



# Dynamical perturbations and critical phenomena in Gauss–Bonnet AdS black holes

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## ABSTRACT

We investigate the perturbations of charged scalar field in 5-dimensional Gauss–Bonnet AdS black hole backgrounds. From the perturbation behaviors we obtain the objective picture on how the high curvature influences the spacetime perturbation and the condensation of the scalar hair. The high curvature effects can also be read from the linear response function such as the susceptibility and the correlation length, when the system approaches the critical point. We find that the Gauss–Bonnet term does not affect the critical exponents of the system and they still take the mean-field values.

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## 1. Introduction

The holographic model of superconductors, which is constructed by a gravitational theory of a Maxwell field coupled to a charged complex scalar field via anti-de Sitter/conformal field theory (AdS/CFT) correspondence [1–3], has been investigated extensively in recent years (for reviews, see Refs. [4–6]). According to the AdS/CFT dictionary, the emergence of the scalar hair in the bulk AdS black hole corresponds to the formation of a charged condensation in the boundary dual CFTs. This brings a remarkable connection between the condensed matter and the gravitational physics which attracts considerable interest for its potential applications to the condensed matter physics [7–31]. At the moment when the condensation occurs in the boundary CFT and in the gravitational counterpart a non-trivial hair for the black hole is triggered, there appears a phase transition [32,33]. The phenomenological signature of this phase transition was recently disclosed in the perturbation around such AdS black holes [28,29].

Motivated by the application of the Mermin–Wagner theorem to the holographic superconductors there were studies of the effects of the curvature corrections on the (3 + 1)-dimensional superconductor [25–27,30,31]. It was found that higher curvature corrections make condensation harder. In addition, the large Gauss–Bonnet factor gives the correction to the disclosed universal value for the conductivity  $\omega_g/T_c \approx 8$  [6] in the probe limit [25–27]. Furthermore Brihaye et al. observed that the decrease of the critical temperature at which condensation sets in is stronger as the Gauss–Bonnet coupling increases, which happens even beyond the probe approximation [31]. In order to get more objective picture on the influence given by the high curvature on the condensation, in this work we are going to study the perturbation in the 5-dimensional Gauss–Bonnet AdS black hole backgrounds. We will concentrate on the bulk high temperature AdS black holes and pay more attention on how the Gauss–Bonnet term influences the perturbation in the bulk background spacetime when the temperature of the black hole drops. Further we are going to study the critical phenomenon once the AdS black hole approaches marginally stable mode and the charged scalar field starts to condensate. We will examine how the Gauss–Bonnet term affects the critical behavior. Recently, Maeda et al. [24] studied most of the static critical exponents of holographic superconductors for a Reissner–Nordström (RN) AdS black hole with planar horizon and found that they take the standard mean-field values. We will generalize their work to the 5-dimensional Gauss–Bonnet AdS black hole configurations and examine the effect of Gauss–Bonnet term on the critical behavior. We will focus our attention on the high-temperature phase for simplicity and study the linear perturbations of the bulk equations of motion in the probe approximation.

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The outline of this work is as follows. In Section 2, we deal with the perturbation equations of the charged scalar field in the 5-dimensional Gauss–Bonnet AdS black hole spacetime. In Section 3, we investigate the perturbation in the bulk and examine the critical phenomenon of the holographic superconductors. We will conclude in the last section with our main results.

## 2. Perturbation equations in the Gauss–Bonnet AdS black hole

Let us begin with the  $D = (p + 2)$ -dimensional charged Gauss–Bonnet black hole described by the metric [34–38]

$$ds^2 = -H(r) dt^2 + H^{-1}(r) dr^2 + \frac{r^2}{l^2} dx_p^2, \quad (1)$$

where the  $U(1)$  gauge field reads

$$A_t = \frac{Q}{4\pi(D-3)} (r_H^{D-3} - r^{D-3}). \quad (2)$$

Here

$$H(r) = \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - \frac{4\alpha}{l^2} \left( 1 - \frac{ml^2}{r^{D-1}} + \frac{Q_0^2 l^2}{r^{2D-4}} \right)} \right], \quad (3)$$

where  $\alpha$  is the Gauss–Bonnet coefficient,  $r_H$  is the horizon radius and  $l$  corresponds to the AdS radius. The gravitational mass  $M$  and the charge  $Q$  are expressed as

$$M = \frac{(D-2)mV_p}{16\pi G_D}, \quad Q^2 = \frac{2\pi(D-2)(D-3)Q_0^2}{G_D},$$

where  $V_p$  is the volume of the  $p$ -dimensional Euclidean space  $dx_p^2$  and  $G_D$  is the  $D$ -dimensional Newton constant. Note that in the asymptotic region ( $r \rightarrow \infty$ ), we have

$$H(r) = \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 - \frac{4\alpha}{l^2}} \right).$$

We can define the effective AdS radius

$$l_{\text{eff}}^2 = \frac{2\alpha}{1 - \sqrt{1 - \frac{4\alpha}{l^2}}} \sim \begin{cases} l^2, & \alpha \rightarrow 0, \\ \frac{l^2}{2}, & \alpha \rightarrow \frac{l^2}{4}. \end{cases} \quad (4)$$

The upper bound of the Gauss–Bonnet coefficient  $\alpha \leq l^2/4$  is known as the Chern–Simons limit. Besides there also exists a lower bound  $\alpha \geq -\frac{(3D-1)(D-3)}{4(D+1)^2}$  by considering the causality of dual field theory on the boundary [38–40]. Using a coordinate transformation, the metric (1) and the potential (2) can be rewritten as

$$ds^2 = \frac{l^2}{u^2} \left[ -\frac{r_H^2(1+c)^{\frac{2}{1-p}} f(u)}{J^2(u)} dt^2 + \frac{J^{\frac{2}{p-1}}(u)}{f(u)} du^2 + r_H^2(1+c)^{\frac{2}{1-p}} J^{\frac{2}{p-1}}(u) dx_p^2 \right], \quad (5)$$

$$A_t = \mu \left[ 1 - \frac{1+c}{J(u)} u^{p-1} \right], \quad (6)$$

with

$$Q_0 = c^{\frac{1}{2}} r_H^{p-2}, \quad u = \frac{r_H}{r(1+c-cr_H/r)^{p-1}}, \quad (7)$$

$$J(u) = 1 + cu^{p-1}, \quad g(u) = J^{\frac{2p}{p-1}}(u) - (1+c)^{\frac{2p}{p-1}} u^{p+1}, \quad f(u) = \frac{J(u)}{2\alpha} \left[ J(u) - \sqrt{J^2(u) - 4\alpha g(u)} \right],$$

where  $\mu$  is the chemical potential and  $c$  is related to the parameter  $Q_0$ . Obviously  $u = 0$  is the AdS boundary and  $u = 1$  is the location of the horizon. The metric (5) will be reduced to the  $D$ -dimensional RN–AdS black hole if we take the limit  $\alpha \rightarrow 0$ . On the other hand, it becomes the neutral Gauss–Bonnet AdS black hole [35] if  $Q_0 = 0$ . There are four parameters  $\alpha$ ,  $c$ ,  $r_H$  and  $\mu$  which parameterize the background (5). In fact, not all of them are independent, they are related by

$$\rho = l^{p-2} r_H^{p-1} \mu, \quad (8)$$

where  $\rho$  is the charge density. The temperature  $T$  and the chemical potential  $\mu$  of the black hole are given by

$$T = \frac{[p+1-(p-1)c]r_H}{4\pi}, \quad (9)$$

$$\mu = \sqrt{\frac{2p}{p-1}} c^{1/2} l r_H. \quad (10)$$

In order to investigate the perturbation in the bulk spacetime and the critical phenomena when the black hole approaches marginally stable from the high temperature phase, we will consider the minimally coupled, charged scalar perturbation,  $\psi_{\varpi,q}(u)e^{-i(\omega t+kx)}$ , with mass  $m$ , which obeys the wave equation

$$\left[ u^p \frac{d}{du} \left( \frac{f}{u^p} \frac{d}{du} \right) + \frac{J^{2p/(p-1)}(\varpi + \aleph)^2}{f(1+c)^{2/(1-p)}} - \frac{q^2}{(1+c)^{2/(1-p)}} - \frac{l^2 m^2 J^{2/(p-1)}}{u^2} \right] \psi_{\varpi,q}(u) = 0, \quad (11)$$

where we have defined four dimensionless quantities:

$$\varpi := \omega/r_H, \quad q := |k|/r_H, \quad \aleph := \frac{eA_t}{r_H} = \sigma \left( \frac{1}{1+c} - \frac{u^{p-1}}{J} \right), \quad \sigma := (1+c)^{p/(p-1)} \frac{e\mu}{r_H}. \quad (12)$$

Using Eq. (10), we can rewrite  $\sigma$  as

$$\sigma = \sqrt{\frac{2p}{p-1}} (le) c^{1/2} (1+c)^{p/(p-1)}. \quad (13)$$

Taking  $e \rightarrow \infty$  and keeping  $\sigma$  fixed, we can employ the probe approximation following [24]. Eq. (13) tells that in the limit  $c \propto (le)^{-2} \rightarrow 0$  the background (5) becomes a neutral Gauss–Bonnet AdS black hole in  $D$  dimensions [35].

Near the AdS boundary  $u \sim 0$ , Eq. (11) becomes

$$\left[ u^p \partial_u \left( \frac{1 - \sqrt{1 - 4\alpha}}{2\alpha} u^{-p} \partial_u \right) - l^2 m^2 u^{-2} \right] \psi_{\varpi,q}(u) = 0, \quad (14)$$

and  $\psi_{\varpi,q}(u)$  has a fall-off behavior as

$$\psi_{\varpi,q}(u) \sim \psi_{\varpi,q}^- u^{\lambda_-} + \psi_{\varpi,q}^+ u^{\lambda_+}, \quad (15)$$

where

$$\lambda_{\pm} := \frac{1}{2} \left[ p + 1 \pm \sqrt{(p+1)^2 + 4m^2 l_{\text{eff}}^2 / l^2} \right]. \quad (16)$$

In the AdS/CFT duality, the order parameter expectation value  $\langle \mathcal{O}_{\varpi,q} \rangle$  corresponds to  $\psi_{\varpi,q}^+$  while the source term is  $\psi_{\varpi,q}^-$ , so the response function can be defined by [24]

$$\chi_{\varpi,q} := \left. \frac{\delta \langle \mathcal{O}_{\varpi,q} \rangle}{\delta \psi_{\varpi,q}^-} \right|_{\psi_{\varpi,q}^- \rightarrow 0} \propto \frac{\psi_{\varpi,q}^+}{\psi_{\varpi,q}^-}. \quad (17)$$

We aim to investigate the perturbation and the critical phenomenon, so that we have to solve the equation of motion of the scalar field, Eq. (11), based on the boundary conditions at the horizon and at the boundary. After obtaining the coefficients  $\psi_{\varpi,q}^{\pm}$ , we can study the behavior of the response function.

Near the horizon  $u \sim 1$ , Eq. (11) becomes

$$f \frac{\partial}{\partial u} \left[ f \frac{\partial}{\partial u} \psi_{\varpi,q}(u) \right] + (1+c) \frac{2p+2}{p-1} \varpi^2 \psi_{\varpi,q}(u) = 0, \quad (18)$$

and its solution is given by  $\psi_{\varpi,q}(u) \sim (1-u)^{\pm i \frac{\varpi}{4\pi T}}$ . We impose the “incoming wave” boundary condition at the horizon, so  $\psi_{\varpi,q}(u) \sim (1-u)^{-i \frac{\varpi}{4\pi T}}$ . Introducing a new variable  $\varphi$  as  $\psi_{\varpi,q}(u) = \aleph(u) \varphi_{\varpi,q}(u)$  and choosing  $\aleph(u) = \exp[i(1+c)^{1/(p-1)} \int_0^u du \frac{J^{p/(p-1)}}{f} (\varpi + \aleph)]$ , we can express the boundary condition at the horizon as  $\varphi_{\varpi,q}(u=1) = \text{const}$ , and Eq. (11) becomes

$$\left\{ \frac{d^2}{du^2} + \left[ \left( \frac{d}{du} \ln \frac{f}{u^p} \right) + 2i \frac{(\varpi + \aleph) J^{p/(p-1)}}{f(1+c)^{1/(1-p)}} \right] \frac{d}{du} - \frac{q^2(1+c)^{2/(p-1)} u^2 + l^2 m^2 J^{2/(p-1)}}{u^2 f} \right. \\ \left. + i \frac{u^p}{f} \frac{d}{du} \left[ \frac{J^{p/(p-1)}(\varpi + \aleph)}{u^p(1+c)^{1/(1-p)}} \right] \right\} \varphi_{\varpi,q}(u) = 0. \quad (19)$$

Near the AdS boundary  $u \sim 0$ ,  $\varphi_{\varpi,q}$  behaves as

$$\varphi_{\varpi,q}(u) \sim \varphi_{\varpi,q}^- u^{\lambda_-} + \varphi_{\varpi,q}^+ u^{\lambda_+}. \quad (20)$$

The boundary conditions at the horizon are now given by

$$\varphi_{\varpi,q}|_{u=1} = 1, \\ \left. \frac{\varphi'_{\varpi,q}}{\varphi_{\varpi,q}} \right|_{u=1} = \frac{q^2(1+c)^{2/(p-1)} + l^2 m^2 J^{2/(p-1)} u^{-2} - i u^p \frac{d}{du} \left[ \frac{J^{p/(p-1)}(\varpi + \aleph)}{u^p(1+c)^{1/(1-p)}} \right]}{\partial_u f - p u^{-1} f + 2i(\varpi + \aleph)(1+c)^{1/(p-1)} J^{p/(p-1)}} \Big|_{u=1}. \quad (21)$$

Eq. (19) is a linear equation and  $\varphi_{\varpi,q}(u)$  must be regular at the horizon. Since we do not concentrate on the amplitude of  $\varphi_{\varpi,q}(u)$ , we can set  $\varphi_{\varpi,q}(u=1) = 1$ .

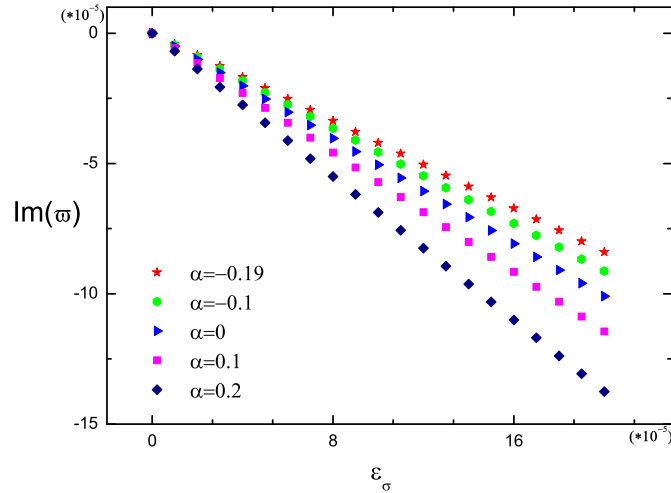


Fig. 1. (Color online.) The trajectories of the imaginary parts of the lowest quasinormal frequency for different values of  $\alpha$ . While the temperature drops to the critical point, the system approaches the marginally stable mode.

### 3. Numerical results

In this section, we will numerically solve Eq. (19) under the boundary conditions (21) in the probe approximation. We will first examine the behavior of the charged scalar field perturbation, which can present us an objective picture on how the black hole approaches the marginally stable mode when the temperature drops. In addition we will determine the coefficients  $\varphi_{\varpi, q}^{\pm}$  from the asymptotic behavior (20), and then study the response function  $\chi_{\varpi, q} \propto \varphi_{\varpi, q}^{+} / \varphi_{\varpi, q}^{-}$ . This can tell us the critical behavior of some physical quantities when the system approaches the critical point. Without loss of generality, hereafter we will set  $e = 1$  and AdS radius  $l = 1$  in our calculation.

To disclose the high curvature influence on the perturbation and the critical phenomenon, we will concentrate on the 5-dimensional ( $p = 3$ ) Gauss–Bonnet AdS black holes with the scalar mass  $m^2 = -3$  and the Gauss–Bonnet coupling parameter within the range  $-\frac{7}{36} \leq \alpha \leq \frac{1}{4}$ .

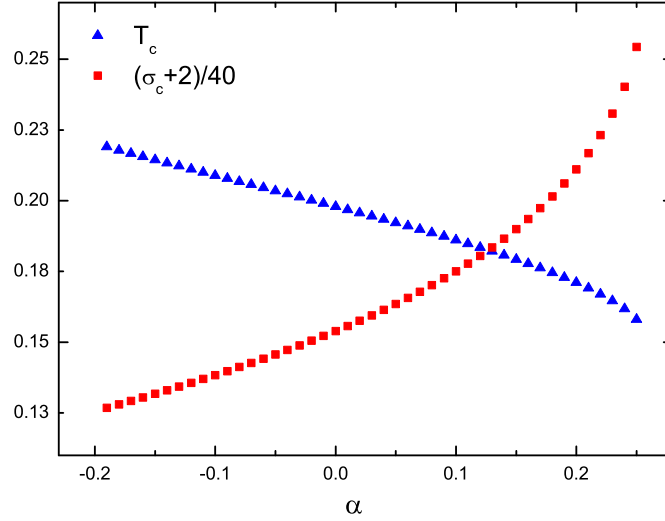
At first we report the influence of the Gauss–Bonnet term on the scalar perturbation behavior. We concentrate on the lowest quasinormal frequency which gives the relaxation time [41]. We can obtain the quasinormal frequencies by solving Eq. (19) based on the boundary conditions (21) at the horizon and  $\varphi_{\varpi, q=0}^{-} = 0$  at the AdS boundary. The objective influence of the Gauss–Bonnet term on the imaginary parts of the lowest quasinormal frequency of the perturbation is shown in Fig. 1. We see that all the imaginary parts of the quasinormal frequencies are negative, which shows that the black hole spacetime is stable. For the larger Gauss–Bonnet coefficient, the imaginary part of the lowest quasinormal frequency has larger deviation from zero. This implies that the higher curvature correction can ensure the system to be more stable and can slow down the process to make the high temperature black hole phase become marginally stable. This objective picture of studying the quasinormal modes is consistent with the observation in [25,26] that the higher curvature correction can hinder the condensation of the scalar hair on the boundary. With the decrease of the black hole temperature, we observe that the lowest quasinormal frequency approaches the origin and vanishes when the temperature of the system reaches the critical value, which indicates that the system approaches marginally stable. The lowest quasinormal frequency approaches the origin with equal spacing and we fit the results for different Gauss–Bonnet coefficient in polynomials as below

$$\begin{aligned}
 \alpha = -0.19, & \quad \varpi_{QNM} \sim (2.57 - 0.69i) \times 10^{-12} + (1.74 - 0.42i)\varepsilon_{\sigma} - (0.27 + 0.49i)\varepsilon_{\sigma}^2, \\
 \alpha = -0.1, & \quad \varpi_{QNM} \sim (1.21 - 1.50i) \times 10^{-13} + (1.96 - 0.46i)\varepsilon_{\sigma} - (0.37 + 0.60i)\varepsilon_{\sigma}^2, \\
 \alpha = 0, & \quad \varpi_{QNM} \sim (-2.92 - 0.81i) \times 10^{-13} + (2.23 - 0.50i)\varepsilon_{\sigma} - (0.52 + 0.76i)\varepsilon_{\sigma}^2, \\
 \alpha = 0.1, & \quad \varpi_{QNM} \sim (-1.05 + 0.23i) \times 10^{-9} + (2.56 - 0.57i)\varepsilon_{\sigma} - (0.72 + 1.00i)\varepsilon_{\sigma}^2, \\
 \alpha = 0.2, & \quad \varpi_{QNM} \sim (1.01 - 3.40i) \times 10^{-13} + (2.97 - 0.69i)\varepsilon_{\sigma} - (1.28 + 1.37i)\varepsilon_{\sigma}^2,
 \end{aligned} \tag{22}$$

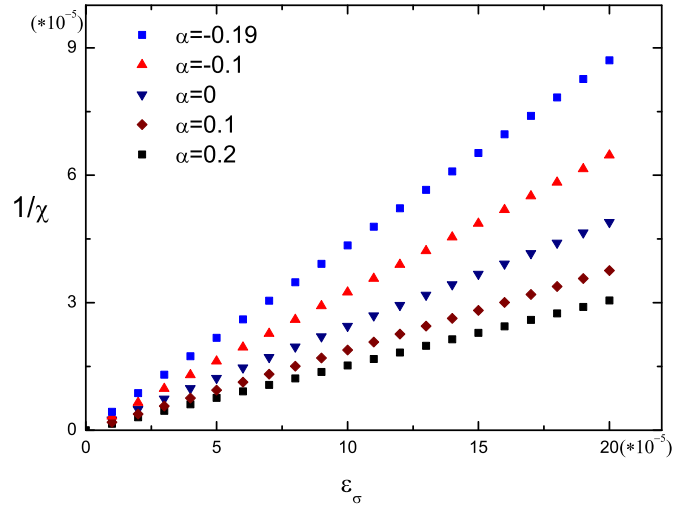
where  $\varepsilon_{\sigma} = 1 - \sigma / \sigma_c$ .

Let’s turn to discussing the thermodynamic susceptibility, which can be obtained numerically by solving Eq. (19) with  $\varpi = q = 0$  under the boundary condition (21) at the horizon and  $\varphi_{\varpi=0, q=0}^{-} = 0$  at the AdS boundary using the shooting method. The dimensionless parameter  $\sigma$  determines the phase structure and its critical value  $\sigma_c$  can be calculated numerically. In Fig. 2 we exhibit the critical point  $\sigma_c$  and the critical temperature  $T_c$  for the background system with different Gauss–Bonnet coefficient. In the 5-dimensional spacetime  $T_c \propto \rho^{1/3}$ . The behavior of  $T_c$  is consistent with the result obtained from the analysis of the condensation in [25,26], which decreases with the increase of the Gauss–Bonnet coefficient. When the Gauss–Bonnet term disappears, our result goes back to that got in [24] for the 5-dimensional RN–AdS background.

To examine the critical behavior of the thermodynamical susceptibility  $\chi$ , we deviate  $\sigma$  away from the critical value  $\sigma_c$  and denote the deviation by  $\varepsilon_{\sigma}$ . After examining  $\varphi_{\varpi=0, q=0}^{\pm}$  as the function of  $\sigma$  near the critical point for different Gauss–Bonnet coefficient, we find as expected that the critical behavior  $\varphi_{\varpi=0, q=0}^{-}$  vanishes while  $\varphi_{\varpi=0, q=0}^{+}$  approaches to a constant when  $\sigma = \sigma_c$ . The Gauss–Bonnet



**Fig. 2.** The values of  $(\sigma_c + 2)/40$  (black line in the web version) and the critical temperature  $T_c$  (blue line in the web version) as a function of the Gauss-Bonnet coefficient  $\alpha$ . It is shown that the critical temperature decreases with the increase of the Gauss-Bonnet coefficient.



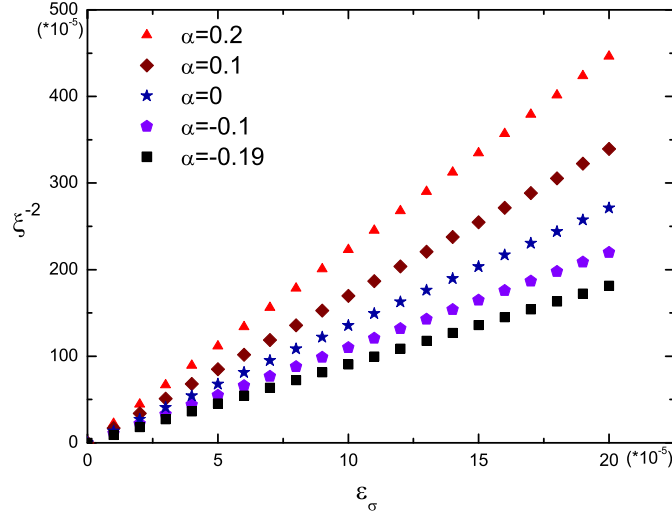
**Fig. 3.** (Color online.) The thermodynamic susceptibility  $\chi$  as a function of  $\varepsilon_\sigma$  for different  $\alpha$ . Plotted are  $1/\chi$  for the deviation  $\varepsilon_\sigma = 10^{-5}n$ ,  $n = 1, 2, \dots, 20$ .

term affects the tendency to the critical behavior which can be observed from the thermodynamical susceptibility for the stationary homogeneous source  $\chi = \chi_{\varpi=0, q=0} \propto \frac{\varphi_{\varpi=0, q=0}^+}{\varphi_{\varpi=0, q=0}^-}$  as shown in Fig. 3. The results are fitted by polynomials as below

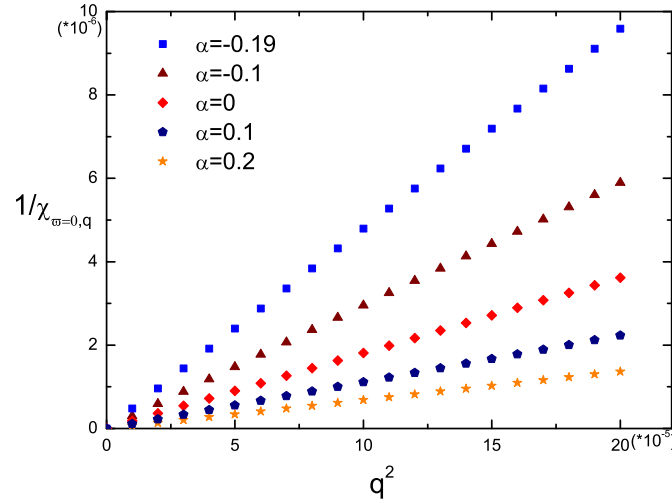
$$\begin{aligned}
 \alpha = -0.19, & \quad 1/\chi \sim 1.04 \times 10^{-9} + 0.44\varepsilon_\sigma - 0.59\varepsilon_\sigma^2, \\
 \alpha = -0.1, & \quad 1/\chi \sim 3.44 \times 10^{-9} + 0.33\varepsilon_\sigma - 1.19\varepsilon_\sigma^2, \\
 \alpha = 0, & \quad 1/\chi \sim 1.74 \times 10^{-8} + 0.25\varepsilon_\sigma - 3.36\varepsilon_\sigma^2, \\
 \alpha = 0.1, & \quad 1/\chi \sim 3.20 \times 10^{-10} + 0.19\varepsilon_\sigma - 7.30\varepsilon_\sigma^2, \\
 \alpha = 0.2, & \quad 1/\chi \sim 6.20 \times 10^{-11} + 0.15\varepsilon_\sigma + 0.75\varepsilon_\sigma^2.
 \end{aligned} \tag{23}$$

At the critical point  $\sigma_c$ ,  $\chi$  diverges as  $\chi \propto 1/\varepsilon_\sigma$  regardless of the values of the Gauss-Bonnet coefficient. Defining  $\chi \propto |\varepsilon_\sigma|^{-\gamma}$ , we find the critical exponent of the thermodynamic susceptibility  $\gamma = 1$  at the critical point. Although the Gauss-Bonnet term cannot modify the critical exponent of the thermodynamic susceptibility, it does influence the tendency of the thermodynamic susceptibility when the critical point is approached. In the vicinity of the critical point, we see that higher curvature correction has bigger thermodynamical susceptibility.

Now we shift our gear to discuss the correlation length and static susceptibility. The critical behavior of the system near the critical point is determined by the large-scale fluctuations. The correlation length is the scale parameter that exists in the system near the phase transition point. It increases while the temperature approaches its critical value and becomes infinite at the moment of the phase transition. Now we check the influence imposed by the Gauss-Bonnet term on the correlation length  $\xi$ , which is defined by  $\xi^2 := -q^{-2}$  [24]. We consider a perturbation with  $\varpi = 0$  for different  $\alpha$ , and solve Eq. (19) with  $\varpi = 0$  under boundary conditions: Eq. (21) at the



**Fig. 4.** (Color online.) The correlation length  $\xi$  as a function of  $\varepsilon_\sigma$  for different  $\alpha$ . Plotted are  $\xi^{-2}$  for the deviation  $\varepsilon_\sigma = 10^{-5}n$ .



**Fig. 5.** (Color online.) The static susceptibility at the critical point  $\chi_{\varpi=0,q}|_{T_c}$  as a function of  $q^2$  for various  $\alpha$ . Plotted are  $1/\chi_{\varpi=0,q}|_{T_c}$  for  $q^2 = 10^{-5}n$ .

horizon and  $\varphi_{\varpi=0,q}^- = 0$  at the AdS boundary. Fig. 4 shows  $\xi^2$  with the interval  $\Delta\varepsilon_\sigma = 10^{-5}$  towards the critical value  $\sigma_c$ . The results can be fitted by polynomials as

$$\begin{aligned}
 \alpha = -0.19, & \quad \xi^{-2} \sim 1.40 \times 10^{-11} + 9.07\varepsilon_\sigma - 6.90\varepsilon_\sigma^2, \\
 \alpha = -0.1, & \quad \xi^{-2} \sim 8.36 \times 10^{-12} + 10.99\varepsilon_\sigma - 8.69\varepsilon_\sigma^2, \\
 \alpha = 0, & \quad \xi^{-2} \sim 1.53 \times 10^{-12} + 13.55\varepsilon_\sigma - 11.16\varepsilon_\sigma^2, \\
 \alpha = 0.1, & \quad \xi^{-2} \sim 2.05 \times 10^{-12} + 16.97\varepsilon_\sigma - 14.50\varepsilon_\sigma^2, \\
 \alpha = 0.2, & \quad \xi^{-2} \sim 2.79 \times 10^{-12} + 22.32\varepsilon_\sigma - 19.63\varepsilon_\sigma^2.
 \end{aligned} \tag{24}$$

This shows that the correlation length  $\xi$  depends on the Gauss-Bonnet coefficient  $\alpha$ . For the smaller  $\alpha$ , the correlation length is bigger for the same deviation from the critical point of the system, which means that it is easier for the system to approach the phase transition point when the Gauss-Bonnet coefficient is smaller. However at the critical point  $\xi^{-2} \propto \varepsilon_\sigma$ , i.e.,  $\xi \propto \varepsilon_\sigma^{-1/2}$  is always true for all chosen Gauss-Bonnet coefficients  $\alpha$ , which shows that the critical exponent  $\nu = 1/2$  is independent of the Gauss-Bonnet term.

The static critical exponent  $\eta$  is determined by the static susceptibility at the critical point  $\chi_{\varpi=0,q}|_{T_c} \propto q^{\eta-2}$  [24] and can be obtained from its  $q$ -dependence. Solving Eq. (19) with  $\varpi = 0$  under the boundary condition (21) at the horizon, we can get  $\chi_{\varpi=0,q}|_{T_c} \propto \frac{\varphi_{\varpi=0,q}^+}{\varphi_{\varpi=0,q}^-}$  from the behavior at the AdS boundary. Fig. 5 shows  $\chi_{\varpi=0,q}|_{T_c}$  as a function of  $q$  and the fitting results are listed below

$$\begin{aligned}
 \alpha = -0.19, & \quad 1/\chi_{\varpi=0,q}|_{T_c} \sim -1.41 \times 10^{-11} + 0.048q^2 - 0.27q^4, \\
 \alpha = -0.1, & \quad 1/\chi_{\varpi=0,q}|_{T_c} \sim -2.38 \times 10^{-10} + 0.030q^2 - 0.65q^4,
 \end{aligned}$$

$$\begin{aligned}
\alpha = 0, & \quad 1/\chi_{\varpi=0,q}|_{T_c} \sim 1.92 \times 10^{-14} + 0.018q^2 + 0.016q^4, \\
\alpha = 0.1, & \quad 1/\chi_{\varpi=0,q}|_{T_c} \sim 2.66 \times 10^{-14} + 0.011q^2 + 0.027q^4, \\
\alpha = 0.2, & \quad 1/\chi_{\varpi=0,q}|_{T_c} \sim 8.96 \times 10^{-14} + 0.0068q^2 - 0.0010q^4.
\end{aligned} \tag{25}$$

We see that the Gauss–Bonnet term affects the slope of the inverse static susceptibility. From the fitting result we obtain  $1/\chi_{\varpi=0,q}|_{T_c} \propto q^{-2}$ , which suggests that the exponent  $\eta = 0$  within numerical errors for various  $\alpha$ .

#### 4. Conclusions and discussions

We investigated the perturbations of charged scalar field in a 5-dimensional Gauss–Bonnet AdS black hole background and paid attention to the effect of the Gauss–Bonnet term on the critical behavior of the system. From the perturbation behavior we obtained the objective picture on how the high curvature influences the spacetime perturbation and the formation of the scalar hair. Our results from the dynamical perturbation support that observed in the study of the condensation phenomena [25,26]. The high curvature will slow down the process for the system with high temperature to approach the marginally stable state and hinder the condensation of the scalar hair. These effects can also be read from the susceptibility and the correlation length in the process when the system approaches the marginally stable moment.

We also calculated the critical exponents for holographic superconductors when the critical point of the system is approached from the high temperature phase. We observed that although the Gauss–Bonnet term affects the processes of the systems to approach the critical moments, they do not change the static critical exponents, namely the static critical exponents still take the mean-field values. This confirmed the conjecture that the critical exponents are determined by the matter fields in the system and are independent of the gravity sector of the system [24]. This is mainly due to the fact that the gravity sector just simply provides a background in the high temperature phase analysis or in the probe approximation, while the matter fields undergo a second-order phase transition (from zero to nonzero condensation) [24]. Note that without the charged scalar field, the system does not have any critical phenomenon. This shows the role of the charged scalar field in the phase transition.

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