PHYSICAL REVIEW D **80,** 041501(R) (2009)

Dynamical scalar degree of freedom in Hořava-Lifshitz gravity

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(Received 13 May 2009; published 17 August 2009)

We investigate the linear cosmological perturbations of Hořava-Lifshitz gravity in a Friedmann-Robertson-Walker universe without any matter. Our results show that a new gauge-invariant dynamical scalar mode emerges, due to the gauge transformation under the "foliation-preserving" diffeomorphism and "projectability condition," and it can produce a scale invariant power spectrum. In the infrared regime with $\lambda = 1$, the dynamical scalar degree of freedom turns to be a nondynamical one at the linear order level.

DOI: 10.1103/PhysRevD.80.041501 PACS numbers: 04.60.−m, 98.80.Bp, 98.80.Cq, 98.80.Qc

Recently, a power-counting renormalizable ultraviolet (UV) complete quantum gravity theory was proposed by Hořava [1–3]. This theory is characterized by the anisotropic scaling between time and space, so the complete diffeomorphism invariance of general relativity (GR) is lost, instead the Hořava-Lifshitz (HL) gravity is invariant under the so-called "foliation-preserving" diffeomorphism. Since this theory was proposed, a great deal of efforts have been made, including studies of cosmology [4–11] and black hole physics [12–14], among others [15–18]. Because of the differences of diffeomorphism group between HL and GR, we expect to see some new dynamical degrees of freedom of gravitational fields in HL gravity. Indeed, a new dynamical scalar degree of freedom of gravitational fields is firstly discussed by Hořava [1] in Minkowski spacetime. The motivation of this paper is to investigate the dynamical behavior of this scalar mode in a cosmological background.

Let us begin with a brief review about the HL gravity [3]. In order to construct a UV renormalizable quantum gravity, one possible way is to introduce high order spatial derivative operators, which make the graviton propagator fall off sufficiently rapidly at large momenta. On the other hand, in order for the theory to be unitary, the Lagrangian can only be quadratic in first time derivatives of the spatial metric. As a consequence, the theory has the anisotropic scaling between space and time. For instance, in $3 + 1$ dimensions the coordinates $(t, \mathbf{x})$ scale as

$$t \to \ell^z t, \qquad \mathbf{x} \to \ell \mathbf{x}, \tag{1}$$

where z is called dynamical critical exponent. In terms of the Arnowitt-Deser-Misner formalism the metric can be written as

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{2}$$

where the spatial metric, lapse function, and shift vector scale as

$$g_{ij} \to g_{ij}, \qquad N \to N, \qquad N_i \to \ell^{z-1} N_i. \tag{3}$$

The action of the nonrelativistic renormalizable gravitational theory proposed by Hořava contains two parts. The part of kinetic term is

$$S_K = \frac{2}{\kappa^2} \int dt d^3x \sqrt{g} N (K_{ij}K^{ij} - \lambda K^2), \tag{4}$$

where the extrinsic curvature reads

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i). \tag{5}$$

The part of the potential term in the so-called "detailed-balance condition" can be written down as

$$\begin{aligned} S_V = \int dt d^3x \sqrt{g} N \bigg[& -\frac{\kappa^2}{2w^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2w^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l{}_k \\ & - \frac{\kappa^2 \mu^2}{8} R_{ij}R^{ij} + \frac{\kappa^2 \mu^2}{8(1 - 3\lambda)} \\ & \times \left(\frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) \bigg], \end{aligned} \tag{6}$$

where λ, κ, μ, w, Λ are coupling constants, ϵ^{ijk} is the antisymmetric tensor defined by $\epsilon_{123} = 1$ and the Cotton tensor reads

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k \left(R^j{}_l - \frac{1}{4} R \delta^j{}_l \right). \tag{7}$$

The Cotton tensor term in the first line of (6), which scales as $z = 3$, is introduced in order for the theory to be power-counting renormalizable in $3 + 1$ dimensions. The other terms in the second and third lines will make the theory undergo a classical flow to $z = 1$ in the infrared (IR) regime, where the coupling constant λ would be expected to flow to $\lambda = 1$. And then GR would be expected to be recovered in the IR regime.

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By combining (4) and (6), the full action $S_1 = S_K + S_V$ can be expressed as

$$S_1 = \int dt d^3x \sqrt{g} N \Big[\alpha_1 (K_{ij}K^{ij} - \lambda K^2) + \beta_1 C_{ij}C^{ij} + \gamma_1 \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l{}_k + \zeta_1 R_{ij}R^{ij} + \eta_1 R^2 + \xi_1 R + \sigma_1 \Big], \tag{8}$$

with coupling constants

$$\alpha_1 = \frac{2}{\kappa^2}, \qquad \beta_1 = -\frac{\kappa^2}{2w^4}, \qquad \gamma_1 = \frac{\kappa^2\mu}{2w^2},$$
$$\zeta_1 = -\frac{\kappa^2\mu^2}{8}, \qquad \eta_1 = \frac{\kappa^2\mu^2}{8(1-3\lambda)}\frac{1-4\lambda}{4}, \tag{9}$$
$$\xi_1 = \frac{\kappa^2\mu^2}{8(1-3\lambda)}\Lambda, \qquad \sigma_1 = \frac{\kappa^2\mu^2}{8(1-3\lambda)}(-3\Lambda^2),$$

where μ and w^2 are real constants, and have their origin as the Newton constant and Chern-Simons coupling of Euclideanized three-dimensional topologically massive gravity [19]. In order to have a real speed of light, Λ must have to be negative for $\lambda > 1/3$ [3], which leads to a negative effective cosmological constant σ_1. This is not consistent with current cosmological observation. To have a positive cosmological constant, one may make an analytic continuation of those parameters [20]

$$\mu \to i\mu, \qquad w^2 \to -iw^2, \tag{10}$$

then the action (8) changes to

$$S_2 = \int dt d^3x \sqrt{g} N \Big[\alpha_2 (K_{ij}K^{ij} - \lambda K^2) + \beta_2 C_{ij}C^{ij} + \gamma_2 \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l{}_k + \zeta_2 R_{ij}R^{ij} + \eta_2 R^2 + \xi_2 R + \sigma_2 \Big], \tag{11}$$

with coefficients

$$\alpha_2 = \alpha_1, \qquad \beta_2 = -\beta_1, \qquad \gamma_2 = -\gamma_1,$$
$$\zeta_2 = -\zeta_1, \qquad \eta_2 = -\eta_1, \qquad \xi_2 = -\xi_1, \tag{12}$$
$$\sigma_2 = -\sigma_1.$$

In addition to the gravitational sector (8) or (11), we can also add some matter sectors to HL theory

$$S_M = \int d^3x dt \sqrt{g} N \mathcal{L}_{\rm matter}(N, N_i, g_{ij}). \tag{13}$$

Before deriving the constraint and dynamical equations of the theory, let us stress that, in order for the theory to be tractable, the lapse function N should be a function of time coordinate t only, otherwise it would lead to difficulties in quantization, at least in the absence of extra gauge symmetries. This prescription is known as the "projectability condition" [3,21].

Because of the "projectability condition" on the lapse function $N(t)$, we can only obtain the spatially integrated Hamiltonian constraint by varying the action with respect to $N(t)$

$$0 = \int d^3x \sqrt{g} \Big\{ -\alpha_m (K_{ij}K^{ij} - \lambda K^2) + \beta_m C_{ij}C^{ij} + \gamma_m \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l{}_k + \zeta_m R_{ij}R^{ij} + \eta_m R^2 + \xi_m R + \sigma_m + J_N \Big\}, \tag{14}$$

with $m = 1, 2$ and

$$J_N = \mathcal{L}_{\rm matter} + N \frac{\delta \mathcal{L}_{\rm matter}}{\delta N}. \tag{15}$$

The equation of motion for N_i gives momentum constraint

$$2\alpha_m (\nabla_j K^{ji} - \lambda \nabla^i K) + N \frac{\delta \mathcal{L}_{\rm matter}}{\delta N_i} = 0. \tag{16}$$

The equation of motion for g_{ij} is very lengthy, and one can find the explicit expression in [4].

Now we investigate HL gravity in a flat Friedmann-Robertson-Walker universe

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{17}$$

with the scale factor $a(t)$. For further simplification we turn off the matter sector (13). In that case, the spatially integrated Hamiltonian constraint (14) reads

$$H^2 = \frac{\sigma_m}{3\alpha_m(1-3\lambda)}, \tag{18}$$

where the Hubble parameter $H = \dot{a}/a$, where a dot denotes the derivative with respect to the cosmic time t, while the momentum constraint is trivially satisfied.

The equation of motion for g_{ij} gives

$$2\alpha_m(3\lambda - 1)\Big[\dot{H} + \frac{3}{2}H^2\Big] = -\sigma_m. \tag{19}$$

By virtue of the above equations, one can easily verify that for the action (8) $H^2 < 0$, while for the action (11) $H^2 > 0$. Namely in the latter case, a de Sitter solution exists. Because of cosmological interest, we will consider the latter case and omit the subscript $m = 2$ in the rest of this paper.

Because of the anisotropic scaling of temporal and spatial coordinates, the time coordinate t plays a privileged role in this theory, and the symmetry of HL theory is smaller than the one of GR. Precisely, HL gravity is invariant under a so-called "foliation-preserving" diffeomorphism, where the coordinate transformations have the form

$$t \to \tilde{t} = f(t), \qquad x^i \to \tilde{x}^i = h^i(t, \mathbf{x}), \tag{20}$$

with $f(t)$ is a function of t only. This is a big difference from the case of GR.

In cosmological perturbation theory, the metric perturbations are usually categorized into three distinct types: scalar, vector, and tensor perturbations

$$\delta g_{00} = -2a^2\phi, \qquad \delta g_{0i} = a^2\partial_i B + a^2 Q_i,$$
$$\delta g_{ij} = a^2 h_{ij} - a^2(\partial_i W_j + \partial_j W_i) - 2a^2(\psi\delta_{ij} - \partial_i\partial_j E), \tag{21}$$

where ϕ, ψ, E, B are four scalar modes, Q_i, W_i are two vector modes which satisfy $\partial^i Q_i = \partial^i W_i = 0$, and h_{ij} is the transverse-traceless tensor mode $h_{ij,j} = h_{ii} = 0$. Under an infinitesimal foliation-preserving coordinate transformation

$$t \to \tilde{t} = t + \epsilon^0(t), \qquad x^i \to \tilde{x}^i = x^i + \epsilon^i(t, \mathbf{x}), \tag{22}$$

and decomposing ϵ^i into

$$\epsilon^i = \partial^i\epsilon(t, \mathbf{x}) + \zeta^i(t, \mathbf{x}), \tag{23}$$

with $\partial_i\zeta^i = 0$, we can obtain the transformation rules

$$\phi \to \tilde{\phi} = \phi - \dot{\epsilon}^0, \tag{24}$$

$$\psi \to \tilde{\psi} = \psi + H\epsilon^0, \tag{25}$$

$$B \to \tilde{B} = B - a\left(\frac{\epsilon}{a^2}\right)^{\cdot}, \tag{26}$$

$$E \to \tilde{E} = E - \frac{1}{a^2}\epsilon, \tag{27}$$

for scalar modes,

$$Q_i \to \tilde{Q}_i = Q_i - a\left(\frac{\zeta_i}{a^2}\right)^{\cdot}, \tag{28}$$

$$W_i \to \tilde{W}_i = W_i + \frac{1}{a^2}\zeta_i, \tag{29}$$

for vector modes, and the transformation for tensor modes is the same as that in GR because of the foliation-preserving diffeomorphism. Unlike what happens in GR [22], however, we can see from (25) that we are forbidden to choose the "spatially flat gauge" in HL gravity since the infinitesimal parameter ϵ^0 is the function of t only. Further, due to the "projectability condition," the lapse function can be set globally to unity, i.e., we can gauge ϕ mode by choosing a proper initial time. And the residual coordinate freedom can be gauged by virtue of (27), thus we can completely fix the coordinates.

With help of those transformation rules, one can easily build up the gauge-invariant variables as

$$\Phi = \phi + \left(\frac{\psi}{H}\right)^{\cdot}, \tag{30}$$

$$\Pi \equiv B - a\dot{E}, \tag{31}$$

$$\Psi = \psi + H\int_{t_0}^{t}\phi dt', \tag{32}$$

where only two of them are independent, since we have $\Phi = (\Psi/H)^{\cdot}$ from (30) and (32). In the gauge ($\phi = 0$, $E = 0$), ψ coincides with the gauge-invariant variable Ψ. Thus it is very convenient to take this gauge in discussing cosmological scalar perturbations. In this gauge, the metric becomes

$$ds^2 = -dt^2 + 2a\partial_i B dt dx^i + a^2(1 - 2\psi)\delta_{ij}dx^i dx^j. \tag{33}$$

Substituting (33) into (11) and performing lots of straightforward but very lengthy calculations, we obtain the action of the scalar perturbations

$$S_K = \int dt d^3x \alpha a^3\bigg\{(1 - 3\lambda)\bigg[6H\psi\dot{\psi} + 3\dot{\psi}^2 + \frac{2}{a}\dot{\psi}\partial^2 B + \frac{9}{2}H^2\psi^2\bigg] + \frac{1 - \lambda}{a^2}B\partial^4 B\bigg\}, \tag{34}$$

$$S_V = \int dt d^3x\bigg\{\frac{2(3\zeta + 8\eta)}{a}\psi\partial^4\psi + \frac{3\sigma}{2}a^3\psi^2 - 2\xi a\psi\partial^2\psi\bigg\}, \tag{35}$$

with $\partial^2 \equiv \delta^{ij}\partial_i\partial_j$. Although the action (11) contains terms such as $C_{ij}C^{ij}$ and $R_{il}\nabla_j R^l{}_k$, the highest order of spatial derivatives in the action is ∂^4 for the sake of the antisymmetric tensor ϵ^{ijk} and flat universe. However, if we abandon the "detailed-balanced condition," ∂^6 terms will appear.

After dropping some surface terms and using the background equations (18) and (19), the spatially integrated Hamiltonian constraint becomes

$$\int d^3x 6\alpha(1 - 3\lambda)H\dot{\psi} = 0. \tag{36}$$

The equation of motion for $\partial_i B$ gives the momentum constraint

$$\partial_i\bigg\{(3\lambda - 1)\dot{\psi} + (\lambda - 1)\frac{1}{a}\partial^2 B\bigg\} = 0, \tag{37}$$

and the dynamical equation of motion for the gravitational field ψ is given by

$$6\alpha(1-3\lambda)\left[\ddot{\psi}+3H\dot{\psi}+\frac{1}{3a}(\partial^2\dot{B}+2H\partial^2 B)\right]+\frac{4\xi}{a^2}\partial^2\psi - \frac{4(3\zeta+8\eta)}{a^4}\partial^4\psi=0. \quad (38)$$

This equation is the main result of this paper.

(i) When $\lambda=1/3$, the first line of (38) vanishes, the coefficients in second line diverge and the Hamiltonian constraint (36) is trivially satisfied. So, the evolution of ψ cannot be determined by the classical equation of motion (38). This strange feature indicates that we need to take account of the quantum renormalization group flows at the UV fixed point.

(ii) When $\lambda\neq 1/3$ and 1, substituting the momentum constraint (37) into the dynamical equation (38), we obtain

$$\ddot{\psi}+3H\dot{\psi}+\frac{(1-\lambda)\xi}{\alpha(1-3\lambda)a^2}\partial^2\psi - \frac{(1-\lambda)(3\zeta+8\eta)}{\alpha(1-3\lambda)a^4}\partial^4\psi=0. \quad (39)$$

(iii) When $\lambda=1$, the momentum constraint (37) becomes

$$\partial_i\dot{\psi}=0, \quad (40)$$

with the general solution

$$\psi(t,\mathbf{x})=f(t)+g(\mathbf{x}). \quad (41)$$

Furthermore, we can obtain $f(t)=\text{const}$ by plugging this solution into the spatially integrated Hamiltonian constraint (36). As a result, ψ is independent of time and thus ψ is not a dynamical degree of freedom at least at the linear order level. Of course, it does not exclude ψ is a dynamical one once higher order perturbations are taken into account.

During the derivation of the above dynamical equations, we have not considered the spatially integrated Hamiltonian constraint (36). Now we explain why we can safely ignore this constraint. It turns out to be more convenient to discuss this issue in Fourier space

$$\psi(t,\mathbf{x})=\frac{1}{(2\pi)^3}\int d^3\mathbf{k}\,\psi_k(t)e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (42)$$

then (36) becomes

$$6\alpha(1-3\lambda)\dot{\psi}_{k=0}=0. \quad (43)$$

we can see from (43) that the spatially integrated Hamiltonian constraint only constrains $k=0$ mode, while this mode can be absorbed into the background. As a result, we only need to consider the momentum constraint (37) in HL gravity.

In what follows we will focus on solving the dynamical equation (39) for the case $\lambda\neq 1/3$ and 1. After introducing a new field $\psi\equiv\chi/a$, (39) can be written in Fourier space as

$$\chi_k''(\tau)+\left[-\frac{(1-\lambda)^2c^2H^2}{(3\lambda-1)\Lambda}k^4\tau^2 + \frac{(1-\lambda)c^2}{3\lambda-1}k^2-\frac{2}{\tau^2}\right]\chi_k(\tau)=0, \quad (44)$$

where prime stands for derivative with respect to conformal time ($d\tau=dt/a$) and c for the speed of light $c\equiv\sqrt{\xi/\alpha}=\sqrt{\kappa^4\mu^2\Lambda/16(3\lambda-1)}$. The requirement that the speed of light be real implies that Λ be positive for $\lambda>1/3$. In the rest of this paper we will concentrate on this case.

Comparing the three terms in the square bracket, we can divide our discussions into three cases.

(i) Case 1: when $k^2\tau^2\gg T_1$ ($T_1\equiv\Lambda/H^2$) and $k^2\tau^2\gg T_2$ ($T_2\equiv\sqrt{\Lambda/c^2H^2}$), k^4 term dominates. In this case, (44) reduces to

$$\chi_k''(\tau)-\frac{(1-\lambda)^2c^2H^2}{(3\lambda-1)\Lambda}k^4\tau^2\chi_k(\tau)=0. \quad (45)$$

This equation has a general solution with form

$$\chi_k(\tau)=C_1\text{Parabolic Cylinder D}[-\tfrac{1}{2},\sqrt{2}\omega^{1/4}k\tau] + C_2\text{Parabolic Cylinder D}[-\tfrac{1}{2},i\sqrt{2}\omega^{1/4}k\tau] \quad (46)$$

where C_1, C_2 are two integration constants, $\omega=(1-\lambda)^2c^2H^2/(3\lambda-1)\Lambda$ and Parabolic Cylinder D$[\alpha,x]$ is the parabolic cylinder function. It is easy to check that both terms in (46) are exponentially decaying modes in the region $-\infty<\tau<0$. In order to see the behavior of this solution more clearly, we can use the WKB approximation to solve (45) in the large k limit. Assuming a trial solution in the form of an asymptotic series expansion

$$\chi_k(\tau)=\exp\left\{\frac{1}{\epsilon}\sum_{n=0}^{\infty}\epsilon^n S_n(\tau)\right\}, \quad (47)$$

and plugging it into (45), at the leading order, the solution reads

$$\chi_k(\tau)=\exp\left\{k^2C_3+k^2\frac{\omega^{1/2}}{2}\tau^2\right\}, \quad (48)$$

where C_3 is an integration constant and the second term in the exponent makes the solution decay exponentially when the conformal time τ evolves from $-\infty$ to 0, which is consistent with the asymptotic behavior of the solution (46).

(ii) Case 2: when $k^2\tau^2\gg T_3$ ($T_3\equiv 1/c^2$) and $k^2\tau^2\ll T_1$, k^2 term will dominate over other two terms

$$\chi_k''(\tau) + c_s^2 k^2 \chi_k(\tau) = 0, \tag{49}$$

where the sound speed $c_s^2 \equiv (1-\lambda)c^2/(3\lambda - 1)$ and a real sound speed requires $1/3 < \lambda < 1$. The solution of (49) is a plane wave solution

$$\chi_k(\tau) = \frac{\exp\{-ikc_s\tau\}}{\sqrt{2kc_s}}. \tag{50}$$

(iii) Case 3: when $k^2\tau^2 \ll T_3$ and $k^2\tau^2 \ll T_2$, k^0 term is dominant. In this case, one has

$$\chi_k''(\tau) - \frac{2}{\tau^2}\chi_k(\tau) = 0, \tag{51}$$

with the solution

$$\chi_k(\tau) = C_4\tau^2 + \frac{C_5}{\tau}, \tag{52}$$

where C_4, C_5 are two integration constants. We can see that, the C_4 term is a decaying mode and the C_5 term is a growing one. For simplicity we neglect the decaying mode by simply setting $C_4 = 0$, then we are able to determine the absolute value of C_5 by matching the absolute value of (50) with it when this mode crosses the sound horizon ($k/aH = -k\tau = 1/c_s$)

$$|C_5| = \frac{1}{\sqrt{2k^3c_s^3}}, \tag{53}$$

thus ψ will be frozen on the superhorizon scales

$$|\psi_k| = \frac{H}{\sqrt{2k^3c_s^3}}. \tag{54}$$

Finally, by the definition of scalar power spectrum $\mathcal{P}_\psi(k) \equiv k^3|\psi|^2/2\pi^2$, we obtain a scale invariant spectrum

$$\mathcal{P}_\psi = \frac{H^2}{4\pi^2c_s^3}. \tag{55}$$

We can understand these results in the following way. On the very small scale (case 1), the fluctuations decay exponentially; on the relatively large scale but still deeply inside the sound horizon (case 2), the fluctuations oscillate until they cross the sound horizon; after they cross the sound horizon (case 3), k^0 term will dominate over the other two terms and the fluctuations χ will grow. However, this growth is compensated by the growth of scale factor, consequently ψ is frozen on the superhorizon scale.

In conclusion, in this paper we investigated the linear cosmological perturbations of HL gravity without any matter in a flat Friedmann-Robertson-Walker universe. We studied the gauge transformation under the foliation-preserving diffeomorphism, derived the rigorous equation of motion for scalar perturbations and discussed the dynamical behavior of this mode. Our results showed that this mode evolved dynamically with time when $1/3 < \lambda < 1$ and could produce a scale invariant spectrum. In the regime with $\lambda = 1$, where GR was expected to be recovered, the dynamical scalar mode became a nondynamical one. It is of great interest to investigate whether this conclusion keeps valid beyond the linear perturbations and some matter sectors are included.

We thank Yi Ling, Yan Liu, Ya-Wen Sun, Anzhong Wang, and Jun'ichi Yokoyama for a lot of valuable discussions. This work was supported in part by the Chinese Academy of Sciences with Grant No. KJCX3-SYW-N2 and the NSFC with Grant No. 10821504 and No. 10525060.

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