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Entanglement Concentration with Different Measurement in a 3-Mode Optomechanical System

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Abstract In this work, we perform a series of phonon counting measurement with different methods in a 3-mode optomechanical system, and we compare the difference of the entanglement after measurement. In this article we focus on the three cases: prefect measurement, imperfect measurement and on-off measurement. We find that whatever measurement you take, the entanglement will increase. The size of entanglement enhancement is the largest in the perfect measurement, second in the imperfect measurement, and it is not obvious in the on-off measurement. We are sure that the more precise measurement information, the larger entanglement concentration.

1 Introduction

In recent years, quantum entanglement[1] has been regarded as a key source in the quantum information processing, for it can apply in terms of quantum cryptography,[2−3] allow the realization of quantum teleportation[4−5] and quantum dense coding.[6] A number of strategies to generate entanglement have been developed in different quantum systems, such as trapped ions,[7] cold atoms,[8] and solid-state qubit.[9] The conventional methods for entanglement photons rely on nonlinear optical process like parameter amplification and second harmonic generation. However, the photons with vastly different frequencies, i.e., microwave photons and optical photons, can not be entangled directly in this way. Nevertheless, optomechanical system[10] provides probability to work out this difficulty and some related work have already been done.[11−18] An optomechanical system suitable to this purpose is based on an optical cavity and a microwave cavity interacting with a mechanical element and such a 3-mode optomechanical system has been realized experimentally.[19−20]

Entanglement will be severely degraded by the channel noise due to it's fragile nature. In order to overcome that decoherence effect, entanglement concentration or entanglement distillation will be utilized. The idea of the standard entanglement concentration is to exact a smaller number of elements with higher entanglement by distillation from a large number of elements with lower entanglement through local operations and classical communication. From 1996 when Bennett *et al.*[21] proposed entanglement concentration

protocol firstly to the present, the various entanglement concentration protocol for discrete-variable[22−23] and continuous-variable[24−25] quantum system have been developed as well as entanglement concentrations have been demonstrated experimentally.[26−27] However, distilling continuous-variable entanglement appears to be significantly harder to achieve than distilling discrete-variable, for one can not distill a Gaussian state by using only Gaussian operations.[25,28−29] Thus non-Gaussian operations, in particular photon counting measurement,[30] are indispensable for Gaussian entangled states distillation. The photon subtraction strategy, one of the available experimental operations beyond the Gaussian regime, is based on this idea. And the non-Gaussian operation with photon counting measurement can be implemented by beam splitters.[31−33]

These ideas motivate us to explore an entanglement concentration protocol based on phonon counting measurement for 3-mode optomechanical system (Fig. 1). In this system, a genuine tripartite entanglement state, where the two cavity output mode and the mechanical output mode are entangled with each other, can be generated.[18] We perform the phonon counting measurement in the mechanical mode (indirectly through auxiliary photon counting) for the genuine tripartite entanglement state with different methods. In previous work,[34] the perfect measurement, i.e., projective measurement, have been considered, but in practice it is difficult to find a measurement device which completely satisfy projective measurement. In this paper, we mainly focus on and get the general result with imperfect measurement and on-off mea-

surement, which is available experimentally at present. While the amount of entanglement after measurement is measured in terms of logarithmic negativity.[35] Numerical result and analytical result show that: (i) Whatever measure you take, the entanglement will increase; (ii) The entanglement enhancement is largest in perfect measurement, while smaller enhancement in imperfect measurement, and it is not obvious in on-off measurement; (iii) We are sure that the more precise measurement information, the larger entanglement concentration.

The remainder of this paper is organized as follows. In Sec. 2, we introduce the physical system and derive the amount of entanglement before concentration, along with the generating of a genuine tripartite entanglement state. In Sec. 3, the definition of logarithmic negativity is briefly summarized and the amount of entanglement after perfect measurement is introduced. Section 4 is devoted to the entanglement distillation with imperfect measurement. We calculate the amount of entanglement after imperfect measurement perturbative order by order and compare the entanglement concentration effect analytically. In Sec. 5, we discuss the on-off measurement and derive the average entanglement after on-off measurement. Finally, we conclude with a discussion and summary about three different measurement strategies numerically and analytically in Sec. 6.

2 Physical System and Measurement Operator

We consider a three-mode optomechanical system: two cavity modes (ω_1 and ω_2) are coupled to a single mechanical mode ω_m (see Fig. 1). The cavities interact with the mechanics via the radiation pressure.[10] The Hamiltonian of the system can be described by

$$\hat{H} = \omega_m \hat{b}^\dagger \hat{b} + \sum_{j=1,2} [\omega_j \hat{a}_j^\dagger \hat{a}_j + g_j(\hat{b}^\dagger + \hat{b})\hat{a}_j^\dagger \hat{a}_j], \quad (1)$$

where $\hat{a}_j$ and $\hat{b}$ are the annihilation operator for cavity $j (j = 1, 2)$ and the mechanical mode respectively. The optomechanical coupling strengths are denoted by g_j. In order to generate steady state entanglement, we assume a strong coherent drive on each cavity, detuned to the red (blue) mechanical sideband for cavity 1(2), i.e., drive frequency $\omega_{d1} = \omega_1 - \omega_m$, $\omega_{d2} = \omega_2 + \omega_m$. We work in an interaction picture with respect to free Hamiltonian and split the cavity field into a classical cavity amplitude $\bar{a}_j$ and a small quantum fluctuation $\hat{d}_j$ with $\hat{d}_j = \hat{a}_j - \bar{a}_j$. Here $\bar{a}_j$ is the average number of photons for each cavity. After performing the rotating wave approximation and linearizing the Hamiltonian independently, we obtain:

$$\hat{H}_{\text{int}} = (G_1 \hat{b}^\dagger \hat{d}_1 + G_2 \hat{b} \hat{d}_2) + \text{h.c.} \quad (2)$$

Here $G_j = g_j \bar{a}_j$ is the dressed coupling. In general, we take $g_j, \bar{a}_j > 0$. Taking the damping and the noise terms

into account, we get the Langevin equation for the optical and mechanical modes operator:[36]

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{b} = -\frac{\gamma}{2}\hat{b} - \mathrm{i}(G_1 \hat{d}_1 + G_2 \hat{d}_2^\dagger) - \sqrt{\gamma}\hat{b}^{\text{in}}(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{d}_1 = -\frac{\kappa_1}{2}\hat{d}_1 - \mathrm{i}G_1 \hat{b} - \sqrt{\kappa_1}\hat{d}_1^{\text{in}}(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{d}_2^\dagger = -\frac{\kappa_2}{2}\hat{d}_2^\dagger + \mathrm{i}G_2 \hat{b} - \sqrt{\kappa_2}\hat{d}_2^{\text{in},\dagger}(t), \quad (3)$$

where κ_j and γ are the damping rate of the cavities and mechanics respectively and i is the imaginary unit. As discussed in Ref. [36], from the Langevin equations and the input-output relation, one can verify that the stationary output state in the Fock state basis $|n_1, n_2, n_m\rangle$, can be expressed as:

$$|\Psi\rangle = \sum_{p,q} \frac{\sqrt{C_{p+q}^p} N_m^{q/2} N_1^{p/2}}{(1 + N_2)^{(p+q+1)/2}} |p, p+q, q\rangle. \quad (4)$$

In Eq. (4), C_{p+q}^p is the binomial coefficients and N_k ($k = 1, 2, m$) is the average photons or phonons number of each output mode with $N_k = \langle(\hat{d}_k^{\text{out}}[0])^\dagger \hat{d}_k^{\text{out}}[0]\rangle$ and they can be given as follow:

$$N_1 = \frac{4C_1 C_2}{(1 + C_1 - C_2)^2}, \quad (5)$$

$$N_2 = \frac{4C_2(C_1 + 1)}{(1 + C_1 - C_2)^2}, \quad (6)$$

$$N_m = \frac{4C_2}{(1 + C_1 - C_2)^2}, \quad (7)$$

with the cooperativity $C_j = 4G_j^2/(\gamma\kappa_j)$. The result was derived under the assumptions of zero temperature and in the limit of narrow bandwidth around the bare frequencies ω_j, ω_m. Note that this is a Gaussian state (more specifically), it is a twice squeezed 3-mode vacuum state,[18] which is a genuine tripartite entangled state. By tracing out the mechanical mode, one obtains a 2-mode squeezed thermal state of the photon output fields, which has entanglement:

$$E_N = \ln \frac{(1 + C_1 - C_2)^2}{A + B + 2C_2(1 + 2C_1) - 4\sqrt{AB}}, \quad (8)$$

with

$$A = C_2(C_1 + C_2), \quad B = (1 + C_1)^2 + C_1 C_2.$$

The entanglement is maximized at the instability point $C_1 \to C_2 - 1$ and remains finite ($E_N \to \ln(2C_1 + 1)$), at variance with the well-known divergence for a parametric amplifier. This is a natural result because the two modes are entangled with the mechanics. Indeed, the divergence is manifested only for the tripartite entanglement.[18] However, as we discuss in the following, a divergence of E_N can be recovered by an ideal measurement. In this sense, the large entanglement of the three-body state is a physical resource which can be used to greatly enhance the bipartite entanglement of the emitted phonons.

In the practice, the mechanics can be connected to a strong damped auxiliary cavity (cf. Fig. 1) such that the mechanical output can be mapped to the optical output, $\hat{d}_{a,\text{out}} = -i\hat{b}_{\text{out}}$.[18] A recent experiment has demonstrated the readout of the phonon number through this mechanism.[37] So in the following text, the measurement of phonon of the mechanical mode is through the measurement of photon of the auxiliary cavity mode. We will simply refer to this method as "measurement of the phonon mode" and quantify its effect on the output entanglement of the two cavities.
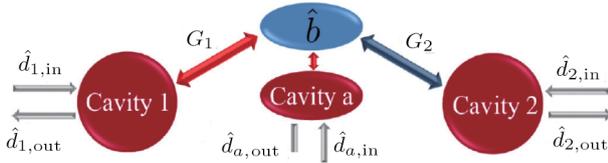

Fig. 1 System schematics. Two driven cavities (cavities 1 and 2) interact with a common mechanical resonator (mode $\hat{b}$). This can generate entanglement in the optical outputs. An auxiliary third cavity (cavity a) can be used to cavity cool the mechanics and to make the mechanical output mode accessible.

In the measurement theory of quantum mechanics, projection operator is a perfect measurement operator

$$\hat{M}_1(q) = |q\rangle\langle q| \,, \qquad (9)$$

but in experiment we often deal with imperfect measurement. A typical imperfect measurement is efficient measurement that the detect efficiency μ is considered.[38] For a single photon detector, the detect efficiency μ can be regarded as the probability for detecting one photon in time t from a one photon field. The explicit form of μ depends on the physical situation, here we just consider a constant value of μ. According to Ref. [38], the operator of measurement with measure outcome q is

$$\hat{M}_\mu(q) = \sum_{n=q}^{\infty} \sqrt{C_n^q}(1-\mu)^{(n-q)/2}\mu^{q/2}|n-q\rangle\langle q| \,. \qquad (10)$$

We find that the imperfect measurement becomes perfect measurement when $\mu = 1$. Another measurement may be on-off measurement. The on case can be interpreted as: we detect photon, but we can not identify the photon number. The off case is that we have not detected photon. In physics, they can be expressed as:[39]

$$\hat{M}_{\text{off}} = |0\rangle\langle 0| \,, \qquad (11)$$

$$\hat{M}_{\text{on}} = \mathbf{I} - |0\rangle\langle 0| = \sum_{k=1}^{\infty} |k\rangle\langle k| \,. \qquad (12)$$

3 Perfect Measurement

We first consider the perfect measurement of the phonon number, described by projection operator $\hat{M}_1(q)$ where q is the outcome of the phonon measurement. Such

measurement increases the entanglement,[34] as it can be computed straightforwardly from the state after measurement:

$$|\Psi_q\rangle = P_q^{-1/2}\hat{M}_1(q)|\Psi\rangle = \sum_{p=0}^{\infty} \sqrt{f_p(q)}|p, p+q\rangle \,. \qquad (13)$$

Here we define

$$f_p(q) = C_{p+q}^p \zeta^p (1-\zeta)^{1+q} \,, \qquad (14)$$

where $\zeta = 4C_1C_2(1+C_1+C_2)^{-2}$, The normalization factor is

$$P_q = \langle\Psi|\hat{M}_1^\dagger(q)\hat{M}_1(q)|\Psi\rangle = \frac{N_{\boldsymbol{m}}^q}{(1+N_{\boldsymbol{m}})^{1+q}} \,. \qquad (15)$$

Although Eq. (13) is not a Gaussian state, we can still quantify the entanglement directly from the definition of logarithmic negativity:[35]

$$E_N \equiv \ln\|\hat{\rho}^{\text{T}}\|_{\mathbf{1}} \,, \qquad (16)$$

where $\hat{\rho}$ is the density matrix of the state being evaluated, $\hat{\rho}^{\text{T}}$ is the partial transpose with respect to one subsystem, $\|.\|_{\mathbf{1}}$ denotes trace norm. Also noticing that for a two mode entangled state written in a Schmidt decompostion as

$$|\varphi\rangle = \sum_n C_n|n_{\boldsymbol{A}}, n_{\boldsymbol{B}}\rangle \,,$$

the entanglement is:

$$E_N = 2\ln\sum_n |C_n| \,.$$

Thus the entanglement of the state in Eq. (13) can be written as:

$$E_N(q) = 2\ln\sum_{p=0}^{\infty} \sqrt{f_p(q)} \,. \qquad (17)$$

A special case is $q = 0$, it means that no phonon has been detected. In this case,

$$E_N(0) = \ln\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}} \,. \qquad (18)$$

To get an analytical expression of the entanglement after measure, an approximation is necessary. Notice that $f_p(q)$ is normalized and it can be regarded as a Gaussian distribution for large q:

$$f_p(q) \approx \frac{1}{\sqrt{2\pi}\sigma(q)} \exp\left[-\frac{[p-\kappa(q)]^2}{2\sigma(q)^2}\right] \,, \qquad (19)$$

with the mean and variance being $\kappa(q) = \zeta(1+q)/(1-\zeta)$, $\sigma(q) = \sqrt{\zeta(1+q)}/(1-\zeta)$. Then:

$$E_N(q) \approx \ln\frac{\sqrt{8\pi\zeta(1+q)}}{1-\zeta} \,. \qquad (20)$$

The entanglement after measurement is found to increase logarithmically with the number of detected phonons (q), and it is larger than the entanglement before measurement, even the measurement outcome is zero (see Fig. 2). It can be found that measurement enhances entanglement.

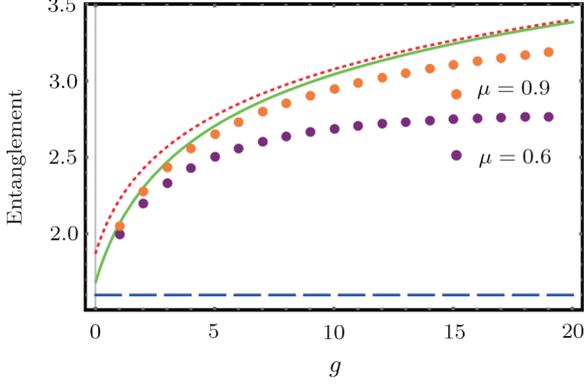This is one of the most important conclusion in the perfect measurement.

Fig. 2 (Color online) Entanglement versus detected phonons(q). Entanglement of the three mode state before measurement E_N (blue dashed line), entanglement after the perfect measurement $E_N(q)$ (green line), entanglement with Gaussian approximation after the perfect measurement (red dotted line), entanglement after the imperfect measurement with $\mu = 0.6$ (purple dots) and entanglement after the imperfect measurement with $\mu = 0.9$ (orange dots). It can be seen even the measurement outcome is zero, $E_N(q)$ is larger than E_N. The entanglement are concentrated. The parameters values are $C_1 = 10$, $C_2 = 2$.

4 Imperfect Measurement

Now we consider the imperfect measurement with the measurement operator $\hat{M}_\mu(q)$. According to Ref. [38], the state after measurement is given by

$$\hat{\rho}_q = \frac{1}{P_\mu(q)} \operatorname{Tr}_q[\hat{M}_\mu(q)\hat{\rho}\hat{M}_\mu^\dagger(q)], \qquad (21)$$

here the trace is for detected mode, and $P_\mu(q)$ is the probability for detecting q phonons from a field with the phonon number distribution P_s:

$$P_\mu(q) = \sum_{s=q}^{\infty} P_s C_s^q \mu^q (1-\mu)^{s-q}. \qquad (22)$$

For the given three mode entangled state $|\Psi\rangle$, when we consider the detect efficiency μ, the state after the imperfect measurement with the detect outcome q is:

$$\hat{\rho}_q = \sum_{s=q}^{\infty} \eta(s)|\Psi_s\rangle\langle\Psi_s|$$
$$= \sum_{p_1=0}^{\infty} \sum_{p_2=0}^{\infty} \sum_{s=q}^{\infty} \sqrt{f_{p_1}(s)f_{p_2}(s)}\,\eta(s)$$
$$\times |p_1, p_1+s\rangle\langle p_2, p_2+s|, \qquad (23)$$

with

$$\eta(s) = \frac{N_{\boldsymbol{m}}^{s-q}(1+N_{\boldsymbol{m}}\mu)^{1+q}}{(1+N_{\boldsymbol{m}})^{1+s}} C_s^q (1-\mu)^{s-q}. \qquad (24)$$

Obvious, $\hat{\rho}_q$ is a mixed state. From Ref. [35], calculating logarithmic entanglement is to find the negativity $N(\hat{\rho}_q^T)$

with

$$\hat{\rho}_q^T = \sum_{p_1=0}^{\infty} \sum_{p_2=0}^{\infty} \sum_{s=q}^{\infty} \sqrt{f_{p_1}(s)f_{p_2}(s)}\,\eta(s)$$
$$\times |p_1, p_2+s\rangle\langle p_2, p_1+s|. \qquad (25)$$

The $\hat{\rho}_q^T$ is a block diagonal matrix by identifying the different total photons $Q = p_1+p_2+s$ for each subblock. When $Q = q$, there is only one matrix element $Q[q] = f_0(q)\eta(q)$ in the subblock matrix. When $Q = q + 1$, it is a 2×2 square matrix with $Q[q + 1]$:

$$Q[q+1] = \begin{bmatrix} \sqrt{f_1(q)f_0(q)}\eta(q) & 0 \\ f_0(q+1)\eta(q+1) & \sqrt{f_0(q)f_1(q)}\eta(q) \end{bmatrix}. \qquad (26)$$

When $Q = q + n$, the sub block matrix $Q[q + n]$ is an $n \times n$ square matrix. Each sub block matrix of $\hat{\rho}_q^T$ is a lower triangular matrix. To calculate $N(\hat{\rho}_q^T)$, we make an approximation. By throwing away most elements and only remaining the main diagonal and the elements which are the nearest to the main diagonal for each sub blocks, then the expression of $\hat{\rho}_q^T$ becomes

$$\hat{\rho}_q^T = \sum_{p_1=0}^{\infty} \sum_{p_2=0}^{\infty} \sum_{s=q}^{q+1} \sqrt{f_{p_1}(s)f_{p_2}(s)}\,\eta(s)$$
$$\times |p_1, p_2+s\rangle\langle p_2, p_1+s|. \qquad (27)$$

Then calculating the entanglement after the imperfect measurement is calculating the eigenvalues of $\hat{\rho}_q^T$. To calculate the eigenvalues of $\hat{\rho}_q^T$, we thought about using perturbation theory. In quantum mechanics, the classical non degenerate stationary state perturbation theory is that for a given $\hat{H}$, we can devide $\hat{H}$ into two part: $\hat{H} = \hat{H}_0 + \hat{H}'$ ($\hat{H}'$ is a perturbation). Here, we just treat $\hat{\rho}_q^T$ as $\hat{H}$ and let $\hat{\rho}_q^T = \hat{H} = \hat{H}_0 + \hat{H}'$ with

$$\hat{H}_0 = \sum_{p_1=0}^{\infty} \sum_{p_2=0}^{\infty} \sqrt{f_{p_1}(q)f_{p_2}(q)}\,\eta(q)$$
$$\times |p_1, p_2+q\rangle\langle p_2, p_1+q|, \qquad (28)$$
$$\hat{H}' = \sum_{p_1=0}^{\infty} \sum_{p_2=0}^{\infty} \sqrt{f_{p_1}(q+1)f_{p_2}(q+1)}\,\eta(q+1)$$
$$\times |p_1, p_2+q+1\rangle\langle p_2, p_1+q+1|. \qquad (29)$$

Also for a classical perturbation theory, the eigen equation of $\hat{H}_0$ is $\hat{H}_0|m\rangle = e_m^{(0)}|m\rangle$, where $e_m^{(0)}$ is eigenvalue in the eigenstate $|m\rangle$. The first-order approximate eigenvalue of $\hat{H}$ in a state $|\phi_m\rangle$ (which is close to $|m\rangle$):

$$e_m^{(1)} = \langle m|\hat{H}'|m\rangle, \qquad (30)$$

and the second-order approximate eigenvalue of $\hat{H}$ in a state $|\phi_m\rangle$:

$$e_m^{(2)} = \sum_{k\neq m} \frac{[\langle m|\hat{H}'|k\rangle]^2}{e_m^{(0)} - e_k^{(0)}}. \qquad (31)$$

Then we have the approximate eigenvalue of $\hat{H}$ in a state $|\phi_m\rangle$

$$e_m = e_m^{(0)} + e_m^{(1)} + e_m^{(2)}. \qquad (32)$$

For $\hat{\rho}_q^{\mathrm{T}}$, its eigenvectors and eigenvalues of $\hat{H}_0$ are

$$
\begin{array}{cccc}
\text{Condition} & \text{Eigenvectors} & \text{Eigenvalues} & \\
p_1 = p_2 = p & |\alpha\rangle & f_p(q)\eta(q) & \\
p_1 < p_2 & |\beta_+\rangle & +\sqrt{f_{p_1}(q)f_{p_2}(q)}\eta(q) & (33) \\
p_1 < p_2 & |\beta_-\rangle & -\sqrt{f_{p_1}(q)f_{p_2}(q)}\eta(q) &
\end{array}
$$

with $|\alpha\rangle = |p,\, p+q\rangle$,

$$
|\beta_\pm\rangle = \frac{|\, p_1, p_2+q\rangle \pm |\, p_2, p_1+q\rangle}{\sqrt{2}}\,.
$$

According to the definition of $N(\hat{\rho}_q^{\mathrm{T}})$, only the eigenvalue of the state $|\beta_-\rangle$ is important. Comparing to the classic formula of perturbation theory, the entanglement after imperfect measurement is:

$$
E_N^\mu(q) = \ln\left[1 + 2\sum_{\beta_-}|E_{\beta_-}|\right], \tag{34}
$$

with

$$
\begin{aligned}
E_{\beta_-} &= E_{\beta_-}^{(0)} + E_{\beta_-}^{(1)} + E_{\beta_-}^{(2)}\,, \\
E_{\beta_-}^{(0)} &= -\sqrt{f_{p_1}(q)f_{p_2}(q)}\,\eta(q)\,, \\
E_{\beta_-}^{(1)} &= \langle\beta_-|\hat{H}'|\beta_-\rangle\,, \\
E_{\beta_-}^{(2)} &= \sum_{m\neq\beta_-}\frac{[\langle\beta_-|\hat{H}'|m\rangle]^2}{E_{\beta_-}^{(0)} - E_m^{(0)}}\,.
\end{aligned} \tag{35}
$$

When only consider the first order, the entanglement can be expressed as:

$$
E_N^\mu(q) = E_N(q) - (q+1)\varepsilon\,, \tag{36}
$$

with $\varepsilon = (1-\mu)N_{\boldsymbol{m}}/(1+N_{\boldsymbol{m}})$. Considering the second order, it is:

$$
E_N^\mu(q) = E_N(q) + (q+1)\varepsilon\left[\frac{q\Omega + \Omega - 1}{2}\varepsilon - 1\right], \tag{37}
$$

with

$$
\Omega = \frac{\sum_{p_1=0}^{\infty}\sum_{p_2=0}^{\infty}g(p_1, p_2)}{(\sum_{p=0}^{\infty}\sqrt{f_p(q)})^2}\,, \tag{38}
$$

$$
g(p_1, p_2) = \frac{f_{p_2}(q+1)f_{p_1}(q+1)}{\sqrt{f_{p_1}(q)f_{p_2+1}(q)} + \sqrt{f_{p_1+1}(q)f_{p_2}(q)}}\,. \tag{39}
$$

In the Gaussian approximation, Ω can get a simple expression, and when q is large, Ω will tend to $1/2$,

$$
\Omega \approx \frac{1}{2}\sqrt{\frac{1+\zeta q}{\zeta + \zeta q}} \approx \frac{1}{2}\,. \tag{40}
$$

Then we get

$$
E_N^\mu(q) = E_N(q) - (q+1)\varepsilon + \frac{q^2-1}{4}\varepsilon^2\,. \tag{41}
$$

In the numerical analysis of the entanglement after the imperfect measurement (see Fig. 2), we can see that the imperfect measurement will be close to the perfect measurement when μ is close to 1. In the analytical analysis, the perturbation approximation is effective when $\mu \to 1$ and the detected phonon number q is small. It can be found that the entanglement after imperfect measurement

is larger than the entanglement before measurement, but smaller than the entanglement after perfect measurement (see Fig. 3). That is to say: perfect measurement is more effective than imperfect measurement in the entanglement concentration, and no matter adopt what kind of measurement method, the entanglement is concentred.
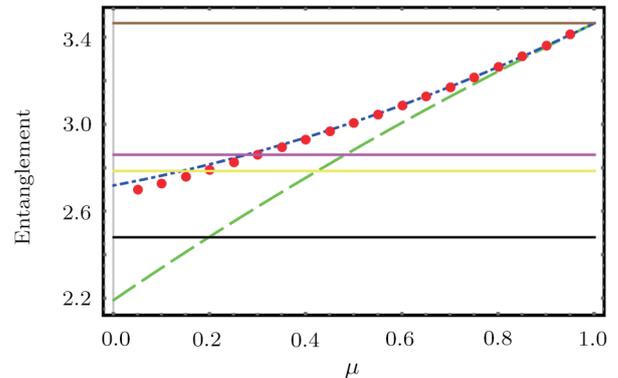

Fig. 3 (Color online) Entanglement versus detect efficiency μ. Entanglement after the perfect measurement $E_N(q)$ (brown line), the three mode state entanglement before measurement E_N (black line), entanglement after on measurement E_N^{on} (pink line), entanglement after off measurement E_N^{off} (yellow line), entanglement with numerical results after the imperfect measurement (red dot), the first order approximation of the entanglement after imperfect measurement (green dashed line) and the second order approximation of the entanglement after imperfect measurement (blue dotted line). It can be seen the perturbation approximation is effective and the imperfect measurement will be close to the perfect measurement when $\mu \to 1$. The parameters values are $C_1 = 10$, $C_2 = 5$, $q = 2$.

5 On-Off Measurement

Now we consider on-off measurement and the state after measure are

$$
\hat{\rho}_{\mathrm{off}} = |\Psi_0\rangle\langle\Psi_0|\,, \tag{42}
$$

$$
\hat{\rho}_{\mathrm{on}} = \sum_{k=1}^{\infty}\frac{1+N_{\boldsymbol{m}}}{N_{\boldsymbol{m}}}P_k|\Psi_k\rangle\langle\Psi_k|\,. \tag{43}
$$

For off case, the entanglement is the same with the perfect measurement when $q = 0$.

$$
E_N^{\mathrm{off}} = E_N(0) = \ln\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}}\,. \tag{44}
$$

For on case, the logarithmic negativity theory still works. We can calculate the $N(\hat{\rho}_{\mathrm{on}}^{\mathrm{T}})$. Then we get $E_N^{\mathrm{on}} = \ln(1 + 2N(\hat{\rho}_{\mathrm{on}}^{\mathrm{T}}))$. In this way it is easy to get a precise numerical solution but difficult to find an analytical expression. Fortunately we find that when make the suitable parameters ($C_1 \gg C_2$), the average entanglement is close to the numerical on measurement entanglement (see Fig. 4)

$$
E_N^{\mathrm{on}} \approx \bar{E}_N^{\mathrm{on}}\,, \tag{45}
$$

and the average entanglement is defined as:

$$\bar{E}_N^{\rm on} = \frac{1+N_m}{N_m}\sum_{k=1}^{\infty} P_k E_N(k)\,. \tag{46}$$

If use Gaussian approximation:

$$\bar{E}_N^{\rm on} \approx \sum_{k=1}^{\infty} \frac{1}{N_m}\Big(\frac{N_m}{1+N_m}\Big)^k \ln\frac{\sqrt{8\pi\zeta(1+k)}}{1-\zeta}\,. \tag{47}$$

We find even in the on-off measurement, the entanglement is concentred, but the effect is weakest comparing to the perfect measurement and the imperfect measurement (see Fig. 3).
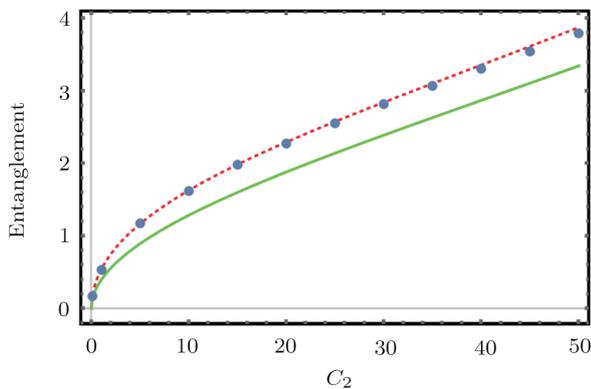

Fig. 4 (Color online) Entanglement versus cooperativities C_2. The numerical result for on measurement entanglement $E_N^{\rm on}$ (blue dots), the average entanglement $\overline{E}_N^{\rm on}$ (red dashed line), the three mode state entanglement before measurement E_N (green line). The parameters value is $C_1 = 100$.

6 Conclusion

In quantum communication, we often need the maximum entangled state. How to achieve maximum entangled state is an important subject of quantum communication. Entanglement concentration is an important method in preparing the maximum entangled state. In the continuous-variable, for the Gaussian entangled state, only the non Gaussian operation is possible in the entanglement concentration. As a non Gaussian operation-quantum measurement, we use three different measurement operators (perfect measurement, imperfect measurement and on-off measurement) to concentrate a 3-mode Gaussian state. Perfect measurement is strongest in the entanglement concentration, imperfect measurement is second, and the on-off measurement is the weakest. But on the other hand, perfect measurement get the most precise measurement information. In this respect, the more precise measurement information, the larger entanglement concentration. But in the experiment the imperfect measurement is relatively easy to implementation, so making an efficient measuring instrument is very important. In the imperfect measurement, we use perturbation theory to calculate the entanglement of a mixed state, the numerical results and analytical results fit well. This has some reference for us to deal with the entanglement of other density matrices. No matter the perfect measurement, imperfect measurement and on-off measurement, they are strong measurement, the entanglement concentration based on weak measurement will be considered in the next work.

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