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Evolution of chirality-odd twist-3 fragmentation functions

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ABSTRACT

We derive the complete set of evolutions of chirality-odd twist-3 fragmentation functions at one-loop level. There are totally nine real twist-3 fragmentation functions, among which seven are independent. The renormalization-scale dependence of the nine functions has an important implication for studies of single transverse-spin asymmetries. We find that the evolutions of the three complex fragmentation functions defined by quark–gluon–quark operator are mixed with themselves. There is no mixing with the fragmentation functions defined only with bilinear quark field operators. In the large-N_c limit the evolutions of the three complex fragmentation functions are simplified and reduced to six homogeneous equations.

Fragmentation Functions (FF's) are important quantities in predictions made with QCD factorization for inclusive hadron production with large momentum transfers, whose generic scale is denoted as Q. Using QCD factorization, the production rate can be predicted as a convolution of inclusive production rate of partons with FF's. The inclusive parton production rate can be calculated with perturbative theory of QCD. The produced partons will fragment into the observed hadron. Fragmentation processes are of long-distance effect and described by FF's.

There are various FF's as ingredients of collinear- and transverse-momentum-Dependent factorization. The properties of FF's are recently reviewed in [1]. In collinear factorization the most well-known FF's are of twist-2 defined with chirality-even QCD operators. These FF's appear in predictions of inclusive hadron production at the leading power of Q^{-1}, where no transverse polarization is observed. The renormalization-scale dependence of twist-2 FF's is governed by DGLAP-type evolutions and is known at two-loop level.

If an initial hadron in Semi-Inclusive DIS and inclusive hadron production in hadron-collisions is transversely polarized, the production rate can contain transverse-spin dependent parts or single transverse-spin asymmetries can appear. Such asymmetries are currently under intensive studies in theory and experiment. The asymmetries at the leading power of Q^{-1} are predicted with twist-3 parton distributions and chirality-odd FF's at twist-3. Because of helicity conservation of perturbative QCD at high energy, contributions from chirality-odd FF's are always combined with the transversity, the parton distribution function of a transversely polarized hadron defined with a twist-2 chirality-odd operator in [2]. This distribution is less known than other standard twist-2 parton distributions. Therefore, studies of chirality-odd FF's are important not only for understanding fragmentation and the asymmetries, but also for extracting information about the transversity.

Although fragmentation processes are in general nonperturbative and FF's at moment can only be extracted from experiment, but the renormalization-scale dependence can be calculated with perturbative QCD. It should be noted that this dependence will determine partly the Q-dependence of the transverse-spin-dependent part of the relevant differential cross-sections, similar to the case of DIS, where the scaling violation is predicted by the evolution of twist-2 parton distribution functions. Unlike the case of the evolution of FF's or PDF's defined with twist-2 operators, whose evolutions are now relatively easy to derive, there is a nontrivial problem in deriving evolutions of higher-twist operators. It is nontrivial to separate contributions at different twists as early studies of higher-twist effects in DIS have

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shown in [3–5]. In this letter, we study the renormalization-scale dependence of twist-3 chirality-odd FF's. Some of them have been studied in [6,7]. Here, we will derive all evolutions of them.

We notice that the same twist-3 operators used to defined chirality-odd FF's are also used to defined chirality-odd parton distributions. Because the time-reversal symmetry, the number of chirality-odd twist-3 parton distributions is smaller than that of chirality-odd twist-3 FF's. The evolutions of chirality-odd twist-3 parton distributions have been studied in [8–13]. The Q-dependence of single transverse-spin asymmetries are governed by the evolution of twist-3 parton distributions and twist-3 FF's combined with the evolution of twist-2 operators. The evolution of chirality-even twist-3 parton distributions can be found in [12,14–18]. With these known evolutions and the evolution studied here, the Q-dependence can be predicted completely. Recently, the evolutions of twist-4 operators have been studied in [19].

For our purpose it is convenient to use the light-cone coordinate system. In this system a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0+a^3)/\sqrt{2}, (a^0-a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. We introduce two light-cone vectors $n^\mu = (0,1,0,0)$ and $l^\mu = (1,0,0,0)$. The transverse metric in the coordinate system is given by $g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu l^\nu - n^\nu l^\mu$. With the metric the transverse part of any vector a^μ is obtained as $a_\perp^\mu = g_\perp^{\mu\nu} a_\nu$.

The definitions of twist-3 chirality-odd FF's have been discussed in [20–22]. We assume that the produced hadron through parton fragmentations moves in the $+$-direction with the momentum $P^\mu = (P^+, 0, 0, 0) = P^+ l^\mu$. We will work in the light-cone gauge $n \cdot G = G^+ = 0$. In this gauge gauge links, which are needed in other gauges to make the definitions gauge-invariant, are always a unit matrix. From two-parton correlation functions one can define three chirality-odd FF's for the unpolarized hadron in d-dimensional space–time:

$$\hat{e}(z) = \frac{z^{d-3} P^+}{4N_c} \int \frac{d\lambda}{2\pi} e^{-i\lambda P^+/z} \sum_X \mathrm{Tr}\langle 0|\psi(0)|hX\rangle\langle hX|\bar{\psi}(\lambda n)|0\rangle,$$

$$\hat{e}_I(z) = \frac{z^{d-3} P^+}{4N_c} \int \frac{d\lambda}{2\pi} e^{-i\lambda P^+/z} \sum_X \mathrm{Tr}\langle 0|\sigma^{-+}\psi(0)|hX\rangle\langle hX|\bar{\psi}(\lambda n)|0\rangle,$$

$$\hat{e}_\partial(z) = \frac{z^{d-3}}{4N_c(d-2)} \int \frac{d\lambda}{2\pi} e^{-i\lambda P^+/z} \sum_X \mathrm{Tr}\langle 0|\gamma^+\gamma_{\perp\mu}(0)\psi(0)|hX\rangle\partial_\perp^\mu\langle hX|\bar{\psi}(\lambda n)|0\rangle. \tag{1}$$

The defined three functions are real and have the support of $0 \le z \le 1$ for fragmentation of a quark. In the case of anti-quark FF's one has $-1 \le z \le 0$. Since the number of γ-matrices sandwiched between two quark fields in Eq. (1) is even, these fields must have different chiralities, or different helicities in the massless case. If chiral-symmetry is an exact symmetry of QCD, these functions are zero. It is noted that time-reversal symmetry of QCD can not be used here to obtain constraints on these functions. In Eq. (1) the state $|hX\rangle$ is an out-state at the time $t=\infty$. Under the transformation of time-reversal, the state becomes an in-state at the time $t=-\infty$. There is in general no simple relation between the in- and out states. If there are no final- or initial state interactions, the out-state can be related to the in-state through a phase factor, or the amplitude $\langle hX|\bar{\psi}|0\rangle$ contains only dispersive part and no absorptive part. In this case, one can use the time-reversal symmetry to show that $\hat{e}_I$ and $\hat{e}_\partial$ are zero. But final-state- or initial-state interactions always exist, the absorptive part of the amplitude $\langle hX|\bar{\psi}|0\rangle$ can be nonzero. Therefore, $\hat{e}_I$ and $\hat{e}_\partial$ are not zero in general.

From the three-parton correlations one can define three complex FF's:

$$\hat{E}_F(z_1,z_2) = -\frac{z_2 g_s}{2(d-2)N_c} \int \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} e^{-i\lambda_1 P^+/z_1 - i\lambda_2 P^+/z_3} \sum_X \mathrm{Tr}\langle 0|i\gamma^+\gamma_{\perp\mu}\psi(0)|hX\rangle\langle hX|\bar{\psi}(\lambda_1 n)G^{+\mu}(\lambda_2 n)|0\rangle,$$

$$\hat{E}_{\bar{F}}(z_1,z_2) = \frac{z_2 g_s}{4N_c}\frac{2}{d-2} \int \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} e^{-i\lambda_1 P^+/z_1 - i\lambda_2 P^+/z_3} \sum_X \mathrm{Tr}\langle 0|\bar{\psi}(0)(i\gamma^+\gamma_{\perp\mu})|hX\rangle\langle hX|G^{+\mu}(\lambda_2 n)\psi(\lambda_1 n)|0\rangle,$$

$$\hat{E}_G(z_1,z_2) = -\frac{z_2 g_s}{4(N_c^2-1)}\frac{2}{d-2} \int \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} e^{i\lambda_1 P^+/z_1 - i\lambda_2 P^+/z_2} \sum_X \mathrm{Tr}\langle 0|\bar{\psi}(\lambda_1 n)i\gamma^+\gamma_{\perp\mu}T^a\psi(0)|hX\rangle\langle hX|G^{a,+\mu}(\lambda_2 n)|0\rangle \tag{2}$$

with $1/z_3 = 1/z_2 - 1/z_1$. $\hat{E}_{\bar{F}}$ is for fragmentation with an anti-quark. In general the three functions are complex. If there is no final state interaction, one can use time-reversal symmetry, as discussed before, to show that they are real. The functions have the support:

$$0 < z_2 < 1, \quad \text{or} \quad z_2 < z_1 < \infty. \tag{3}$$

In [23] it is shown that these functions are zero at $z_1 = z_2$ or $1/z_1 = 0$.

The introduced FF's are defined with chirality-odd QCD operators of twist-3. They are not independent. From QCD equation of motion one can derive the relations [6,21,22]:

$$2z_2\hat{e}_\partial(z_2) - \hat{e}_I(z_2) = z_2^2 \int \frac{dz_1}{z_1} P\frac{1}{z_2 - z_1}\mathrm{Im}\hat{E}_F(z_1,z_2), \quad \hat{e}(z_2) = z_2^2 \int \frac{dz_1}{z_1} P\frac{1}{z_2-z_1}\mathrm{Re}\hat{E}_F(z_1,z_2). \tag{4}$$

With these relations we can take $\hat{e}$ and $\hat{e}_I$ as redundant. This is a reasonable choice as we will explain later. It is interesting to compare the defined FF's with parton distributions defined with the same chirality-odd operators. Taking $\hat{E}_F$ as an example, one can define a parton distribution with the same operator in $\hat{E}_F$ as its matrix element of a single hadron. Because the state is of a single stable hadron, the in- and out state are related through a phase factor. One can use time-reversal symmetry of QCD to show that the defined parton distribution is real. For $\hat{E}_F$ itself, the symmetry does not give any constraint. This results in that $\hat{E}_F$ is in general a complex function.

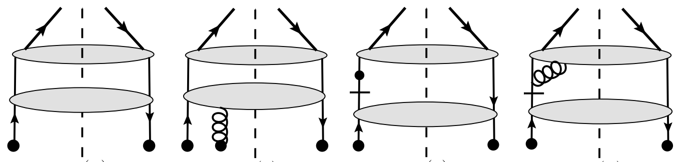

Fig. 1. The structure of diagrams for evolutions.

The defined FF's in Eqs. (1), (2) depend not only on momentum fractions, but also on the renormalization scale μ implicitly. The μ-dependence or evolution of the FF's is the subject of our study. In [6] the evolutions of the real parts of $\hat{E}_{F,G}$ have been studied. The evolution of $\hat{e}_\partial$ is obtained in [7]. In general different FF's can be mixed under evolutions. We can divide FF's into two groups. One group contains $\hat{e}$ and the real part of $\hat{E}_{F,\bar{F},G}$. These FF's can be called as T-even, while another group contains $\hat{e}_I$, $\hat{e}_\partial$ and the imaginary parts of $\hat{E}_{F,\bar{F},G}$. The FF's in this group can be called as T-odd because they are zero in the absence of final-state interactions. Under the evolution two groups of FF's can not be mixed with each other, because the evolution kernels do not contain absorptive parts, at least at one-loop level. For T-even FF's we may only need to determine the evolution of $\text{Re}\hat{E}_{F,\bar{F},G}$. The evolution of $\hat{e}$ may be determined by using the second relation in Eq. (4). For T-odd FF's, one may determine the evolution of $\hat{e}_I$ with the first relation in Eq. (4), once one knows the evolution of $\text{Im}\hat{E}_F$ and $\hat{e}_\partial$. In this letter, we will directly derive the evolution of each FF and use the relations to check our results.

To derive the evolutions of FF's, we essentially need to study, e.g., contributions of Fig. 1a and Fig. 1b. Fig. 1a and Fig. 1b represent the generic structure of diagrams for the contributions from two-parton FF's to the evolution of two-parton- and three-parton FF's, respectively. The black dots there denote the insertion of the field operators with given projections of γ-matrices in the definitions of FF's. The middle bubbles represent diagrams of parton scattering. The top bubbles denote the two-parton density matrix Γ, which is defined and can be decomposed as:

$$\Gamma_{ji}(q) = \int \frac{d^d\xi}{(2\pi)^d} e^{-i\xi\cdot q} \sum_X \langle 0|\psi_j(0)|PX\rangle\langle XP|\bar{\psi}_i(\xi)|0\rangle$$
$$= \frac{\delta(q^-)}{z^{d-3}P^+}\left[\delta^{d-2}(q_\perp)\left(\gamma^- P^+ d(z) + \hat{e}(z) + \sigma^{+-}\hat{e}_I(z)\right) - i\gamma^-\gamma_\perp^\mu P^+ \hat{e}_\partial(z)\frac{\partial}{\partial q_\perp^\mu}\delta^{d-2}(q_\perp)\right]_{ji} + \cdots, \tag{5}$$

where $\cdots$ denote terms at twist higher than 3. The plus component of q is fixed as $q^+ = P^+/z$. $d(z)$ is the standard twist-2 FF defined in [24]. Other three functions are those FF's defined in Eq. (1). When we take the middle part at the tree-level, i.e., at order of α_s^0, or delete the middle bubbles, from Fig. 1a one simply obtains corresponding two-parton FF's themselves, and from Fig. 1b the three-parton FF's receive no contributions from two-parton FF's. When the middle part is at one-loop level or at order of α_s, there are U.V. divergences. Renormalization is hence needed and the renormalization-scale dependence appears. This dependence determines the evolutions.

As mentioned at the beginning, there is a nontrivial problem of how to consistently separate contributions at different twists. The problem can be explained with Fig. 1a or Fig. 1b. With the decomposition of Γ in Eq. (5) one can see that Fig. 1a or Fig. 1b contains the contributions at twist-2. However the above decomposition can not be used here for finding the twist-3 contributions, because the so-called bad components of quark fields in Γ appear for twist-3 contributions. They appear in the definition of $\hat{e}$ and $\hat{e}_I$. The bad component is defined by decomposing the quark field ψ as:

$$\psi(x) = \psi_+(x) + \psi_-(x), \quad \psi_-(x) = \frac{1}{2}\gamma^+\gamma^-\psi(x), \quad \psi_+(x) = \frac{1}{2}\gamma^-\gamma^+\psi(x), \tag{6}$$

where ψ_- is called as the bad component and ψ_+ is the good component. It is easy to realize that the field ψ_- can not be interpreted as a field for creating- or annihilating a quark moving in the +-direction. The propagator containing ψ_- does not propagate. Therefore, ψ_- can not be used with perturbation theory here directly. That is the reason for the un-efficiency of using the decomposition to separate the contributions at different twists, especially, beyond tree-level.

The solution of the problem is to express the bad component with the good component combined with gluon fields through equation of motion in the light-cone gauge:

$$2\partial^+\psi_- + \gamma^+\gamma_\perp\cdot D_\perp\psi_+ = 0, \quad 2D^-\psi_+ + \gamma^-\gamma_\perp\cdot D_\perp\psi_- = 0 \tag{7}$$

with the covariant derivative $D^\mu = \partial^\mu + ig_s G^\mu$. By using $\psi_\pm$ the operator $\psi(0)\bar{\psi}(\xi)$ in Γ can be written with various combinations of $\psi_\pm$ and these combinations give the different parts in the second line of Eq. (5):

$$\Gamma(q) = \Gamma^{(++)}(q) + \Gamma^{(+-)}(q) + \Gamma^{(+-)\dagger}(q) + \Gamma^{(--)}(q),$$
$$\Gamma^{(++)}(q) = \int \frac{d^d\xi}{(2\pi)^d} e^{-i\xi\cdot q} \sum_X \langle 0|\psi_+(0)|PX\rangle\langle XP|\bar{\psi}_+(\xi)|0\rangle$$
$$= \frac{\delta(q^-)}{z^{d-3}P^+}\left(\delta^{d-2}(q_\perp)\gamma^- P^+ d(z) - i\gamma^-\gamma_\perp^\mu P^+\hat{e}_\partial(z)\frac{\partial}{\partial q_\perp^\mu}\delta^{d-2}(q_\perp)\right) + \cdots,$$
$$\Gamma^{(+-)}(q) = \int \frac{d^d\xi}{(2\pi)^d} e^{-i\xi\cdot q} \sum_X \langle 0|\psi_+(0)|PX\rangle\langle XP|\bar{\psi}_-(\xi)|0\rangle,$$

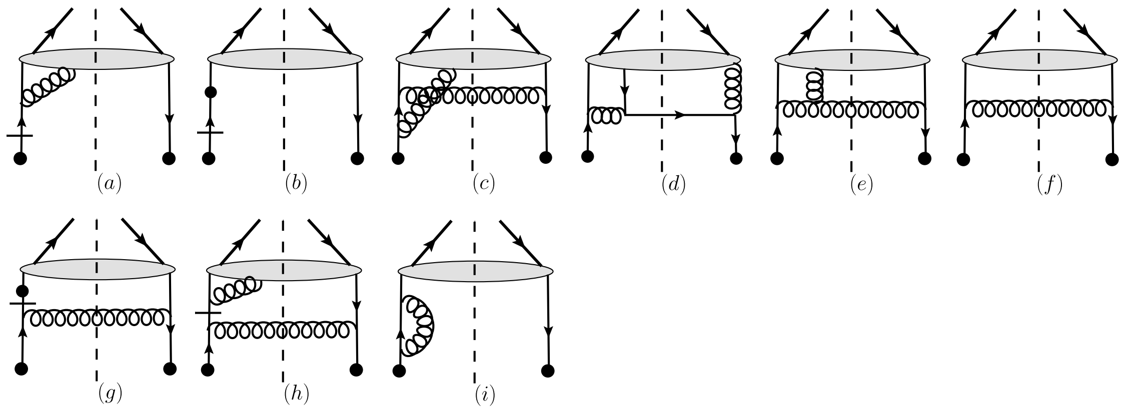

Fig. 2. Diagrams for the evolution of 2-parton FF's.

$$\Gamma^{(+-)}(q) + \left(\Gamma^{(+-)}(q)\right)^\dagger = \frac{\delta(q^-)}{z^{d-3}P^+}\delta^{d-2}(q_\perp)\left(\hat{e}(z) + \sigma^{+-}\hat{e}_I(z)\right) + \cdots. \tag{8}$$

$\Gamma^{(--)}$ stands for the combination with $\psi_-(0)\bar{\psi}_-(\xi)$ and is of twist higher than 3.

We use the equation of motion in Eq. (7) to express $\bar{\psi}_-$ in $\Gamma^{(+-)}$ with $\bar{\psi}_+$:

$$\Gamma^{(+-)}(q) = \left[\int \frac{d^d\xi}{(2\pi)^d} e^{-i\xi\cdot q}\sum_X \langle 0|\psi_+(0)|PX\rangle\langle XP|\partial_\perp^\mu \bar{\psi}_+(\xi)|0\rangle\right]\gamma_{\perp\mu}\frac{i\gamma^+}{2q^+} + \int d^dk\left[\int \frac{d^d\xi d^d\xi_g}{(2\pi)^{2d}}\right.$$
$$\left. e^{-i\xi\cdot k - i\xi_g\cdot(q-k)}\sum_X \langle 0|\psi_+(0)|PX\rangle\langle XP|\bar{\psi}_+(\xi)G_\perp^{a,\mu}(\xi_g)|0\rangle\right](-ig_s\gamma_\mu T^a)\frac{i\gamma^+}{2q^+}, \tag{9}$$

where irrelevant terms are neglected. In Eq. (9) the first term can be expressed by Fig. 1c, where the top-bubble represents the correlation function in $[\cdots]$ in the first term. The black dot near the top bubble denotes the vertex $\gamma_\perp^\mu$. The second term can be expressed with Fig. 1d, where the top-bubble represents the quark–gluon correlation function in $[\cdots]$. In Fig. 1c and 1d the quark line with the short bar denotes the special propagator $i\gamma^+/(2q^+)$. The propagator is just the contraction of $\bar{\psi}_-$ with ψ, it has no pole at $q^2 = 0$ [4].

With the above discussion it is now clear how to correctly separate contributions at different twists from Fig. 1a. The twist-3 contributions from 2-parton FF's can be obtained by calculating Fig. 1a with the top-bubble represented by $\Gamma^{(++)}$. The contributions from Fig. 1c and Fig. 1d have to be included with the rules discussed in the above. In Fig. 1a, 1b and 1c, the top-bubbles only contain good components of quark fields. The contributions from Fig. 1b are calculated in the similar way. There are contributions with an additional gluon-line connecting two bubbles. Their leading contributions are already at twist-3 and have no bad components in the top bubble. Since we will only deal with diagrams whose top bubbles contain good components of quark fields only, it is natural to take $\hat{e}$ and $\hat{e}_I$ as redundant in evolutions.

We first turn to the evolutions of 2-parton FF's. At one-loop we need to study contributions from diagrams given in Fig. 2. A direct calculation of Fig. 2a and 2b gives the results as the relations in Eq. (4) with $\hat{e}_\partial = 0$. There is no contribution to $\hat{e}_\partial$ from Fig. 2a and 2b. Inserting one-loop correction to propagators and vertices into Fig. 2a and Fig. 2b, we obtain the explicit renormalization-scale dependence, hence the contributions to evolutions. The contributions to evolutions from the remaining diagrams can be calculated directly. In calculations all contributions are proportional to the integral

$$\mu^\epsilon \int \frac{d^{d-2}k_\perp}{(2\pi)^{d-2}}\frac{1}{k_\perp^2} = \frac{1}{4\pi}\left(\frac{2}{\epsilon} - \gamma + \ln(4\pi) - \frac{2}{\epsilon_c} + \ln\frac{e^\gamma \mu^2}{4\pi\mu_c^2}\right), \tag{10}$$

which is regularized with the dimensional regularization in $d = 4-\epsilon$ space–time. The first pole of $1/\epsilon$ is from the U.V. divergence, which is subtracted by renormalization. μ is the renormalization scale. The pole in $\epsilon_c = 4-d$ is from collinear divergence and μ_c is the related scale. The contributions from Fig. 2a and Fig. 2b by inserting one-loop correction to propagators and vertices, and those from Fig. 2i can be called as the virtual part, because there is no parton crossing the cut. The contributions from the remaining diagrams are called as the real part, where there is one gluon crossing the cut.

In calculating the virtual part, we find that Fig. 2b and Fig. 2i give no contribution to the evolution of $\hat{e}$. Fig. 2a and Fig. 2b give no contribution to the evolution of $\hat{e}_\partial$ because of $\gamma^+\gamma^+ = 0$. We have the total virtual part of the evolutions of two-parton FF's:

$$\mu\frac{\partial \hat{e}(z)}{\partial\mu}\bigg|_{vir.} = \int \frac{dx_1}{x_1}\mathcal{K}_v(z,x_1)\frac{z^2}{z-x_1}\mathrm{Re}\hat{E}_F(x_1,z),$$
$$\mu\frac{\partial \hat{e}_I(z)}{\partial\mu}\bigg|_{vir.} = -\int \frac{dx_1}{x_1}\mathcal{K}_v(z,x_1)\frac{z^2}{z-x_1}\mathrm{Im}\hat{E}_F(x_1,z) + \frac{\alpha_s C_F}{\pi}\left(3 + 4\ln z - 4\int_0^1 \frac{dy}{y}\right)\hat{e}_\partial(z),$$
$$\mu\frac{\partial \hat{e}_\partial(z)}{\partial\mu}\bigg|_{vir.} = \frac{\alpha_s C_F}{2\pi}\left(3 + 4\ln z - 4\int_0^1 \frac{dy}{y}\right)\hat{e}_\partial(z), \tag{11}$$

with

$$\mathcal{K}_v(z,x_1)=\frac{\alpha_s}{\pi}\left\{C_F\left(\frac{1}{2}+\ln(zx_1)-2\int_0^1\frac{dy}{y}\right)+\frac{1}{2N_c}\frac{z}{x_1-z}\ln\frac{z}{x_1}+\frac{N_c}{2}\ln\frac{z}{x_1}\right\}. \tag{12}$$

The above results contain a divergence in the integral of y as the momentum fraction in unit of P^+. In the light-cone gauge the gluon propagator becomes:

$$\frac{i}{k^2+i\varepsilon}\left(-g^{\mu\nu}+\frac{n^\mu k^\nu+k^\mu n^\nu}{n\cdot k}\right). \tag{13}$$

The divergence comes from the part with $1/n\cdot k$. This divergence is the so-called light-cone divergence. It will be cancelled in the final results.

The calculation of the real part is straightforward. Adding the real part to the virtual part we have the evolutions of two-parton FF's at one-loop:

$$\begin{aligned}
\mu\frac{\partial\hat e(z,\mu)}{\partial\mu}&=\frac{z\alpha_s}{\pi}\int\frac{d\xi_1 d\xi_2}{\xi_1\xi_2}\Big\{\mathrm{Re}\hat E_F(x_1,x_2)\Big[-C_F\frac{2\xi_1\xi_2}{(1-\xi_2)_+(\xi_2-\xi_1)}+\frac{N_c}{2}\frac{\xi_1}{\xi_2-\xi_1}+\frac{1}{2N_c}\\
&\quad-\frac{\xi_1\delta(\xi_2-1)}{2(1-\xi_1)}\left(C_F+\frac{\ln\xi_1}{N_c(1-\xi_1)}\right)\Big]+\frac{C_F}{N_c}\mathrm{Re}\hat E_G(x_1,x_2)\frac{1-\xi_2}{1-\xi_2+\xi_1}\Big\},\\
\mu\frac{\partial\hat e_\partial(z,\mu)}{\partial\mu}&=\frac{\alpha_s C_F}{\pi}\left[\frac{3}{2}\hat e_\partial(z)+2\int_z^1\frac{d\xi}{\xi}\frac{1}{(1-\xi)_+}\hat e_\partial(x)\right]+\frac{\alpha_s}{\pi}\int\frac{d\xi_1 d\xi_2}{\xi_1\xi_2}\Big\{\mathrm{Im}\hat E_F(x_1,x_2)\\
&\quad\Big[\frac{N_c\xi_2}{2(\xi_2-\xi_1)^2}\left(-\xi_1-\frac{\xi_2(1-\xi_2)}{1-\xi_1}\right)-C_F\left(\xi_2+\frac{\xi_2}{\xi_1-\xi_2}\right)\Big]-\frac{C_F}{N_c}\mathrm{Im}\hat E_G(x_1,x_2)\frac{(1-\xi_2)^2}{1-\xi_2+\xi_1}\Big]\Big\},\\
\mu\frac{\partial\hat e_I(z,\mu)}{\partial\mu}&=\frac{\alpha_s}{\pi}\left\{3zC_F\hat e_\partial(z)+4zC_F\int_z^1\frac{d\xi}{\xi}\frac{1}{(1-\xi)_+}\hat e_\partial(x)\right\}+\frac{z\alpha_s}{\pi}\int\frac{d\xi_1 d\xi_2}{\xi_1\xi_2}\Big\{\mathrm{Im}\hat E_F(x_1,x_2)\\
&\quad\Big[C_F\frac{2\xi_1(\xi_2^2-\xi_1)}{(1-\xi_2)_+(\xi_2-\xi_1)^2}-\frac{1}{2N_c}\frac{\xi_2(\xi_1+\xi_2)}{(\xi_2-\xi_1)^2}+\frac{\xi_1\delta(\xi_2-1)}{2(1-\xi_1)}\left(C_F+\frac{\ln\xi_1}{N_c(1-\xi_1)}\right)\\
&\quad-C_F\frac{\xi_1}{\xi_2-\xi_1}\Big]-\frac{C_F}{N_c}\mathrm{Im}\hat E_G(x_1,x_2)\frac{1-\xi_2}{1-\xi_2+\xi_1}\Big\},
\end{aligned} \tag{14}$$

with the notation and the integration measure:

$$\xi=\frac{z}{x},\quad \xi_{1,2}=\frac{z}{x_{1,2}},\quad \int\frac{d\xi_1 d\xi_2}{\xi_1\xi_2}=\int_z^1\frac{d\xi_2}{\xi_2}\int_0^{\xi_2}\frac{d\xi_1}{\xi_1}=\int_z^1\frac{dx_2}{x_2}\int_{x_2}^\infty\frac{dx_1}{x_1}. \tag{15}$$

In the above results the light-cone singularities are cancelled. After the cancellation the standard +-distribution appears, which is defined as:

$$\int_0^1 dz\frac{\theta(z-x)}{(1-z)_+}f(z)=\int_x^1 dz\frac{f(z)-f(1)}{1-z}+f(1)\ln(1-x). \tag{16}$$

Now we turn to the evolution of three-parton FF's. There are contributions from two-parton FF's to the evolution given by Fig. 3. Interestingly, the sum of the contributions is zero. E.g., from Fig. 3a and Fig. 3e

$$\begin{aligned}
\mu\frac{\partial\hat E_F(z_1,z_2)}{\partial\mu}\bigg|_{3a}&=2iz_2\frac{\alpha_s C_F}{\pi}\frac{z_1-z_2}{z_1^2}\hat e_\partial(z_2),\\
\mu\frac{\partial\hat E_F(z_1,z_2)}{\partial\mu}\bigg|_{3e}&=-2iz_2\frac{\alpha_s C_F}{2\pi}\frac{z_1-z_2}{z_1^2}\hat e_\partial(z_2).
\end{aligned} \tag{17}$$

The sum of the two diagrams is zero. The cancellation for other diagrams happens in a similar way. Therefore, the evolution of $\hat E_{F,G}$ only involves three-parton FF's. There is no mixing with two-parton FF's at one-loop.

The virtual part of the evolutions of $\hat E_{F,G}$ is simple. It is from the quark- or gluon self-energy correction, and the μ-dependence of g_s in the definitions of $\hat E_{F,G}$. We have:

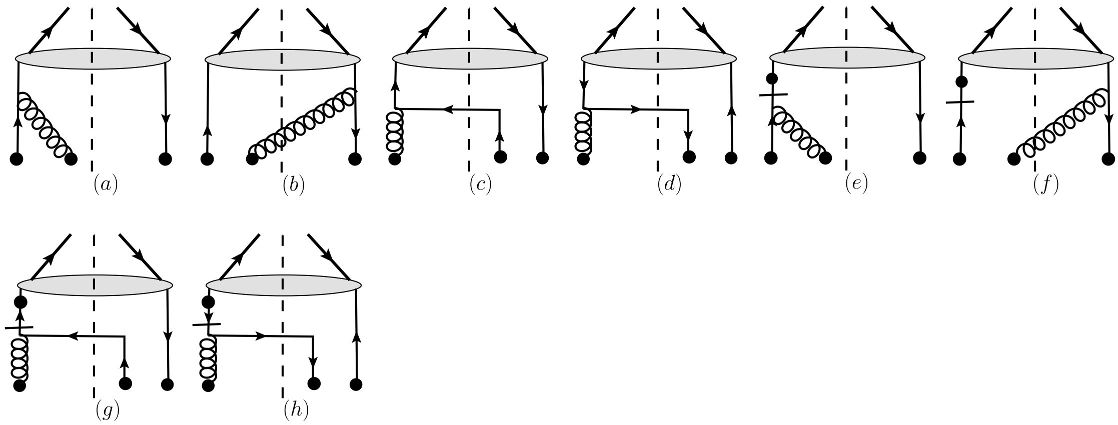

Fig. 3. Diagrams for the evolution of three-parton FF's.

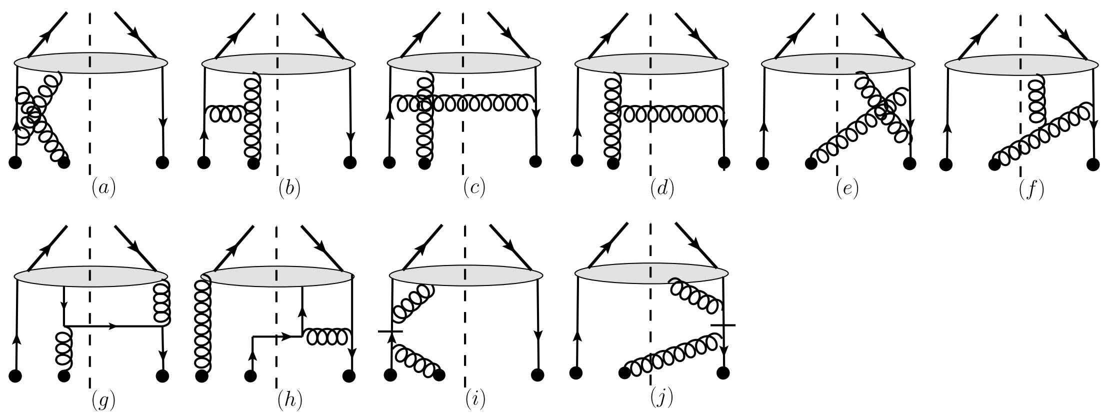

Fig. 4. Diagrams for the $\hat{E}_F$-evolution.

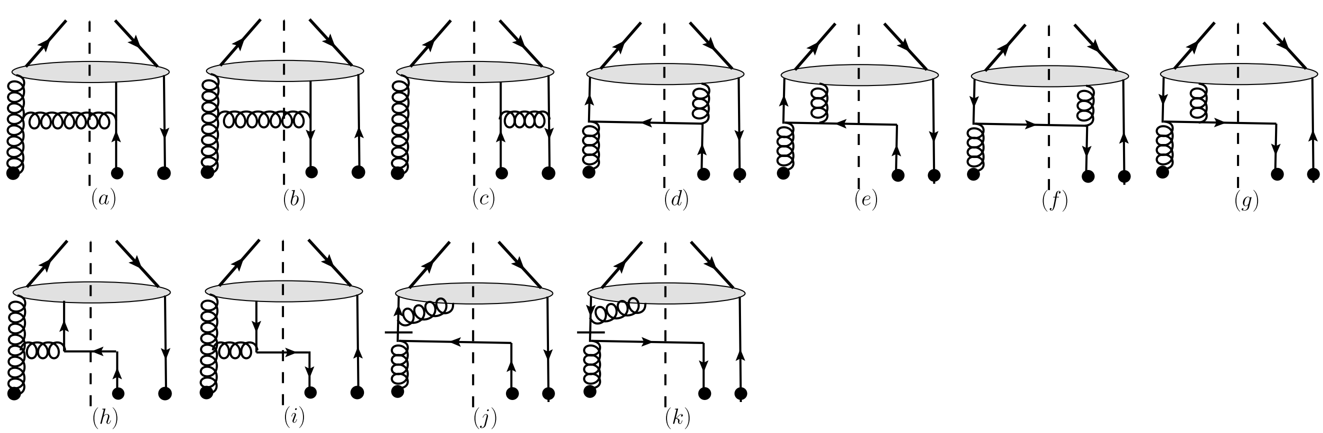

Fig. 5. Diagrams for the evolution of $\hat{E}_G$.

$$\mu \frac{\partial \hat{E}_F(z_1,z_2)}{\partial \mu}\bigg|_{vir.} = \frac{\alpha_s}{\pi}\left[C_F\left(\frac{3}{2}+\ln z_1 z_2 - 2\int_0^1 \frac{dy}{y}\right) + N_c\left(\ln z_3 - \int_0^1 \frac{dy}{y}\right)\right]\hat{E}_F(z_1,z_2),$$
$$\mu \frac{\partial \hat{E}_G(z_1,z_2)}{\partial \mu}\bigg|_{vir.} = \frac{\alpha_s}{\pi}\left[C_F\left(\frac{3}{2}+\ln z_1 z_3 - 2\int_0^1 \frac{dy}{y}\right) + N_c\left(\ln z_2 - \int_0^1 \frac{dy}{y}\right)\right]\hat{E}_G(z_1,z_2). \tag{18}$$

Again, in this part there are light-cone singularities.

The real part of the evolution of $\hat{E}_F$ or $\hat{E}_G$ is from Fig. 4 or Fig. 5, respectively. The calculations of these diagrams are straightforward. Adding the real part, the light-cone singularity in the virtual part will be cancelled. We introduce the following notations for giving our results:

$$\frac{1}{x_{13}}=\frac{1}{x_1}+\frac{1}{z_3},\quad \frac{1}{x_{11}}=\frac{1}{x_1}+\frac{1}{z_1},\quad \xi_1=\frac{z_1}{x_1},\quad \xi_2=\frac{z_2}{x_2}. \tag{19}$$

The evolution of $\hat{E}_F$ reads:

$$\begin{aligned}
\mu\frac{\partial \hat{E}_F(z_1,z_2)}{\partial\mu} &= \frac{\alpha_s}{\pi}\Bigg\{\left(C_F\left(\frac{3}{2}+\ln\frac{z_2}{z_1}\right)+N_c\ln\frac{z_3}{z_2}\right)\hat{E}_F(z_1,z_2)+N_c\int_{z_2/z_1}^{1}\frac{d\xi_2}{(1-\xi_2)_+}\hat{E}_F(z_1,x_2)\\
&+\int_0^1\frac{d\xi_1}{(1-\xi_1)_+}\left(N_c\hat{E}_F(x_1,z_2)-\frac{1}{N_c}\hat{E}_F(x_1,x_{13})\right)-\frac{N_c}{2}\int_0^{z_1/z_2}\frac{d\xi_1}{1-\xi_1}\hat{E}_F(x_1,z_2)\\
&+\frac{1}{z_3}\int\frac{dx_1}{x_1}\Bigg\{\hat{E}_F(x_1,z_2)\Bigg[\frac{1}{2N_c}\frac{z_2^2(x_1-z_3)}{z_1(x_1-z_2)}\left(\theta(z_3-x_1)\frac{z_1}{z_3}+\theta(x_1-z_3)\frac{z_2}{x_1-z_2}\right)\\
&+\frac{N_c}{2}\left(\theta(x_1-z_1)\frac{z_2^2(-x_1z_2-x_1z_3+z_2z_3)}{z_1(x_1-z_2)^2}-\theta(z_1-x_1)\frac{z_2^2(x_1-z_3)}{z_3(x_1-z_2)}\right)\\
&-C_F\frac{z_2^3}{z_1(x_1-z_1)}\Bigg]-\frac{1}{2N_c}\hat{E}_F(x_1,x_{13})z_2\theta(x_1-z_1)\\
&+\frac{1}{2}\hat{E}_F^*(x_1,z_1)\frac{z_1z_2}{z_3(x_1-z_1)}\left[2C_F(x_1+z_3)+N_c\frac{x_1^3}{(x_1-z_2)(x_1-z_1)}\right]\Bigg\}\\
&+\frac{N_c}{2}\int\frac{dx_2}{x_2}\hat{E}_F(z_1,x_2)\theta(x_2-z_2)\left(-\frac{z_2}{x_2}+\frac{z_1(z_1x_2+z_3(z_1-x_2))}{z_3(x_2-z_1)^2}\right)\\
&-\frac{C_F}{N_c}\int\frac{dx_1}{x_1}\left[\hat{E}_G^*(x_1,x_{11})\frac{(x_{11}-z_2)^2}{x_{11}z_2}\theta(x_1-z_3)+\hat{E}_G(x_1,z_3)\frac{x_1z_2}{z_3(x_1+z_1)}\right]\Bigg\}.
\end{aligned}\tag{20}$$

The principal description of the last integral in the second line in Eq. (20) is implied. The evolution of $\hat{E}_F$ involves not only $\hat{E}_F$ and $\hat{E}_G$, but also their complex conjugate. It is noted that under the one-loop evolution of Re$\hat{E}_F$ or Im$\hat{E}_F$ will not be mixed with the imaginary- or real parts of three-parton FF's, respectively. The individual evolution of the real- and imaginary part of $\hat{E}_F$ can be read from the above equation directly.

Summing the contributions from Fig. 5 and the virtual part in Eq. (18), we obtain the evolution of $\hat{E}_G$:

$$\begin{aligned}
\mu\frac{\partial \hat{E}_G(z_1,z_2)}{\partial\mu} &= \frac{\alpha_s}{\pi}\Bigg\{C_F\left(\frac{3}{2}+\ln\frac{z_3}{z_1}\right)\hat{E}_G(z_1,z_2)+N_c\int_{z_2/z_1}^{1}\frac{d\xi_2}{(1-\xi_2)_+}\hat{E}_G(z_1,x_2)\\
&+\int_0^1\frac{d\xi_1}{(1-\xi_1)_+}\left(N_c\hat{E}_G(x_1,x_{13})-\frac{1}{N_c}\hat{E}_G(x_1,z_2)\right)+\frac{1}{2N_c}\int_0^{z_1/z_2}\frac{d\xi_1}{1-\xi_1}\hat{E}_G(x_1,z_2)\\
&+\frac{N_c}{2}\int_{z_1}^{\infty}\frac{dx_1}{x_1}\hat{E}_G(x_1,x_{13})\frac{x_1^2z_1+x_1z_2^2-z_2^2z_1}{x_1(x_1(z_1-z_2)+z_1z_2)}+\frac{1}{N_c}\int\frac{dx_1}{x_1}\hat{E}_G(x_1,z_2)\Bigg[\theta(z_1-x_1)\\
&+\theta(x_1-z_1)\frac{z_1}{x_1-z_2}\Bigg]-\frac{N_c}{2}\int_{z_2}^{z_1}\frac{dx_2}{x_2}\hat{E}_G(z_1,x_2)\left[\frac{z_1^2}{z_2(x_2-z_1)}+\frac{x_2(z_1-z_2)+z_2^2}{x_2z_2}\right]\\
&-\frac{1}{2(N_c^2-1)}\Bigg[\int_{z_2}^{z_3}\frac{dx_2}{x_2}\left(\hat{E}_F^*(z_3,x_2)+\hat{E}_{\bar{F}}^*(z_3,x_2)\right)\frac{z_3^2(x_2-z_2)^2}{x_2z_2(z_3-x_2)^2}\\
&+\int\frac{dx_1}{x_1}\left(\hat{E}_F(x_1,z_3)+\hat{E}_{\bar{F}}(x_1,z_3)\right)\left(\frac{z_3^2(z_3+z_2-2x_1)}{z_1(x_1-z_3)^2}-N_c^2\frac{x_1z_1z_3+z_2^2(2z_1+z_3)}{z_1z_2(x_1+z_1)}\right)\Bigg]\Bigg\}.
\end{aligned}\tag{21}$$

Here, the evolution involves $\hat{E}_{\bar{F}}$. The evolution of $\hat{E}_{\bar{F}}$ can be derived from Eq. (20) with the symmetry of charge-conjugation. The results in Eq. (14) and Eqs. (20), (21) are our main results. These results satisfy the relations in Eq. (4). This is an important check of our results.

Our results in Eqs. (20), (21) and the evolution of $\hat{E}_{\bar{F}}$ form a closed system of differential-integral equations. The system contains six equations because $\hat{E}_{F,\bar{F},G}$ are complex functions. In the large-N_c limit, the system is simplified. Because of the normalization factor of color in the FF's definitions of Eq. (2), one can expect that all 3-parton FF's are at the same order of N_c in the limit of $N_c\to\infty$. With this expectation one can easily find the evolutions in the limit from Eq. (14) and Eq. (20). E.g., in the limit, the evolution of $\hat{E}_F$ becomes:

$$\mu \frac{\partial \hat{E}_F(z_1,z_2)}{\partial \mu} = \frac{\alpha_s N_c}{2\pi} \Bigg\{ \left(\frac{3}{2} + \ln \frac{z_3^2}{z_1 z_2} \right) \hat{E}_F(z_1,z_2) + 2 \int_{z_2/z_1}^{1} \frac{d\xi_2}{(1-\xi_2)_+} \hat{E}_F(z_1,x_2)$$
$$+2 \int_0^1 \frac{d\xi_1}{(1-\xi_1)_+} \hat{E}_F(x_1,z_2) - \int_0^{z_1/z_2} \frac{d\xi_1}{1-\xi_1} \hat{E}_F(x_1,z_2)$$
$$+\frac{1}{z_3} \int \frac{dx_1}{x_1} \Bigg\{ \hat{E}_F(x_1,z_2) \Bigg[\Bigg(\theta(x_1-z_1) \frac{z_2^2(-x_1 z_2 - x_1 z_3 + z_2 z_3)}{z_1(x_1-z_2)^2}$$
$$-\theta(z_1-x_1) \frac{z_2^2(x_1-z_3)}{z_3(x_1-z_2)} \Bigg) - \frac{z_2^3}{z_1(x_1-z_1)} \Bigg]$$
$$+\hat{E}_F^*(x_1,z_1) \frac{z_1 z_2}{z_3(x_1-z_1)} \left[(x_1+z_3) + \frac{x_1^3}{(x_1-z_2)(x_1-z_1)} \right] \Bigg\}$$
$$+ \int \frac{dx_2}{x_2} \hat{E}_F(z_1,x_2) \theta(x_2-z_2) \left(-\frac{z_2}{x_2} + \frac{z_1(z_1 x_2 + z_3(z_1-x_2))}{z_3(x_2-z_1)^2} \right) \Bigg\} + \mathcal{O}(N_c^0), \tag{22}$$

from the above and the result for $\hat{E}_G$ we realize that there is no mixing between three-parton FF's under the evolution in the large-N_c limit. The system become six independent homogeneous equations. This observation has been first made in [6] for the real parts of three-parton FF's. From the results in Eq. (14) two-parton FF's do not be mixed with $\hat{E}_G$ in the large-N_c limit.

Our results about the evolution of $\hat{e}_I$ and imaginary parts of $\hat{E}_{F,\bar{F},G}$ are new. The evolution of $\hat{e}_\partial$ has been derived in [7]. The result agrees with our result of $\hat{e}_\partial$, except the contribution from Im$\hat{E}_G$, which is missing in [7]. The evolution of $\hat{e}$ and real parts of $\hat{E}_{F,\bar{F},G}$ has been first given in [6]. But, we are unable to find a complete agreement with our results.

To summarize: We have derived the evolutions of all chirality-odd twist-3 FF's at one-loop level. These evolutions satisfy the constraints from QCD equation of motion. From our results, the three-parton FF's will only be mixed with themselves under the evolutions, but not with two-parton FF's. In the large-N_c limit, the mixing disappears and the evolutions of three-parton FF's are governed by six homogeneous equations. With our results combined with evolutions existing in literature, the Q-dependence of single transverse-spin asymmetries can be now predicted completely.

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