

## Intertwined Order and Holography: The Case of Parity Breaking Pair Density Waves

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We present a minimal bottom-up extension of the Chern-Simons bulk action for holographic translational symmetry breaking that naturally gives rise to pair density waves. We construct stationary inhomogeneous black hole solutions in which both the U(1) symmetry and spatially translational symmetry are spontaneously broken at a finite temperature and charge density. This novel solution provides a dual description of a superconducting phase intertwined with charge, current, and parity orders.

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*Introduction.*—The explanation of states of matter that break symmetry spontaneously in circumstances where quantum effects are dominating has been a traditional mainstay of condensed matter physics. Famous examples are the BCS theory of conventional superconductivity as well as the Peierls theory explaining charge and spin density waves in terms of the nesting of the Fermi surface. As an outcome of a long empirical development, it was discovered that the strongly interacting systems realized in underdoped cuprate superconductors seem to realize a remarkably complex form of “intertwined” orders [1,2]. There are indications for exotic, quantum physical forms of order that are not found elsewhere. A first example is the pattern of *spontaneous electronic “orbital” currents* encircling the plaquettes of the copper oxide planes [3–5], while there is definitive evidence for the breaking of parity [6]. Another example is the *pair density wave* (PDW): a superconducting state that does break translations in a zero magnetic field, for which experimental evidence was reported in spin-stripped 214 superconductors [7,8] and very recently in a charge ordered 2212 “BISCO” superconductor [9]. Departing from the established repertoire of condensed matter theories, it appears to be quite difficult to explain the origin for the observed patterns of symmetry breaking in cuprates [10,11]. The details of the conventional symmetry breaking (like the periodicity of the charge order) are far from being understood [1], while a convincing explanation of the mechanisms leading to spontaneous currents and PDWs is still lacking. Last but not least, it appears that these orders occur simultaneously in a specially orchestrated intertwined relationship, for reasons that are largely mysterious [2].

Holographic duality has a track record as a useful theoretical laboratory to explore the rich structure of quantum matter characterized by densely entangled degrees of freedom (d.o.f.) [12,13]. It maps the quantum problem in the “boundary” onto classical gravitational physics in a

“bulk” space-time with one extra dimension. The rules of the effective field theory (EFT) based on symmetry and locality are applied to the bulk, and this dualizes in a set of robust and universal phenomenological theories describing the physics in the boundary. The application of holography to condensed matter was jump-started by the discovery of holographic superconductivity [14] showing that spontaneous symmetry breaking has a stunningly elegant gravitational dual [12]: The black holes carry a halo of charged hair which spontaneously breaks a global U(1) symmetry of the dual field theory. This shares similarities with BCS superconductivity in the boundary [12] but yet is quite different [15] given that the normal state is a holographic *strange metal* [12] instead of a Fermi liquid. We will demonstrate here that, by further developing the bulk theory using the EFT rule book, a pattern of intertwined order emerges in the boundary sharing intriguing similarities with those realized in the cuprates. This suggests that the peculiar intertwined nature of the low-temperature order may be rooted in the densely entangled nature of the quantum critical metal. The next ingredient is bulk *topological* terms—Chern-Simons (CS) [16] and theta terms [17] in odd and even dimensions, respectively. Intriguingly, these describe the emergence of spontaneous *current order intertwined with the crystallization of charge*.

We present here a minimal extension of the CS-type bulk theory to incorporate holographic superconductivity. The outcome is that the superconductivity generically turns into a pair density wave, automatically implying the simultaneous spontaneous breaking of parity [6]. Representative results are shown in Fig. 2 for a unidirectional and a “tetragonal” crystallization in two space dimensions. The rules governing these patterns are most easily inferred from the unidirectional case [Figs. 2(a) and 2(b)]. The charge accumulates in “stripes” [bright areas in Fig. 2(b)] where the current densities are maximal. The currents run in the perpendicular  $y$  direction, and these form a staggered

pattern with the maxima being coincident with the maximal charge density, vanishing in the middle of the low density domains. The superfluid density [Fig. 2(a)] exhibits the same periodicity as the current order, but it is precisely out of phase with the latter: It has nodes where the current and charge density are maximal, while it oscillates from maximal negative to positive values in the low density domains. This is precisely what is meant with a pair density wave. These rules repeats themselves in the tetragonally (“checkerboard”) ordered case. The charge order [Fig. 2(d)] is now accompanied by spontaneous staggered current patterns [18] similar to the “ $d$ -density wave” [19–22], having quite a history in the condensed matter literature. This is now accompanied with a 2D PDW having twice the periodicity of the charge order, changing sign from one “interstitial” region to the next where it acquires a maximum absolute value. Let us now discuss in more detail how we arrive at these results.

*Gravity setup.*—Our starting point is a  $(3+1)$ -dimensional gravity theory dual to the  $(2+1)$ -dimensional boundary theory containing a gauge field  $A_\mu$  and two scalar fields  $\chi$  and  $\theta$ , assuming the coordinate system  $(t, x, y, z)$  in which the anti-de Sitter (AdS) boundary is located at  $z = 0$ :

$$S = \int d^4x \sqrt{-g} \left( \mathcal{R} + 6 - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \mathcal{F}(\chi) (\partial_\mu \theta - A_\mu)^2 - \frac{Z(\chi)}{4} F^2 - V(\chi) - \vartheta(\chi) \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu} F^{\lambda\sigma} \right). \quad (1)$$

Ignoring the fourth term that involves  $\theta$ , this reveals the canonical holographic mechanism for spontaneous translational symmetry breaking. One recognizes the theta term  $\sim \chi F \wedge F$  (with  $\epsilon_{txyz} = \sqrt{-g}$ ) as inspired on top-down holography by the consistent truncation of 11D supergravity [23], while fully backreacted geometries can be obtained [24–27]. Viewed from a bottom-up perspective, such a topological term is generic, and it can be added to the bulk in order to respect general symmetry principles. The topological term has the effect to shift the instability (Breitenlohner-Freedman (BF) bound violation in the bulk) to a finite wave vector, but for this to happen one needs simultaneous intertwined vacuum expectation values (VEVs) of the currents and the charge density. This is responsible for the intertwined charge-current order in Fig. 2. We notice that in any other holographic setup the breaking of translations involves meticulous fine-tuning (e.g., [28]), and in this sense the intertwinement of charge and current orders follows naturally from holography.

Our novelty is to include the  $\theta$  field, that represents an easy way to include the minimal coupling of  $\chi$  to the gauge field  $A_\mu$  by a Stückelberg term (known as “Josephson action” in the condensed matter literature). This has the advantage of admitting less restrictive scalar couplings:

In this generalized class of theories, the two real d.o.f.  $\chi$  and  $\theta$  are not necessarily associated with the magnitude and phase of a complex scalar, respectively. Holographic superconductors in similar setups have been studied, for instance, in Refs. [29–33]. The bulk solution with non-trivial  $(\chi, \theta)$  corresponds to the U(1) broken phase. Although there is no top-down justification presently available, the structure of this action suggests a minimal way to merge the superconductivity and the flux phase. As  $\theta$  can be absorbed by gauge fixing, the condensation of the scalar operator dual to  $\chi$  could play the role of the superconducting order parameter. The bulk Stückelberg term implies the breaking of U(1) in the boundary, and the presence of the superconducting condensate is revealed by the characteristic delta function at zero frequency in the optical conductivity (see Supplemental Material [34]).

In this Letter, we focus on the particular model with

$$\begin{aligned} Z(\chi) &= \frac{1}{\cosh(\sqrt{3}\chi)}, & V(\chi) &= 1 - \cosh(\sqrt{2}\chi), \\ \mathcal{F}(\chi) &= \cosh(\chi) - 1, & \vartheta(\chi) &= \frac{1}{4\sqrt{3}} \tanh(\sqrt{3}\chi). \end{aligned} \quad (2)$$

The scalar operator  $\mathcal{O}$  dual to  $\chi$  has therefore dimension 2. The normal phase with  $\chi = 0$  is described by the AdS Reissner-Nordström (RN) black brane [36]:

$$\begin{aligned} ds^2 &= \frac{1}{z^2} \left[ -H(z) dt^2 + \frac{1}{H(z)} dz^2 + (dx^2 + dy^2) \right], \\ H(z) &= (1-z) \left( 1 + z + z^2 - \frac{\mu^2 z^3}{4} \right), \\ A_t &= \mu(1-z), \end{aligned} \quad (3)$$

where  $\mu$  is the chemical potential, rescaling the coordinates so that the horizon is at  $z = 1$ . The temperature is given by  $T/\mu = (12 - \mu^2)/16\pi\mu$ . Departing from the extremal RN AdS<sub>2</sub>  $\times$   $\mathbb{R}^2$  background, it follows from a perturbative analysis that a homogeneous U(1) broken solution is excluded for the model (2) [37]. The last term plays a crucial role, imposing that the breaking of the U(1) symmetry goes hand in hand with the breaking of translational invariance. It follows immediately from this bulk action that parity is broken as well in the simultaneous presence of the superconducting, charge, and current orders with a VEV proportional to the U(1) order parameter [34].

*Intertwined black holes.*—Let us now construct the inhomogeneous intertwined black holes, focusing first on the unidirectional “striped” solutions. The procedure leading to the fully crystallized solutions is just a straightforward generalization involving two spatial directions. We seek for unidirectional solutions within the following ansatz:

$$ds^2 = \frac{1}{z^2} \left[ -H(z)U_1 dt^2 + \frac{U_2}{H(z)} dz^2 + U_3(dx + z^2 U_5 dz)^2 + U_4(dy + (1-z)U_6 dt)^2 \right],$$

$$A = A_t dt + A_y dy, \quad \chi = z\psi, \quad (4)$$

with nine functions  $(\psi, A_t, A_y, U_i)$ ,  $i = 1, \dots, 6$ , depending on  $z$  and the spatial coordinate  $x$  involved in the symmetry breaking, while  $H(z)$  is given in Eq. (3). This results in a system of equations of motion involving nine partial differential equations (PDEs) in terms of the variables  $z$  and  $x$ . We employ the DeTurck method [38] to solve the system. For the tetragonal case, we deal with 15 functions that all depend on the radial coordinate  $z$  and two spatial coordinates  $(x, y)$ . To solve such a set of cohomogeneity-three PDEs is challenging due to a dramatic increase in computational complexity.

Spontaneous symmetry breaking means that all sources should be turned off, while fixing the boundary metric to be asymptotically AdS at the UV boundary  $z = 0$ ,

$$U_1(0, x) = U_2(0, x) = U_3(0, x) = U_4(0, x) = 1,$$

$$\psi(0, x) = A_y(0, x) = U_5(0, x) = U_6(0, x) = 0,$$

$$A_t(0, x) = \mu. \quad (5)$$

We impose the regularity condition at the horizon  $z = 1$ , and therefore all the functions have an analytic expansion in powers of  $(1 - z)$ . On the spatial boundary, one imposes a periodicity condition. With those boundary conditions, the set of inhomogeneous solutions is parametrized in terms of the temperature  $T/\mu$  and wave number  $k/\mu$ , determining the periodicity in the symmetry broken  $x$  direction.

We show the critical temperature as a function of the wave number in Fig. 1(a) derived from a linear stability analysis of the normal state. This curve is peaked at a nonzero value  $k_c \approx 0.75\mu$  with a  $T_c \approx 0.03\mu$ : This is where the continuous thermal phase transition occurs from the normal system to a unidirectional striped phase with periodicity  $k_c$ . At temperatures below  $T_c$ , nonlinearities become important, and the full set of nine (or 15 for tetragonal case) coupled PDEs has to be solved. We have numerically constructed the solutions parameterized in terms of  $T/\mu$  and  $k/\mu$ , finding that the free energy is minimized at a weakly temperature-dependent ordering wave vector [24,25] indicated by the red dotted line in Fig. 1(a) for the unidirectional case, with similar results for the tetragonal case [37].

In Fig. 1(b), we show the temperature dependence of the various order parameters as a function of the temperature. The current and PDW order parameters have a similar temperature dependence indicative of a Landau mean-field second-order transition  $\sim \sqrt{T_c - T}$ . The charge order grows more slowly  $\sim (T_c - T)$ ; this is indicative of the current

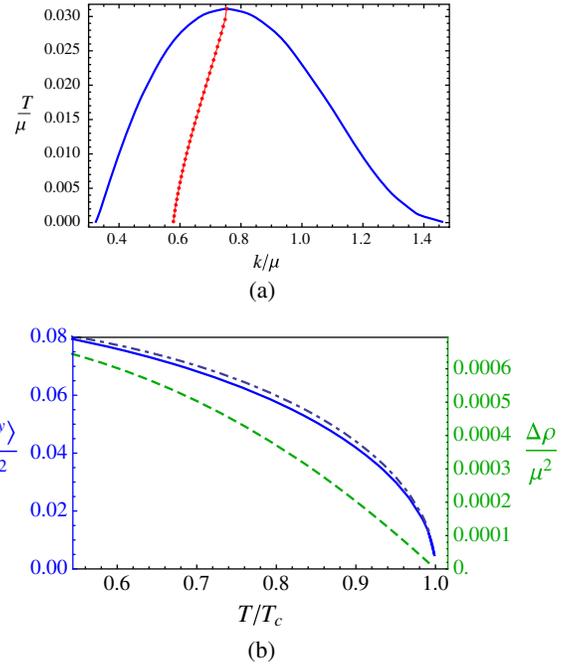


FIG. 1. (a) The stability “dome” (blue line) following from a linear stability analysis in the temperature ( $T/\mu$ ) ordering wave vector ( $k/\mu$ ) plane ( $\mu$  is the chemical potential) for the unidirectional order. The actual second-order transition is associated with the maximal  $T_c$ , and the red dotted line shows the temperature dependence of the ordering wave vector of the fully backreacted intertwined order. (b) The temperature dependence of the charge modulation  $\Delta\rho$  (dashed line), current  $\langle J^y \rangle$  (dash-dotted line), and pair density wave  $\langle O \rangle$  (full line) VEVs of the unidirectional phase. The dominating current and pair density wave VEVs show a characteristic mean field temperature evolution, while the “parasitic” charge density modulation  $\Delta\rho$  increases linearly. Nontrivial configurations of  $\langle O \rangle$  also imply a spontaneous breaking of parity, which was very recently observed in the cuprates [6]. Similar results are obtained for the full tetragonal order in Fig. 2.

order or PDW being the *dominant* order parameter while the charge modulation is induced parasitically. This is a generic affair which is well understood in terms of the Landau theory as of relevance also to the ordering of spin stripes [39]. The lowest-order invariant coupling the order parameters is  $\rho|\langle O \rangle|^2$  ( $\rho|\langle J^y \rangle|^2$ ). When the order occurs at a finite wave number, this immediately implies that the period of the charge order is twice that of the current or PDW order. However, it also implies that when the dominant (mean-field) instability is associated with the currents and the PDW, the charge density modulation will grow linearly with the temperature below  $T_c$ . Interestingly, this is accomplished in the bulk by the leading nonlinear correction near  $T_c$ . The current and condensate modes are in combination violating the BF bound and drive the instability. In linear order the charge density is decoupled, but in next-to-leading order the modulated  $A_y$  and  $\chi$  fields induce a charge density modulation growing linearly with

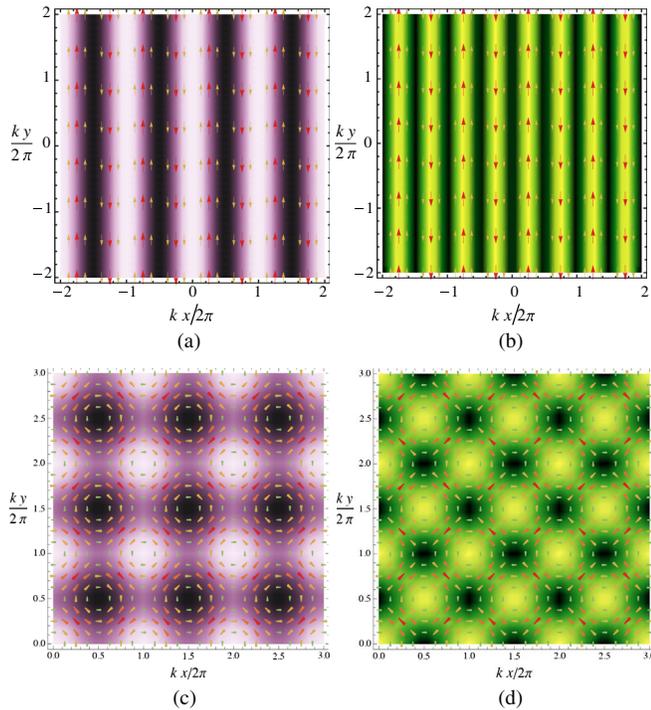


FIG. 2. The density plots of (a) condensate and (b) charge density distribution for a unidirectional phase at  $T/T_c \approx 0.64$  and that of (c) condensate and (d) charge density distribution for the square checkerboard at  $T/T_c \approx 0.94$ . The arrows denote integral curves of the currents ( $\langle J^x \rangle$ ,  $\langle J^y \rangle$ ). The bright parts correspond to large positive values while the dark region to the negative values. In the unidirectional phase the charge density oscillates at twice the frequency of the currents and condensate. The latter two orders are precisely out of phase. The checkerboard is a tetragonal charge crystal going hand in hand with a staggered pattern of current fluxes circling around the plaquettes, which is similar to the  $d$ -density waves of condensed matter physics.

the temperature. Eventually, at low temperatures, nonlinearities encapsulated by the fully backreacted bulk solutions become quite important, giving rise to the intertwined ordering pattern shown in Fig. 2 which we already discussed.

*Discussion.*—There is obviously still a long way to go in order to address the ordering patterns realized in underdoped cuprates in any detail. At the present stage, the outcomes of the holographic exercise presented in the above offer no more than a rough cartoon. However, the cartoon is suggestive with regard to generalities. Departing from a strange metal normal state, holographic “naturalness” seems to insist that the spontaneous symmetry breaking at a low temperature should necessarily give rise to intertwined order involving besides charge order also spontaneous currents, parity breaking, and pair density waves. This rests on applying the rules of EFT to the gravitation dual: The simplest way to break translations is by invoking the topological (theta and CS) terms that automatically intertwine charge and current orders. We have

presented here the most minimal gravitational theory that resurrects the superconductivity, turning it automatically in a pair density wave, remarkably at the “expense” of an unavoidable spontaneous breaking of parity.

Is there room to make this more realistic? There are some intrinsic limitations. The temperature dependence of order parameters (Fig. 1) should be taken with a grain of salt. Because of the matrix large- $N$  limit associated with holography, thermal fluctuations are completely suppressed, while it is well established that these are important for both the PDW [7,8] and the current order [40,41]. A crucial missing ingredient is the periodic background potential. The present construction lives in the Galilean continuum, while in experiments the ionic background potential is playing an important role. This complicates the bulk physics further, and only very recently have first results become available showing a commensurate lock-in between the spontaneous translational order and a background periodic potential [42]. There is a wealth of phenomena to explore. Is it possible to construct holographically the current-loop order living inside the  $\text{CuO}_2$  unit cells by applying an appropriate background potential? It has been all along clear that the cuprate stripes and charge density waves are eventually rooted in incommensuration effects associated with the competition between the periodicity of the background and spontaneous “lattices” [43,44].

Another major limitation of state of the art holography is related to the truly relativistic nature of the boundary field theory. The matter described in the boundary is formed from massless d.o.f. with the ramification that spin and orbital motions are locked together in helical or chiral d.o.f. as for Dirac fermions in two or three dimensions. There are no “separate” spin d.o.f. that can be used to construct Heisenberg antiferromagnets and the spin systems required for the spin stripes. The way to go is to further enumerate *nonrelativistic* holographic setups, so that one can contemplate holographic constructions involving spin order of the kind that is ubiquitous in condensed matter systems. This theme of intertwined order illustrates in an effective manner that there is still much realistic quantum matter physics to be explored in the holography laboratory.

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