



# Triangle singularity and a possible $\phi p$ resonance in the $\Lambda_c^+ \rightarrow \pi^0 \phi p$ decay



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## ABSTRACT

We study the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  decay by considering a triangle singularity mechanism. In this mechanism, the  $\Lambda_c^+$  decays into the  $K^* \Sigma^*(1385)$ , the  $\Sigma^*(1385)$  decays into the  $\pi^0 \Sigma$  (or  $\Lambda$ ), and then the  $K^* \Sigma$  (or  $\Lambda$ ) interact to produce the  $\phi p$  in the final state. This mechanism produces a peak structure around 2020 MeV. In addition, the possibility that there is a hidden-strange pentaquark-like state is also considered by taking into account the final state interactions of  $K^* \Lambda$ ,  $K^* \Sigma$ , and  $\phi p$ . We conclude that it is difficult to search for the hidden-strange analogue of the  $P_c$  states in this decay. However, we do expect nontrivial behavior in the  $\phi p$  invariant mass distribution. The predictions can be tested by experiments such as BESIII, LHCb and Belle-II.

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## 1. Introduction

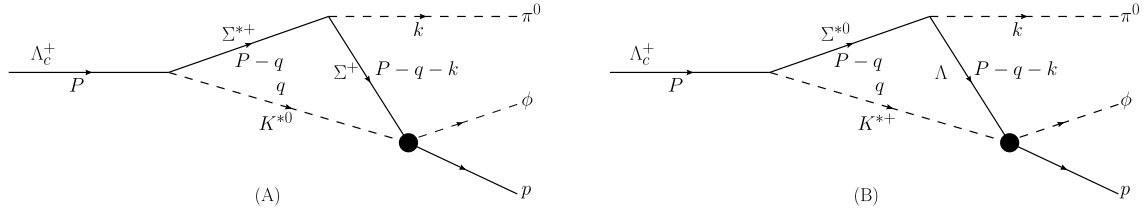
In 2015, two hidden-charm pentaquark-like structures,  $P_c(4380)$  and  $P_c(4450)$ , were observed in the  $J/\psi p$  invariant mass spectrum via the  $\Lambda_b^0 \rightarrow K^- J/\psi p$  decay by the LHCb Collaboration [1]. After they were observed, the two  $J/\psi p$  resonances were investigated within multiple theoretical schemes with the aim to explain their nature (for more details and references, see the recent reviews [2,3]). The existence of pentaquarks with hidden charm in that mass region was already predicted in Refs. [4–8] by studying the interactions of anticharm mesons and charm baryons using different models. Furthermore, it was pointed out in Ref. [9] (see also Refs. [10,11]) that a triangle singularity is located very close to the  $\chi_{c1} p$  threshold,  $\simeq 4.45$  GeV, and thus at the  $P_c(4450)$  mass. Such a singularity could produce a narrow peak mimicking the behavior of a narrow resonance, which requires the  $\chi_{c1}$  and proton to rescatter in an  $S$  wave into the final state  $J/\psi p$  [12]. This would require quantum numbers  $J^P = 1/2^+$  or  $3/2^+$  for the peak. Notice that although the  $3/2^-$  and  $5/2^+$  were reported as the most preferred quantum numbers in the original LHCb publication [1],  $3/2^+$  remains one of the favored possibilities in a later experimental analysis using an extended model [13]. Clearly, further investigations on the  $P_c(4380)$  and  $P_c(4450)$  structures, in particular from more processes and more experiments, are needed. Since the  $P_c$

structures were observed in the decay mode  $J/\psi p$ , it is natural to expect that these states, were they hadronic resonances, can be produced in photo-production process  $\gamma p \rightarrow P_c^+ \rightarrow J/\psi p$  where they will appear as  $s$ -channel resonances [14–17].

Analogous to the hidden-charm pentaquark states, one may consider the possible existence of hidden-strange pentaquarks  $P_s$ , in which the  $c\bar{c}$  pair is replaced by  $s\bar{s}$ . In fact, in the light flavor sector below 2 GeV, understanding the nature of the  $N^*(1535)$  resonance with spin parity  $J^P = 1/2^-$  is very challenging [18,19]. One peculiar property of the  $N^*(1535)$  is that it couples strongly to the channels with strangeness, such as the  $\eta N$  and  $K\Lambda$ , which is difficult to understand in the classical three-constituent-quark models. This finds an explanation within the chiral unitary approach in the work of Ref. [20]. The strange decay properties of the  $N^*(1535)$  resonance can also be easily understood by considering large five-quark components in it [21–24]. Within this pentaquark picture, the  $N^*(1535)$  resonance could be the lowest  $L = 1$  orbitally excited  $uud$  state with a large admixture of  $[ud][us]\bar{s}$  pentaquark component. This makes the  $N^*(1535)$  heavier than the  $N^*(1440)$  and also gives a natural explanation of its large couplings to the channels with strangeness [25]. In a very recent quark model study [26], a  $J^P = 1/2^-$  state with a mass varying from 1873 to 1881 MeV is obtained, and its main component is  $\eta N$ . This state could correspond to the resonance  $N^*(1895)$  which has only an overall two-star status according to the review by the Particle Data Group (PDG) [27]. Its existence is supported by the analysis of the new  $\eta$  photo-production data [28,29], which finds

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**Fig. 1.** Triangle diagrams for the  $\Lambda_c^+ \rightarrow \pi^0 p \phi$  decay. (A):  $\Sigma^+$ -exchange. (B):  $\Lambda$ -exchange. The definitions of the kinematical variables ( $P, q, k$ ) are also shown.

that the  $N^*(1895)$  with  $J^P = 1/2^-$  is crucial in order to describe the cusp observed in the  $\eta$  photo-production at around 1896 MeV as well as the fast near-threshold rise of the total cross section of the  $\gamma p \rightarrow \eta' p$  reaction [28]. In Refs. [28,29], it was also pointed out that the  $N^*(1895)$  has strong couplings to both the  $\eta N$  and  $\eta' N$  channels.

At around 2 GeV, a  $\phi N$  bound state is predicted in several models [30,31,26]. Such a  $\phi N$  state can be viewed as a  $P_s$  pentaquark. In Ref. [26], a  $J^P = 3/2^-$  state dominated by the  $\phi N$  component is obtained with a mass varying from 1949 to 1957 MeV. Independently, a  $J^P = 3/2^-$   $N^*$  resonance<sup>1</sup> with a mass about 2.1 GeV is proposed to explain the experimental results [33–43] on the associated strangeness production reactions  $\gamma p \rightarrow p\phi$ ,  $\gamma p \rightarrow K^+ \Lambda(1520)$ ,  $\gamma p \rightarrow K^* \Lambda$  and  $\gamma d \rightarrow d\phi$ . The forward-direction enhancement at around  $W = 2.1$  GeV in the  $\gamma p \rightarrow p\phi$  reaction can be also reproduced by including a special correlated five-quark configuration of a color-antitriplet ( $su$ ) diquark and a color-triplet [ $\bar{s}(ud)$ ], which subsequently hadronize into the  $\phi$  and proton [44, 45]. However, it is pointed that such a five-quark configuration is not literally a resonant pentaquark state [44,45]. In Ref. [46], it is proposed that the  $J^P = 3/2^-$  states  $N^*(1875)$  and  $N^*(2100)$  in the  $\phi$  photo-production are hadronic molecular states from the  $\Sigma^* K$  and  $\Sigma K^*$  interactions, respectively, and they can be regarded as the hidden-strange partners of the LHCb pentaquarks.

The  $P_c$  structures were produced in the process  $\Lambda_b^0 \rightarrow K^- J/\psi p$ . Analogously, one may study the possible  $P_s$  states in the singly Cabibbo suppressed process  $\Lambda_c^+ \rightarrow \pi^0 \phi p$ . As pointed out in Ref. [45], the  $\Lambda_c^+ \rightarrow \pi^0 P_s^+ \rightarrow \pi^0 \phi p$  and  $\Lambda_b^0 \rightarrow K^- P_c^+ \rightarrow K^- J/\psi p$  are entirely comparable if one substitutes  $V_{cb} V_{cs}^* \rightarrow V_{cs}^* V_{us}$ . One important difference between the two processes is that the former has a much smaller phase space—the  $\Lambda_c^+$  is above the  $\pi^0 \phi p$  three-body threshold only by 193 MeV. Such a small phase space for the  $\Lambda_c^+$  decay restricts that only the neutral pion is possible in the final state, for a hadronic decay, if we want to produce in addition a  $p\phi$  pair. Because of the small phase space and the weak  $\pi^0 \phi$  interaction,<sup>2</sup> no other resonances except for the possible  $P_s$  contribute to the process. The first experimental measurement of the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  process has been reported by the Belle Collaboration [47] very recently. No significant signal was observed with an upper limit on the branching fraction of  $\mathcal{B}(\Lambda_c^+ \rightarrow \pi^0 p \phi) < 15.3 \times 10^{-5}$  at a 90% confidence level [47].

In this paper, we will show that the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  also receives a contribution from triangle singularities close to the physical region. A triangle singularity appears on the physical boundary in a particular situation when all the intermediate states are on shell, and

all the particles move along the same direction (parallel or anti-parallel) such that the interactions at all three vertices can happen as classical processes [48]. Such a physical picture can be easily seen following the analysis of Ref. [12]. In addition to the works related to the  $P_c$  structures mentioned above, the role played by triangle singularities has been broadly investigated recently in the literature [49–65]. Along this line, we will calculate the triangle singularity contribution to the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  decay, where the  $\Lambda_c^+$  decays into  $K^* \Sigma^*(1385)$ , the  $\Sigma^*(1385)$  ( $\equiv \Sigma^*$ ) decays to the  $\pi^0 \Sigma$  (or  $\Lambda$ ) and the  $K^* \Sigma$  (or  $\Lambda$ ) rescatter into  $\phi p$  in the final state, see Fig. 1. In addition to the effects of the triangle mechanism, we consider also the final state interaction (FSI) of  $K^* \Lambda \rightarrow \phi p$  and  $K^* \Sigma \rightarrow \phi p$ . Were there a  $P_s$  resonance, it must couple to both the  $\phi p$  and  $K^* \Sigma/\Lambda$  and thus may be manifest in the Dalitz plot or in the  $\phi p$  invariant mass distribution. Yet, because of the small phase space and depending on the mass and width of such a  $P_s$  state, it could be difficult to search for it. As will be shown in this paper, on one hand the triangle singularity contribution can enhance the production of such a resonance, on the other hand it makes the identification of the  $P_s$  signal more difficult if its mass is around 2.02 GeV.

This paper is organized as follows. In Sec. 2, we discuss the triangle diagrams and how a  $P_s$  is included in our model. The numerical results are presented in Sec. 3, and finally a short summary is given in Sec. 4.

## 2. Formalism

The decay  $\Lambda_c^+ \rightarrow \pi^0 p \phi$  can proceed through the triangle diagrams depicted in Fig. 1.<sup>3</sup> Given the masses of the initial state  $\Lambda_c$ , the neutral pion in the final state and two of the intermediate states, for example the  $K^*$  and  $\Sigma/\Lambda$ , the region for the  $\Sigma^*$  mass in order to produce a triangle singularity at the physical boundary, i.e., in the physical region<sup>4</sup> can be worked out [9,55]. Using the central values for all of the mentioned hadron masses, the region can be obtained as [1386.6, 1390.1] MeV for diagram (A), while the measured mass of the  $\Sigma^{*+}$ ,  $(1382.80 \pm 0.35)$  MeV, is 4 MeV below. The region is [1384.8, 1394.3] MeV for the  $\Lambda$ -exchange in diagram (B), while the measured mass of the  $\Sigma^{*0}$   $(1383.7 \pm 1.0)$  MeV almost reaches the lower bound. In this case, the triangle singularities still have sizable influence on the physical decay amplitude. The lower bound of that region means that the triangle singularity in the  $\phi p$  invariant mass is located exactly at the two-body threshold of the two particles which rescatter into the  $\phi p$ . Thus, one expects that the triangle singularity induced effects would be mainly because of the  $\Lambda$ -exchange diagram which can produce an enhancement around the  $K^{*+} \Lambda$  threshold at

<sup>1</sup> In the editions of the PDG review before 2012, all the evidence for a  $J^P = 3/2^-$  state with a mass above 1800 MeV was filed under a two-star  $N^*(2080)$ . There is now evidence [32] of two states in this region, and the PDG has associate the older data (according to masses) to two states: a three-star  $N^*(1875)$  and a two-star  $N^*(2120)$  [27].

<sup>2</sup> The  $\pi^0 \phi$  interaction should be very weak for two reasons. Firstly, the pion and the  $\phi$  meson do not have the same quark flavors, which leads to an Okubo-Zweig-lizuka (OZI) suppression. Secondly, the small phase space means that the pion is soft, and the interaction between a soft pion and matter fields is weak because of the spontaneous breaking of chiral symmetry in quantum chromodynamics.

<sup>3</sup> Replacing the  $\Lambda$  by the  $\Sigma^0$  leads to vanishing contribution because the  $\Sigma^{*0}$  cannot couple to the  $\Sigma^0 \pi^0$ .

<sup>4</sup> The triangle singularity of course cannot be exactly in the physical region since otherwise one would get a logarithmically divergent amplitude. It is shifted into the complex plane because of the finite decay width of at least one of the intermediate states. The amplitude in the physical region is well defined without any divergence.

2007 MeV. In the following, we give the details of the calculation which shows explicitly the enhancement around that energy.

The decay of  $\Lambda_c^+ \rightarrow (\Sigma^* K^*)^+$  can proceed by  $W$ -exchange diagram [66–68]:  $(cd)u \rightarrow (du)u$ , and the  $duu$  are hadronized, together with a  $s\bar{s}$  pair with the vacuum quantum numbers, into the  $\Sigma^* K^*$ .

The evaluation of the diagram in Fig. 1 requires us first to provide an expression for the  $\Lambda_c^+ \rightarrow (\Sigma^* K^*)^+$  vertex. Because the  $\Sigma^* K^*$  threshold ( $\simeq 2277$  MeV) is very close to the mass of  $\Lambda_c^+$ , we consider only the  $S$ -wave coupling. Then we can write

$$t_{\Lambda_c^+ \rightarrow \Sigma^* K^*} = f_I g_{\Lambda_c \Sigma^* K^*} \bar{u}^\mu (P - q) u(P) \varepsilon_\mu(q), \quad (1)$$

where  $f_I$  is the isospin factor with  $f_I = \sqrt{2/3}$  for the  $\Sigma^{*+} K^{*0}$  and  $-\sqrt{1/3}$  for the  $\Sigma^{*0} K^{*+}$ , and  $g_{\Lambda_c \Sigma^* K^*}$  is an effective coupling constant which can be obtained, in general, from the branching ratio of  $\Lambda_c^+ \rightarrow \Sigma^* K^*$ .

The decays of  $\Sigma^* \rightarrow \pi \Sigma$  and  $\pi \Lambda$  are in  $P$  waves, then we can easily write with  $SU(3)$  symmetry

$$t_{\Sigma^{*+} \rightarrow \pi^0 \Sigma^+} = \frac{g}{m_\pi} \bar{u}(P - q - k) u^\mu (P - q) k_\mu, \quad (2)$$

$$t_{\Sigma^{*0} \rightarrow \pi^0 \Lambda} = \sqrt{3} \frac{g}{m_\pi} \bar{u}(P - q - k) u^\mu (P - q) k_\mu, \quad (3)$$

with  $g = 0.69$  obtained from the total decay width  $\Gamma_{\Sigma^*} = 37.13$  MeV and the branching fraction  $\text{Br}[\Sigma^* \rightarrow \pi \Sigma] = 0.117$ .

After the production of the  $K^{*0} \Sigma^+$  and  $K^{*+} \Lambda$ , they rescatter into the  $\phi p$  in the final state, as shown in Fig. 1. The total decay amplitude for the processes shown in Fig. 1 can be written as

$$t = \frac{g_{\Lambda_c \Sigma^* K^*} g}{m_\pi} \bar{\epsilon}_\phi \cdot \vec{k} \sum_{i=\Sigma, \Lambda} C_i \int \frac{d^4 q}{(2\pi)^4} \times \frac{i 2 m_{\Sigma^*}}{(P - q)^2 - m_{\Sigma^*}^2 + i m_{\Sigma^*} \Gamma_{\Sigma^*}} \frac{i}{q^2 - m_{K^*}^2 + i m_{K^*} \Gamma_{K^*}} \times \frac{i 2 m_i}{(P - q - k)^2 - m_i^2 + i \epsilon}, \quad (4)$$

where we have defined  $C_\Sigma = \frac{\sqrt{6}}{3} t_{K^{*0} \Sigma^+ \rightarrow \phi p}$  and  $C_\Lambda = -t_{K^{*+} \Lambda \rightarrow \phi p}$ , and  $t_{K^{*0} \Sigma^+ \rightarrow \phi p}$  and  $t_{K^{*+} \Lambda \rightarrow \phi p}$  are  $T$ -matrix elements for the rescattering processes, which will be discussed in the next section. We notice that the  $K^* \Sigma^*$  mass threshold is close to the mass of  $\Lambda_c^+$  and the range of the  $\phi p$  invariant mass for the decay of interest, [1957.7, 2141.5] MeV, allows us to make nonrelativistic approximation for all the involved baryons and vector mesons. Therefore, we can consider only  $S$  waves for the rescattering. Furthermore, we can make the approximation

$$\sum |\bar{\epsilon}_\phi \cdot \vec{k}| \simeq |\vec{k}|^2, \quad (5)$$

where the sum runs over the polarizations of the  $\phi$  meson.

After performing the contour integration over the temporal component  $q^0$  in Eq. (6), in the same way as shown in Refs. [12, 69], and including the finite widths of the  $\Sigma^*$  and  $K^*$  resonances, we get

$$t = -\frac{g_{\Lambda_c \Sigma^* K^*} g}{m_\pi} \bar{\epsilon}_\phi \cdot \vec{k} m_{\Sigma^*} t_T, \quad (6)$$

$$t_T = \sum_{i=\Sigma, \Lambda} C_i m_i \int \frac{d^3 q}{(2\pi)^3} \frac{1}{\omega_{K^*} E_{\Sigma^*} E_i} \times \frac{1}{k^0 - E_i - E_{\Sigma^*} + i \Gamma_{\Sigma^*}/2} \frac{1}{P^0 + \omega_{K^*} + E_i - k^0} \times \frac{1}{P^0 - \omega_{K^*} - E_i - k^0 + i \Gamma_{K^*}/2} \times \frac{1}{[P^0 \omega_{K^*} + k^0 E_i - (\omega_{K^*} + E_i)(\omega_{K^*} + E_i + E_{\Sigma^*})]} \times \frac{1}{P^0 - E_{\Sigma^*} - \omega_{K^*} + i \Gamma_{\Sigma^*}/2},$$

where  $\omega_{K^*} = \sqrt{m_{K^*}^2 + |\vec{q}|^2}$ ,  $E_{\Sigma^*} = \sqrt{m_{\Sigma^*}^2 + |\vec{q}|^2}$ ,  $P^0 = M_{\Lambda_c^+}$ ,  $k^0 = \sqrt{m_{\pi^0}^2 + |\vec{k}|^2} = (M_{\Lambda_c^+}^2 + m_{\pi^0}^2 - M_{\phi p}^2)/(2M_{\Lambda_c^+})$ , and  $E_i = \sqrt{m_i^2 + |\vec{q} + \vec{k}|^2}$  with  $i = \Sigma$  or  $\Lambda$ . Because the  $S$ -wave vertices attached to the  $\Lambda_c$  initial state and the  $\phi p$  final state do not introduce any momentum dependence into the loop amplitude, and the  $P$ -wave pionic vertices result in a factor of the pion momentum, the above loop integral is ultraviolet convergent.

The  $\phi p$  invariant mass distribution for the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  decay then reads

$$\frac{d\Gamma}{dM_{\phi p}} = \frac{m_p m_{\Sigma^*}^2 g_{\Lambda_c \Sigma^* K^*}^2 g^2}{16\pi^3 M_{\Lambda_c^+} m_\pi^2} |\vec{k}|^3 |\vec{p}_\phi| |t_T|^2, \quad (7)$$

where  $\vec{k}$  is the  $\pi^0$  momentum in the rest frame of the  $\Lambda_c^+$ , and  $\vec{p}_\phi$  is the  $\phi$  momentum in the center-of-mass frame of the  $\phi p$  system. They are given by

$$|\vec{k}| = \frac{\sqrt{[M_{\Lambda_c^+}^2 - (m_{\pi^0} + M_{\phi p})^2][M_{\Lambda_c^+}^2 - (m_{\pi^0} - M_{\phi p})^2]}}{2M_{\Lambda_c^+}},$$

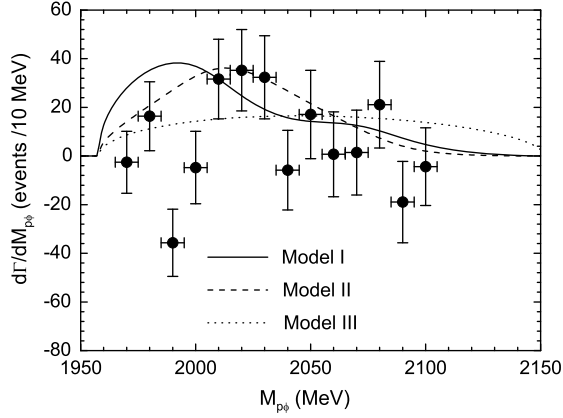
$$|\vec{p}_\phi| = \frac{\sqrt{[M_{\phi p}^2 - (m_\phi + m_p)^2][M_{\phi p}^2 - (m_\phi - m_p)^2]}}{2M_{\phi p}},$$

with  $m_{\pi^0} = 134.98$  MeV,  $m_\phi = 1019.46$  MeV, and  $m_p = 939.27$  MeV. Finally, the partial decay width of the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  decay is obtained by integrating Eq. (7) over  $M_{\phi p}$ ,

$$\Gamma = \int_{m_\phi + m_p}^{M_{\Lambda_c^+} - m_{\pi^0}} dM_{\phi p} \frac{d\Gamma}{dM_{\phi p}}. \quad (8)$$

### 3. Numerical results

So far we have not specified the input for the rescattering  $T$ -matrix elements. In principle, because of the very small phase space and the closeness of the thresholds, all of the involved hadrons,  $K^*$ ,  $\Sigma$  or  $\Lambda$ ,  $\phi$  and proton, can be treated nonrelativistically. Thus, one may construct a nonrelativistic effective field theory describing the interaction between vector mesons and baryons with the leading order defined by a few constant contact terms. However, it does not make much sense doing it in that manner because of the lack of experimental information. We will thus take the model of Ref. [70] where the interaction of the vector mesons with the  $SU(3)$  octet baryons is studied in the local hidden gauge formalism using a coupled-channel unitary approach. In that model, a degenerate pair of resonances with  $J^P = 1/2^-$  and  $3/2^-$  which couple strongly to  $K^* \Sigma$ ,  $K^* \Lambda$  and  $\phi p$  is obtained, and the pole is at  $(1977 + i55)$  MeV [70]. They can be regarded as the  $P_s$  states. The prediction in this model was updated in light of the  $\gamma p \rightarrow K^0 \Sigma^+$  data [71] in Ref. [72] to get resonance parameters with a mass of 2035 MeV and a width of 125 MeV. However, this



**Fig. 2.** Invariant mass distribution of the  $\Lambda_c^+ \rightarrow \pi^0 p \phi$  decay. The experimental data are taken from Ref. [47].

state only shows up in the transitions involving the  $K^* \Sigma$  channel. One may regard that model as providing a special set of parameters for the nonrelativistic effective field theory mentioned above. By adjusting the interaction strengths, one can in principle investigate the possibility of  $P_s$  with other masses and as well as the possibility without any  $P_s$ , i.e. no pole around the  $\phi p$  threshold.

Here we present the numerical results for the  $\phi p$  invariant mass distribution for three different cases, which are denoted as Model I, II and III, in Fig. 2. Model I represents the calculation of the triangle diagrams in Fig. 1 with the FSI taken from Ref. [72], which includes the contribution of a  $P_s$  state with properties specified in that model, which could be well different in other models and in reality, see above. Model II is different from Model I by modeling the FSI by a constant, and it thus represents the case without any  $P_s$  resonance. For comparison, we show the phase space without any special dynamics as Model III. To be more explicit, for these three cases, the total decay amplitudes  $t_j$  ( $j = I, II$ , and III) are given by

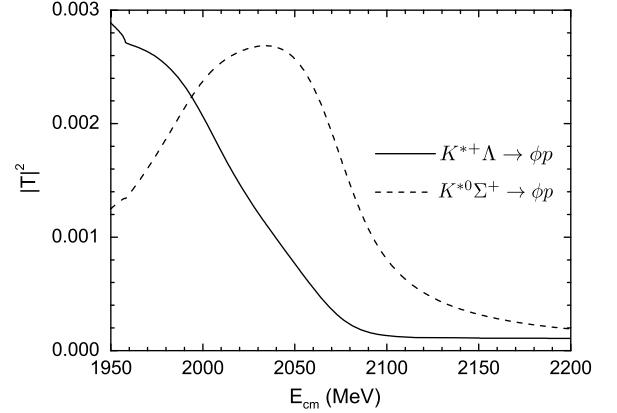
$$t_I = t,$$

$$t_{II} = t, \text{ but with } t_{K^* \Lambda \rightarrow \phi p} = \frac{\sqrt{6}}{2} t_{K^* \Sigma^+ \rightarrow \phi p} \\ = c_1 \frac{\sqrt{6}}{2} \frac{E_{K^*} + E_\phi}{4F_\pi^2}, \quad (9)$$

$$t_{III} = c_2, \quad (10)$$

where  $t$  is the amplitude shown in Eq. (6),  $c_1$  and  $c_2$  are normalization constants to be adjusted to match the measured event distribution,  $F_\pi = 92.2$  MeV is the pion decay constant,  $E_{K^*}$  and  $E_\phi$  are the energies of the  $K^*$  and  $\phi$  mesons in the  $\phi p$  center-of-mass frame. Here we take  $E_{K^*} = 891.66$  MeV and  $E_\phi = 1043.26$  MeV, which are obtained at the  $K^{*+} \Lambda$  mass threshold.

In Fig. 2, the solid, dashed, and dotted curves represent the results of Model I, II, and III, respectively. The parameter  $c_1$  of Model II has been adjusted to the strength of the experimental data reported by the Belle Collaboration [47] at its peak around  $M_{\phi p} = 2020$  MeV. The results of Model I and III are normalized such as to have the same integrated partial width as Model II. Model II clearly shows a peak structure around 2.02 GeV. The origin of this peak is the triangle diagrams, in particular the  $\Lambda$ -exchange in Fig. 1 (B) which has a triangle singularity close to the  $K^* \Lambda$  threshold ( $\simeq 2.01$  GeV). The width of this peak is comparable with the width of the  $K^*$ , which is about 50 MeV. This is a quite natural consequence as the  $\phi p$  invariant mass is the same as the  $K^* \Lambda$  invariant mass so that its distribution inherits the width of the



**Fig. 3.** The squared norm of the  $T$ -matrix elements for  $K^{*+} \Lambda \rightarrow \phi p$  and  $K^{*0} \Sigma^+ \rightarrow \phi p$  as a function of the meson–baryon invariant mass  $E_{cm}$  in the model of Ref. [72].

$K^*$ . Were the  $K^*$  width much smaller, one would get a much narrower peak. For Model I, one might think that there should be also a bump structure around 2035 MeV which is the mass of the generated resonance in the vector meson–baryon interaction model we are using [72]. However, the triangle diagram involving the  $K^* \Lambda \rightarrow \phi p$  transition is the predominant contribution in the present case because its triangle singularity is closer to the physical region, while the resonance peak only shows up in the channels involving the  $K^* \Sigma$ . Here, the FSI results in a near-threshold enhancement, see Fig. 3 where the kink in the solid line is located at the  $\phi p$  threshold. It could be that in other models the resonance couples to these vector meson–baryon channels in a different pattern so as to show up as a near-threshold peak in the  $\phi p$  invariant mass distribution. Identifying such an enhancement in experiments is difficult as it requires the data to have a high statistics. In particular, it becomes much more difficult if the  $P_s$  mass is close to the  $K^* \Lambda$  threshold because of the presence of kinematic singularities there. In any case, the phase space shown as Model III is very different from both Model I and Model II. Despite the low statistics of the current Belle data, the curve of Model II, whose shape is completely fixed, has a remarkable agreement with the data. In particular, the data seem to indeed have a peak around  $K^* \Lambda$  threshold. More data are welcome to clarify the situation.

Using the value of  $g_{\Lambda_c \Sigma^* K^*}$  estimated in Appendix A, we can get an estimate of the branching fraction of the three-body decay  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  by integrating over the  $\phi p$  invariant mass distribution. For Model II with  $c_1 = 1$ , the result is

$$\text{Br}(\Lambda_c^+ \rightarrow \pi^0 \phi p)_{II} = \mathcal{O}(10^{-4}), \quad (11)$$

which is of the same order as the Belle upper limit [47]. Thus, although not all of the contributions to the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  decay are from this mechanism we expect the actual branching fraction is of this order.

#### 4. Summary

We study the  $\Lambda_c^+ \rightarrow \pi^0 \phi p$  decay by considering a triangle singularity mechanism. The decay was proposed to be a channel to search for the hidden-strange partner of the  $P_c$  states. The mechanism is such that the  $\Lambda_c^+$  decays into the  $K^* \Sigma^*$ , the  $\Sigma^*$  subsequently decays into the  $\pi^0 \Sigma$  (or  $\Lambda$ ), and the  $K^*$  then interacts with the  $\Sigma$  (or  $\Lambda$ ) to produce the  $\phi p$  in the final state. In the  $K^* \Sigma/\Lambda \rightarrow \phi p$  FSI, we consider cases with and without a  $P_s$  state. For the case with the  $P_s$ , we take the model of Refs. [70,72] which produces a resonance at around 2 GeV. The triangle singularities considered in this paper are close to the physical region, and can



produce a peak at around 2.02 GeV with a width similar to that of the  $K^*$  resonance. The obtained  $\phi p$  invariant mass distribution agrees with the existing Belle data. Were there a  $P_s$  state, it could distort the distribution. However, it is difficult to be identified in the decay under study because of the small phase space and the presence of triangle singularities. We look forward to more data on this channel as well as the relevant  $K^*\Sigma^*$  channel from the BESIII, Belle-II and LHCb experiments in the future, which will be decisive to illuminate the role played by triangle singularities in this decay.

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## Appendix A. Estimate of the $\Lambda_c \Sigma^* K^*$ coupling constant

The branching fraction for  $\Lambda_c^+ \rightarrow \Sigma^* K^*$  has not been estimated so far. Yet, an upper limit has been reported as  $\text{Br}(\Lambda_c^+ \rightarrow \Lambda K^+ \pi^+ \pi^-) < 5 \times 10^{-4}$  [27]. Because the  $\Sigma^*$  and  $K^*$  decay dominantly into the  $\Lambda\pi$  and  $K\pi$ , we thus take  $10^{-4}$  as an order-of-magnitude estimate for  $\Lambda_c^+ \rightarrow \Sigma^* K^*$  to estimate the coupling constant  $g_{\Lambda_c \Sigma^* K^*}$  using the following decay width formula

$$\Gamma[\Lambda_c^+ \rightarrow \Sigma^* K^*] = \frac{g_{\Lambda_c \Sigma^* K^*}^2 |\vec{p}_1|}{48\pi} \left( 1 + \frac{m_{\Sigma^*} + m_{K^*}}{M_{\Lambda_c^+}} \right) \times \left( 1 + \frac{m_{\Sigma^*} - m_{K^*}}{M_{\Lambda_c^+}} \right) \left( 8 + \frac{(M_{\Lambda_c^+}^2 - m_{\Sigma^*}^2 - m_{K^*}^2)^2}{m_{\Sigma^*}^2 m_{K^*}^2} \right), \quad (\text{A.1})$$

with

$$|\vec{p}_1| = \frac{\sqrt{[M_{\Lambda_c^+}^2 - (m_{\Sigma^*} + m_{K^*})^2][M_{\Lambda_c^+}^2 - (m_{\Sigma^*} - m_{K^*})^2]}}{2M_{\Lambda_c^+}}.$$

Using the measured masses  $M_{\Lambda_c^+} = 2286$  MeV,  $m_{\Sigma^*} = 1385$  MeV,  $m_{K^*} = 893$  MeV and the total decay width of  $\Gamma_{\Lambda_c^+} = 3 \times 10^{-9}$  MeV, we get

$$g_{\Lambda_c \Sigma^* K^*} \sim 2 \times 10^{-7}. \quad (\text{A.2})$$

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