, **114** (2015) 038701.
[23] Buldyrev S. V., Parshani R., Paul G., Stanley H. E. and Havlin S., *Nature*, **464** (2010) 1025.
[24] Adhikari B. M., Epstein C. M. and Dhamala M., *Phys. Rev. E*, **88** (2013) 030701(R).
[25] Louzada V. H. P., Araujo N. A. M., Andrade J. S. jr. and Herramnn H. J., *Sci. Rep.*, **2** (2012) 658.
[26] Hong H. and Strogatz S. H., *Phys. Rev. Lett.*, **106** (2011) 054102.
[27] Hong H. and Strogatz S. H., *Phys. Rev. E*, **84** (2011) 046202.
[28] Borgers C. and Kopell N., *Neural Comput.*, **15** (2003) 509.
[29] Restrepo J. G., Ott E. and Hunt B. R., *Chaos*, **16** (2006) 015107.
[30] Zanette D. H., *Europhys. Lett.*, **72** (2005) 190.
[31] Zhang X., Ruan Z. and Liu Z., *Chaos*, **23** (2013) 033135.
[32] Wang H. and Li X., *Phys. Rev. E*, **83** (2011) 066214.
[33] Matthews P. C., Mirollo R. E. and Strogatz S. H., *Physica D*, **52** (1991) 293.
[34] `http://www.wormatlas.org/`.
[35] Freund T. F. and Buzsaki G., *Hippocampus*, **6** (1996) 347.
[36] McBain C. J. and Fisahn A., *Nat. Rev. Neurosci.*, **2** (2001) 11.
[37] Klausberger T. and Somogyi P., *Science*, **321** (2008) 53.
[38] Xu K., Zhang X., Wang C. and Liu Z., *Sci. Rep.*, **4** (2014) 7568.
[39] Restrepo J. G., Ott E. and Hunt B. R., *Phys. Rev. E*, **71** (2005) 036151.
[40] Ichinomiya T., *Phys. Rev. E*, **70** (2004) 026116.
[41] Daido H., *J. Stat. Phys.*, **60** (1990) 753.
[42] Strogatz S. H. and Mirollo R. E., *J. Stat. Phys.*, **63** (1991) 613.